Hot Gas Ball Curvature Effect on Expansion Rate

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Nomenclature

C_1	constant
c_p	specific heat at constant pressure
k	thermal conductivity
n	$n = 1$ for cylindrical, $n = 2$ for spherical case
q	heat flux
r	radius of gas ball
T	temperature
u	flow velocity
α	thermal diffusivity, $\alpha = k/c_p\,\rho$
β	nondimensional parameter
ρ	density
η	nondimensional radius, $\eta = r/r_0$
θ	nondimensional temperature, $\theta = (T - T_0)/(T_\infty - T_0)$
κ	stretch rate of expanding gas ball, $\kappa = \frac{n\,u}{r}$

Subscripts

c	critical boundary
l	limited boundary
0	outer boundary
∞	inner boundary

1 Introduction

Flame temperature decreases from the hot gas ball to the surrounding gas along radius. If a line source heats gas, a hot gas ball is cylindrical symmetry. If a point source heats gas, where the radius of hot gas ball is under microgravity condition, hot gas ball is spherical symmetry. Hot gas production rate [1], flame propagation rate for premixed mixture, flame spread rate along combustible surface, or heating rate of ignition source increases the radius of hot gas ball. Heat transfer from the hot gas ball to the surroundings occurs to convective and diffusive heat transfers [1]. Heat transfer rate depends radius,expansion rate, and thermal diffusivity. If the heat transfer rate increases with the radius, the expansion terminates at a radius where the heat transfer rate exceeds a limiting value. The conductive and convective heat are the governing mechanisms, a simple model is proposed [1]. Ball geometry is considered for cylindrical and spherical cases.

2 Model of hot gas ball

Assuming the radius of hot gas ball r, the surrounding region is heated by the heat transferred from the hot gas ball. The flow velocity on the hot gas ball surface is the expansion rate u. The governing equation at steady state [2], [3] is (1)

$$u\frac{dT}{dr} - \frac{1}{\rho\, r^n}\frac{k}{c_p}\frac{d(r^n\dfrac{dT}{dr})}{dr} = 0, \tag{1}$$

$n = 1$ for cylindrical, $n = 2$ for spherical case. Using the thermal diffusivity $\alpha = k/c_p\,\rho$, temperature $\theta = (T - T_0)/(T_\infty - T_0)$, radius $\eta = r/r_0$, the equation of heat conservation at steady state is (2),

$$u\frac{d\theta}{d\eta} - \frac{\alpha}{r_0}\frac{1}{\eta^n}\frac{d(\eta^n\dfrac{d\theta}{d\eta})}{d\eta} = 0. \tag{2}$$

The boundary conditions are (3),(4),

$$\eta = 1, \theta = 0, \tag{3}$$

$$\lim_{\eta\to 0}\theta = 1. \tag{4}$$

Arranging (2),

$$(\frac{u\,r_0}{\alpha} - \frac{n}{\eta})\frac{d\theta}{d\eta} - \frac{d^2\theta}{d\eta^2} = 0. \tag{5}$$

The term $d\theta/d\eta$ vanishes where

$$\frac{u\,r_0}{\alpha} - \frac{n}{\eta_c} = 0, \tag{6}$$

the critical radius η_c is

$$\eta_c = \frac{n\,\alpha}{u\,r_0}. \tag{7}$$

Gas ball expands for $\eta > \eta_c$. No steady solution exists for $\eta < \eta_c$. This indicates hot gas ball heated ignition source needs to expand beyond η_c to form a flame kernel. $\eta_c < 1$ is satisfied, a minimum of $-d\theta/d\eta$ is between $\eta = 0$ and $\eta = 1$. The integration from the boundary (3) to $\eta = \eta_c$ no zero condition $(u\,r_0/\alpha - n/\eta)$ term (5) for integration. The modified boundary condition of (4) is

$$\lim_{\eta\to\eta_c}\theta = 1. \tag{8}$$

$\eta_c > 1$ is satisfied, $n\,\alpha > u\,r_0$ and a monotonous decrease curve appears for u or r_0. By integrating (5) for $\eta > 0$,

$$\ln(|\frac{d\theta}{d\eta}|) = \frac{u\,r_0}{\alpha}\eta - n\ln(\eta) + C_0. \tag{9}$$

The non dimensional temperature gradient $d\theta/d\eta$ is

$$\frac{d\theta}{d\eta} = C_1 \exp(\frac{u\, r_0}{\alpha}\eta)\eta^{-n}. \tag{10}$$

With parameter $u\, r_0/\alpha$, a constant C_1 determined with the boundary conditions,(3,4,8).

2 Numerical integration of temperature gradient

By integrating $d\theta/d\eta$ numerically, temperature θ is given for radius η. With β in (11), temperature is integrated for $0 < \eta < 1$.

$$\beta = \frac{u\, r_0}{\alpha} \tag{11}$$

The heat flux q is expressed by (12).

$$q = -k\frac{dT}{dr} = \frac{k(T_\infty - T_0)}{r_0}(-\frac{d\theta}{d\eta}) \tag{12}$$

$$-\frac{d\theta}{d\eta} = \frac{q\, r_0}{k(T_\infty - T_0)} \tag{13}$$

By numerically differentiating temperature θ with η, the heat flux q is determined (13).

3 Results and Discussion

3 Cylindrical case

Figure 1 temperature θ for radius η with parameter β from $1/e^3$ to e^3 and $n = 1$ cylindrical case. Temperature θ decreases monotonically for $\beta < 1$. As β from 1 to e^3, the critical radius η_c from 1 to 0. As β increases to e^3, area expands toward $\eta = 1$ is the surface. Temperature gradient $-d\theta/d\eta$ is

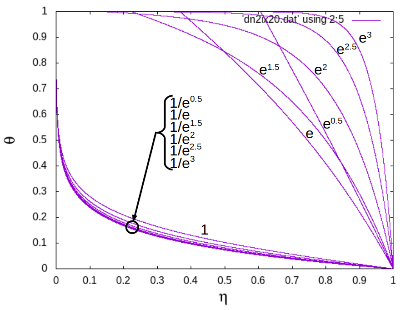

Figure 1: Temperature θ.

in figure 2 cylindrical case. Temperature gradient $-d\theta/d\eta$ decreases monotonically with η for $\beta < 1$. Temperature gradient $-d\theta/d\eta$ of $\beta = e^{0.5}$ is larger that that of $\beta = e$ to small $(1 - \eta_0)$. Temperature gradient $-d\theta/d\eta$ increases rapidly as η increases to 1 for $\beta > e^{1.5}$. As β increases to e^3, temperature gradient area expands toward $\eta = 1$. For a limited heat loss, a limited radius η_l exists. For a limiting heat loss $2q_0$ case, η_l is less than 0.9. For a limiting heat loss $2q_0$ case, hot gas ball larger than η_c expands to η_l due to low heat loss.

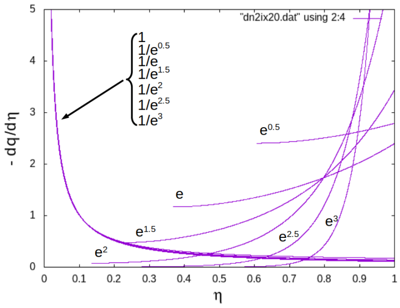

Figure 2: Temperature gradient $-\frac{d\theta}{d\eta}$.

3 Spherical case

Figure 3 temperature θ for radius η with parameter β from $1/e^3$ to e^3 and $n = 2$ for spherical case. Temperature θ decreases monotonically for $\beta < e^{0.5}$. As β from e to e^3, the critical radius η_c from 1 to 0. As β increases to e^3, area expands toward $\eta = 1$ is the surface. Temperature gradient $-d\theta/d\eta$ is in figure 4 for spherical case. Temperature gradient $-d\theta/d\eta$ decreases monotonically with η for $\beta < e^{0.5}$. Temperature gradient $-d\theta/d\eta$ of $\beta = e$ is larger than that of $\beta = e^{1.5}$ to small $(1 - \eta_0)$. Temperature gradient $-d\theta/d\eta$ increases rapidly as η increases for $\beta > e^2$.

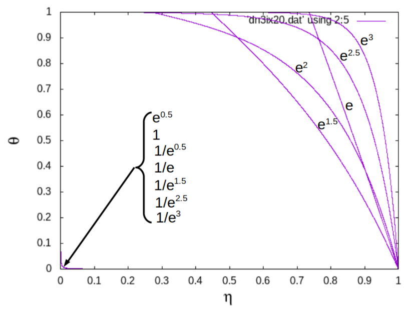

Figure 3: Temperature θ.

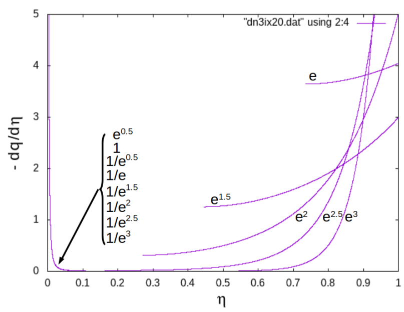

Figure 4: Temperature gradient $-\frac{d\theta}{d\eta}$.

3 Effect of radius on the heat flux

The heat flux depends radius of gas ball. The stretch rate of expanding gas ball κ is

$$\kappa = \frac{n\,u}{r} = \frac{n\,u}{r_0}\frac{1}{\eta}. \tag{14}$$

For a given u, the stretch rate is proportional to $1/\eta$. Temperature gradient $-d\theta/d\eta$ for cylindrical and spherical cases are in figure 5 with $1/\eta$ log. Curves are "dn2ix20.dat" cylindrical and "dn3ix20.dat" spherical cases. Two types are in this figure, one increases with $1/\eta$ the other decreases with $1/\eta$. For small β, line from $1/\eta = 1$ to $1/\eta = 10$ is for both cylindrical and spherical cases. Temperature gradient increases with the stretch rate. Line spherical case is lower than that cylindrical case. For $\beta > 1$ for cylindrical case, $\beta > e^{0.5}$ for spherical case, temperature gradient $-d\theta/d\eta$ decreases as $1/\eta$ increases. Temperature gradient decreases with the stretch rate. High non dimensional temperature gradients near $1/\eta = 1$ due β where u or r_0 large. Comparing curves same β, cylindrical case is lower than that spherical case of $\beta > e$. Hot gas ball cylindrically at $\beta > e$ deforms spherical shape near $1/\eta = 1$.

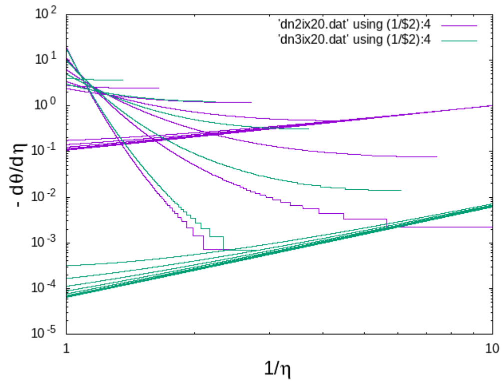

Figure 5: Temperature gradient $-\frac{d\theta}{d\eta}$.

4 Conclusions

The heat transfer from a hot gas ball surrounding examined with a simple model of convective and conductive heat transfers in cylindrical and spherical cases. A critical radius η_c exists for hot gas ball. Hot gas ball beyond the critical non dimensional radius η_c expands to the limiting radius η_l for β. The decrease β forms nearly steady hot gas ball.

5 Acknowledgements

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References

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