

A Matlab code for the design of shock tube driver insert

Yaqin Tan, Zongtai Li, Rémy Mével
Tsinghua University
Beijing, China

1 Introduction

To develop accurate detailed chemical kinetic mechanism, high-quality experimental data are of the primary importance. Among the different laboratory-scale experimental facilities that can be used to provide the experimental data for reaction model development, the shock tube technique plays an important role since it can provide high-temperature ignition delay times and species concentrations. Ideally, the pressure behind reflected shock wave should stay constant during the full length of the test time. However, there are always non-ideal pressure rise in the test section, which affects the ignition delay-time [1]. Several approaches in either simulations or experiments have been proposed to account for or eliminate such non-ideal pressure rise: 1) in simulations: use CHEMSHOCK-like approach [2] to take into account the experimental pressure increase rate dP/dt during the modeling step; 2) in experiments: introduce an insert in the driver section of the shock tube. Dumitrescu [3] was the first to propose that the non-ideal pressure rise due to the boundary-layer induced shocks attenuation could be compensated by using a tapered driver section. The expansion wave reflected from the driver insert catches up with the shock propagating towards the test section and compensates or cancels out the pressure increase. Following the approach of Dumitrescu, Hong et al. [4] developed a method to design driver insert to suppress the pressure rise behind reflected shock wave. They determined the beginning and end location of area change of the driver insert and calculated the rate of area change along the insert surface. Finally, they modified the smooth driver insert shape and proposed to use a step-like shape. Their method was validated through some sample cases in their shock tube. Despite the insights provided by the study of Hong et al., the data they provided in their summary figures can be employed for designing insert only in their specific shock tube. Grogan and Ihme [5] developed a quasi-one-dimensional model named Stanshock that considers the non-ideal effects to simulate the wave propagation process in shock tube. The non-ideal pressure rise behind the reflected shock wave was well captured and the design of the driver insert could also be performed using their model. The utilization of Stanshock was validated and showed good agreement with the study of Hong et al. [4]. Nevertheless, the approach of Grogan and Ihme does not provide any control of the non-ideal pressure-rise rate, which only depends on the shock tube dimensions and the boundary layer model. In this paper, it is proposed to develop a simple and rapid Matlab code to design driver insert for shock tube.

2 Gas dynamics in shock tube

Under tailored interface conditions, the gas dynamics in a shock tube can be represented by the trajectories of the waves shown in Figure 1. At the membrane burst, an expansion fan and a shock wave are created. To indicate the different regions in the shock tube, we have adopted the following notations: 1 refers to the initial conditions of the driven gas, 2 refers to the conditions behind the incident shock

wave, 3 refers to the conditions in the driver gas behind the contact surface, 4 refers to the initial conditions of the driver gas, and 5 refers to the conditions behind the reflected shock wave, i.e., the conditions in the test section, marked as red numbers in Figure 1. Several characteristic times can be defined in the figure to emphasize the main steps of the propagation process. The time t_4 is the time for the incident expansion head to travel from the diaphragm to the end wall of the driver section. The time t_5 is the time at which the expansion tail and the reflected expansion head intersect. The time t_2 is the time for the incident shock wave to travel from the diaphragm to the end wall of the driven section. The time t_i is the time at which the reflected shock intersects with the tailored contact surface. The time t_3^* is the time at which the reflected expansion head intersects with the transmitted reflected shock wave. The time t_3 is the time at which the reflected expansion head intersects with the tailored contact surface. Finally, the time t_6 is the time at which the reflected expansion head reflects at the end wall of the driver section. The test time corresponds to the time during which the reservoir properties are constant, i.e., between the arrival of the reflected expansion head and incident shock wave, $t_6 - t_2$.

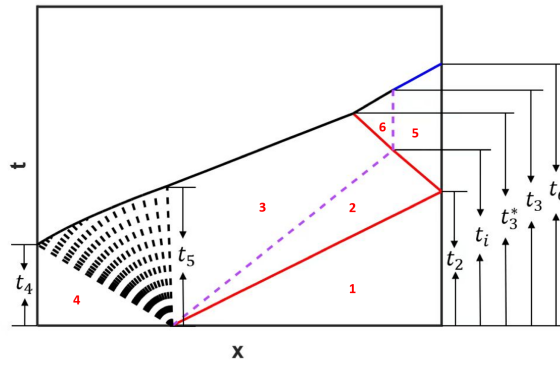


Figure 1: An example of x - t wave trajectory diagram. Red solid lines: incident and reflected shocks, black dashed lines: expansion fan, black solid line: reflected expansion head, purple dashed line: contact surface, blue solid line: reflected expansion head after interaction with the contact surface.

3 Calculation procedures

In this section, a model is developed to design a shock tube driver insert that could compensate the pressure rise at a particular rate in the test section. In this model, the following assumptions are made: (a) all gases are perfect, (b) there are no viscous effects, (c) the test gas is in thermodynamic equilibrium. Following the approach of Hong et al. [4], the design of the driver insert is based on two main aspects: (i) the length of the varying area part of the driver insert, which is dependent on the test time; and (ii) the relationship between the area changing rate of insert and reflected pressure change.

The main steps of the insert design approach are listed as follows:

1. Choose the target test conditions including the pressure P_5 , temperature T_5 , the initial temperature T_1 as well as T_4 and composition of driven gas.
2. Calculate the initial pressure P_1 for the driven gas and the tailored composition of the driver mixture.
3. With the given size of the shock tube, the wave trajectories and the characteristic times can be calculated using basic thermodynamic and jump equations, as well as the method of characteristics.

4. Determine the position of driver insert tip as well as the end of area change of the insert, depending on the target ideal test time. The tip position is when the time difference $t_6 - t_2$ is zero and is obtained by iteration by changing the length of the driver section. The end position is when the time difference $t_6 - t_2$ reaches the target test time value and no area change is needed anymore.
5. Calculate the insert area at the end of area change, depending on the ideal pressure rise rate to be compensated during the test time.
6. Calculate the relationship between the changing area ratio A_4/A_1 and the pressure change $\Delta P_5/(P_5)$ using the theory of Alpher and White [6].

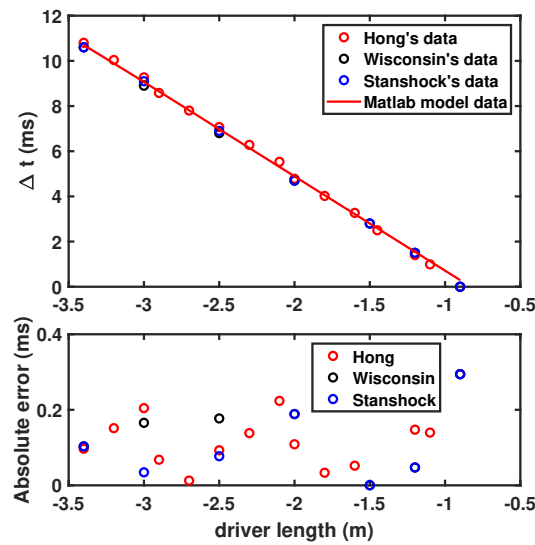


Figure 2: Time interval Δt as a function of the driver length. $T_5=900$ K, $T_1=T_4=300$ K. $P_5=404$ kPa. The driven gas is Ar, and the tailored driver gas mixture is 50.5% He/49.5%N₂. The driven section length L is 8.5 m. The absolute error between the sets of data and Hong's data is shown in the bottom figure. Δt is the time interval between the arrival of the incident shock and the arrival of the reflected expansion head at the test section location.

4 Validations

Following the approach described in subsection 3, a Matlab code was established. To validate the model, calculations were made for conditions corresponding to the examples shown in Hong et al. [4]. A shock tube with a driver length of 3.4 m, a driven length of 8.5 m, an inner diameter of 15 cm was considered. The initial temperature is 300 K in both the driver and driven sections. The target test conditions were $T_5=900$ K and $P_5=404$ kPa, the test time was 10.8 ms, and the pressure increase rate of $dP_5^*/dt=0.022/\text{ms}$ for a duration of 7 ms to be compensated, where $dP_5^*=dP_5^*/P_5^*$. The calculated tailored driver mixture composition was $X_{He}=0.505$ and $X_{N_2}=0.495$, and the driven gas was pure Ar. Following the calculation procedures, the first step is to validate the calculations of the wave trajectories. The time interval between the arrival of reflected expansion head and incident shock wave at the end wall, $\Delta t = t_6 - t_2$, as a function of the driver length, i.e., the position of driver insert tip, was obtained using several approaches: data from Hong et al. [4], calculated using the one-dimensional shock tube

code of Wisconsin Shock Tube Laboratory [7] and calculated using Stanshock [5]. Figure 2 shows the comparison between the three sets of data and the Matlab model data. The absolute error is below 0.3 ms for all three sets of data. The maximum error is around 0.29 ms when the driver length is 0.9 m. It is noted that part of the discrepancies between the data of Hong et al. and our Matlab code could be attributed to the error introduced during the data digitizing process.

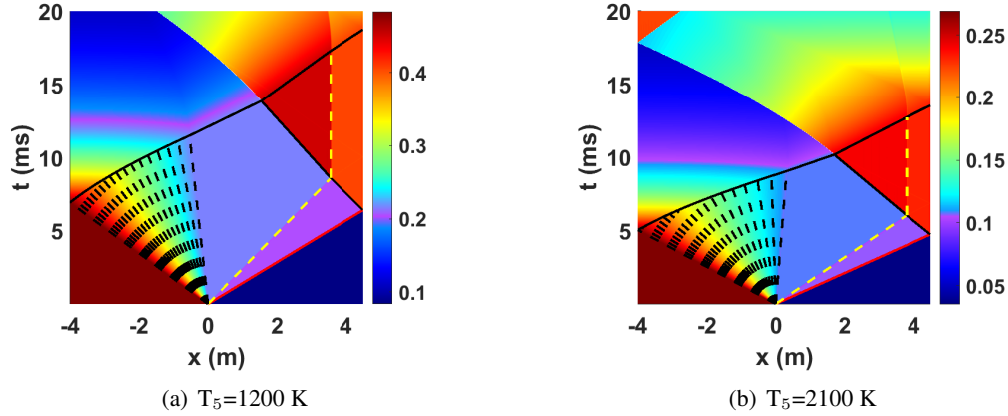


Figure 3: x-t diagrams and density field for tailored interface cases without driver insert obtained from Stanshock [5] superimposed with the trajectories obtained from the Matlab model. The target pressure is $P_5 = 101$ kPa. The driver gas is a tailored mixture of helium and nitrogen, while the driven gas is argon. The initial temperature in shock tube is 300 K. The length of driver and driven section are 4 m and 4.5 m, respectively. The uniform shock tube diameter is 11.4 cm.

Table 1: Characteristic times validation between the Matlab model and Stanshock. The target pressure and temperatures are $P_5 = 101$ kPa and $T_5=1200$, and 2100 K. The driver gas is a tailored mixture of helium and nitrogen, while the driven gas is argon. The initial temperature is taken to be 300 K. The uniform driver and driven diameter is 11.4 cm. The length of driver and driven section is 4 m and 4.5 m, respectively.

Time	$T_5=1200$ K			$T_5=2100$ K		
	Matlab	Stanshock	Error	Matlab	Stanshock	Error
$t_2(s)$	0.0064	0.0064	0%	0.0047	0.0047	0%
$t_i(s)$	0.0086	0.0086	0%	0.0060	0.0060	0%
$t_3^*(s)$	0.0139	0.0140	0.714%	0.0102	0.0103	0.971%
$t_5(s)$	0.0115	0.0115	0%	0.0090	0.0091	1.10%
$t_4(s)$	0.0069	0.0069	0%	0.0050	0.0050	0%

To further demonstrate that the Matlab model could predict the wave trajectories, the Stanshock program [5] was used to obtain the x-t diagram for several cases, neglecting the effect of the boundary layer effect. Calculations for a range of T_5 were performed. The initial temperature and velocity were set at 300 K and 0 m/s, respectively, throughout the domain. Argon was employed as the driven gas, and a tailored mixture of helium and nitrogen was utilized for the driver gas in a shock tube with a uniform cross section, where the driver mixture composition was calculated by our Matlab model to meet the tailoring conditions. The wave trajectory obtained from our Matlab model is superimposed on the density flow field obtained from the Stanshock program. A series of comparison are shown in Figure 3. It is shown that the differences between the results of our model and those of Stanshock are quite low.

The trajectories of the waves obtained from the two methods are essentially overlapping. Typical values of the characteristic times shown in Figure 1 are summed up and the errors are listed in Table 1. With a maximum error of 1.02%, it can be concluded that the Matlab model can predict the wave trajectories accurately.

To further validate our approach, the pressure variation in P_5 , $\Delta P_5/P_5$, as a function of A_4/A_1 is validated against the data of Hong et al. [4] as shown in Figure 4. The maximum error is around 1.5%. It is concluded that our Matlab model can also determine the pressure variation behaviors as a function of the area ratio.

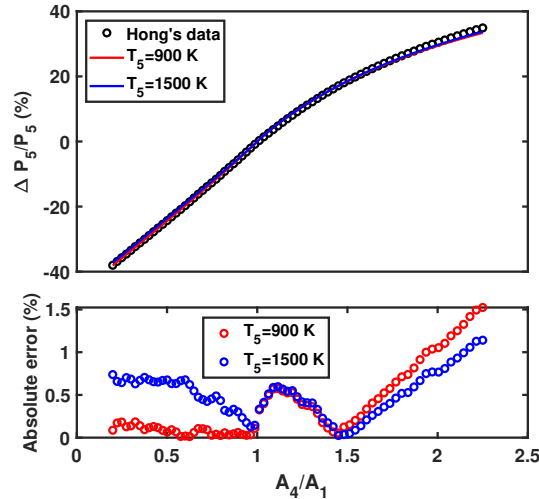


Figure 4: $\Delta P_5/P_5$ as a function of A_4/A_1 . $T_5=900$ K, $T_1=T_4=300$ K. $P_5=404$ kPa. The driven gas is Ar, and the tailored driver gas mixture is 50.5% He/49.5%N₂. The length of driver and driven sections are 3.4 m and 8.5 m, respectively.

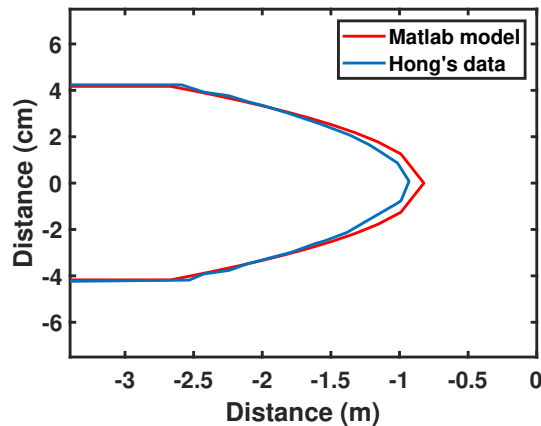


Figure 5: The axial cross-section of the shock tube driver section modified with a driver insert. $T_5=900$ K, $T_1=T_4=300$ K. $P_5=4$ atm. The driven gas is Ar, and the tailored driver gas mixture is 50.5% He/49.5%N₂. Driven section length is 8.5 m and driver length is 3.4 m. The inner diameter of the driver section is 15 cm. Compensated pressure rise, dP_5^*/dt , behind the reflected shock wave is 0.022/ms for the duration of 7 ms.

Based on the validation results shown above, the Matlab model we developed could be utilized to design driver insert. Figure 5 shows the calculated insert shape compared with the insert data from Hong et al. [4]. The difference between the unchanged area section of the insert might be partly attributed to the error resulting from the data extraction process. The relative error for the end position of the varying area is 3.06% and that for the tip position is 11.57%, which have been shown in Figure 2. Overall, the insert designed by the Matlab model is reasonable and consistent with Hong's paper [4].

5 Conclusion

A Matlab model was established to design shock tube driver insert to compensate the non-ideal pressure rise due to the boundary layer effects. The wave trajectories and pressure rate of change as a function of driver to driven section area ratio calculated by the model were validated against three approaches, and a driver insert design could therefore be obtained, which is also validated against previous studies. The Matlab model is quite fast and enables to obtain the shape of the driver within 14 seconds.

References

- [1] Pang GA., Davidson DF., Hanson RK. (2009). Experimental study and modeling of shock tube ignition delay times for hydrogen–oxygen–argon mixtures at low temperatures. *Proc. Combust. Inst.* 32: 181-188.
- [2] Li H., Owens ZC., Davidson DF., Hanson RK. (2008). A simple reactive gasdynamic model for the computation of gas temperature and species concentrations behind reflected shock waves. *Int. J. Chem. Kinet.* 40: 189-198.
- [3] Dumitrescu LZ. (1972). An Attenuation-Free Shock Tube. *Phys. Fluids.* 15: 207-209.
- [4] Hong Z., Pang GA., Vasu SS., Davidson DF., Hanson RK. (2009). The use of driver inserts to reduce non-ideal pressure variations behind reflected shock waves. *Shock Waves.* 19: 113-123.
- [5] Grogan K., Ihme M. (2020). StanShock: a gas-dynamic model for shock tube simulations with non-ideal effects and chemical kinetics. *Shock Waves.* 30: 425-438.
- [6] Alpher RA., White DR. (2006). Flow in shock tubes with area change at the diaphragm section. *J. Fluid. Mech.* 3: 457-470.
- [7] Bonazza, R.. x-t Diagram Tools. Available at:<http://silver.neep.wisc.edu/shock/tools/xt.html>.