Gravity effect on steady, 1-D propagation through dust clouds

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1 Introduction

Ignition and explosion can occur wherever combustible dust is handled. Accurate risk assessment is a key to prevent accidental dust explosions, and insights into combustion characteristics such as propagation speed and extinction limit are essential for quantitative risk assessment [1]. Although extensive experimental data are available [2, 3], establishing accurate predictive methods is crucial as the combustion characteristics vary depending on the condition. This paper presents a simple model that can assess the effect of gravity level on propagation characteristics.

Lam et al. [4] tested two different modeling approaches, those are, continuum and discrete models. The former treats the dust cloud as a continuum phase rather than resolving individual particles. In contrast, the latter model considers heat generation from individual particles by superimposing Green's functions of heat conduction equation. Although the discrete model can be applied to a broader range of conditions than the continuum model, its use of Green's functions makes it difficult to couple with the Navier-Stokes (NS) equation. On the other hand, the prediction of the continuum model (that can be easily coupled with the NS equation) agrees well with that of the discrete model unless the reaction time is much shorter than the interparticle heat-conduction time [4]. Further, the continuum model allows one-dimensional modeling that can provide a basic understanding of the phenomenon.

This study thus adopts the continuum modeling approach and discusses the influence of gravity level on the speed and the limit condition of one-dimensional propagation. Modifications are made so that the present model can consider the difference between dust-cloud and gas-phase velocities as well as the thermal expansion of the gas phase.

2 Model

The present model considers one-way coupling; the fluid flow affects the dust cloud, whereas the influence of the dust cloud on the gas phase is neglected. It is thus valid for lean conditions. Similar to [4], this study assumes thermal equilibrium between the gas phase and the dust cloud, and therefore the energy equation is solved only for the gas phase. While Lam et al. [4] assumed a zeroth-order reaction

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(i.e., a d^3-law) above an ignition temperature, this study considers a first-order reaction with respect to the dust-cloud concentration to evaluate the influence of its motion due to gravity on the reaction rate.

One-dimensional, steady traveling-wave solutions are sought in the moving coordinate x attached to the reaction front, which propagates in the $-x$ direction. The following are the basic equations solved in this study:

$$\rho_g v_g = \rho_{g0} v_{g0} \tag{1}$$

$$v_p \frac{dv_p}{dx} = \frac{v_g - v_p}{\tau_r} + g \tag{2}$$

$$\frac{dv_p C_p}{dx} = -k_0 H(T - T_{ig}) C_p \tag{3}$$

$$c_{g0} \frac{d\rho_g v_g T}{dx} = \lambda_{g0} \frac{d^2 T}{dx^2} + Q k_0 H(T - T_{ig}) C_p \tag{4}$$

$$\rho_g T = \rho_{g0} T_0 \tag{5}$$

where ρ is the density, v the velocity, τ_r the relaxation time, g the acceleration of gravity, C_p the mass concentration of dust cloud, k_0 the reaction rate constant, c the specific heat, T the temperature, λ the thermal conductivity, and Q the heat of combustion. Subscript g and p denote the gas phase and the dust cloud, respectively, whereas subscript 0 is used to emphasize that the quantity is a constant. Equations (1)-(5) express the gas-phase continuity, the equation of motion of the dust cloud, the dust-cloud mass conservation, the gas-phase energy conservation, and the equation of state, respectively.

In Eqs. (3) and (4), the stepwise ignition-temperature kinetics [5] is used (H is the Heaviside step function, and T_{ig} is the ignition temperature) because the ignition temperatures have been measured for various types of dust clouds, while establishing global Arrhenius parameters has difficulties as they can depend on dust-cloud characteristics. The origin $x = 0$ of the moving coordinate is defined such that $T(0) = T_{ig}$.

The following dimensionless quantities are introduced:

$$\bar{x} = \left(\frac{k_0}{\alpha_{g0}}\right)^{1/2} x, \qquad \bar{v} = \frac{v}{(\alpha_{g0} k_0)^{1/2}}, \qquad \bar{\rho}_g = \frac{\rho_g}{\rho_{g0}}$$
$$\bar{C}_p = \frac{C_p}{C_{p0}}, \qquad \bar{T} = \frac{T - T_0}{T_{ad} - T_0} \tag{6}$$

Here, $\alpha_{g0} = \lambda_{g0}/\rho_{g0} c_{g0}$ is the gas-phase thermal diffusivity, and $T_{ad} = T_0 + Q C_{p0}/\rho_{g0} c_{g0}$ the adiabatic combustion temperature when $g = 0$. Basic equations (1)-(5) are then rewritten in the following dimensionless form:

$$\bar{\rho}_g \bar{v}_g = \bar{v}_{g0} \tag{7}$$

$$\bar{v}_p \frac{d\bar{v}_p}{d\bar{x}} = \frac{\bar{v}_g - \bar{v}_p}{\bar{\tau}_r} + \bar{g} \tag{8}$$

$$\frac{d\bar{v}_p \bar{C}_p}{d\bar{x}} = -H(\bar{T} - \bar{T}_{ig}) \bar{C}_p \tag{9}$$

$$\bar{v}_{\text{g0}}\frac{d\bar{T}}{d\bar{x}}=\frac{d^2\bar{T}}{d\bar{x}^2}+H\left(\bar{T}-\bar{T}_{\text{ig}}\right)\bar{C}_{\text{p}} \tag{10}$$

$$\bar{\rho}_{\text{g}}[(\sigma-1)\bar{T}+1]=1 \tag{11}$$

where $\sigma = T_{\text{ad}}/T_0$ is the gas-phase expansion ratio. The following are the boundary conditions:

At $\bar{x} = -\infty$ (unburned condition)

$$\bar{v}_{\text{p}}=\bar{v}_{\text{g0}}+\bar{\tau}_{\text{r}}\bar{g}, \qquad \bar{C}_{\text{p}}=1, \qquad \bar{T}=0$$

At $\bar{x} = \infty$ (burned condition) (12)

$$\bar{T}=1+\frac{\bar{\tau}_{\text{r}}\bar{g}}{\bar{v}_{\text{g0}}}$$

Note that the burned temperature can be different from the adiabatic value ($\bar{T} = 1$) as gravity changes the dust-cloud flux. Quantity $\bar{v}_{\text{g0}}$ is the propagation velocity with respect to the unburned gas phase, and its value shall be determined as the eigenvalue of the system. As the relaxation time is proportional to the particle diameter squared, it is expressed as follows:

$$\bar{\tau}_{\text{r}}=C_\tau\bar{d}_{\text{p}}^2 \tag{13}$$

where $\bar{d}_{\text{p}}$ is the dimensionless particle diameter.

3 Results and Discussion

Equations (7)-(11) were solved with boundary conditions (12). First, the case of $\sigma = 1$ and $\bar{g} = 0$ (constant density and no gravity) was considered as it allows analytical solutions. Then, $\bar{\rho}_{\text{g}} = 1$, $\bar{v}_{\text{g}} = \bar{v}_{\text{p}} = \bar{v}_{\text{g0}}$, and the eigenvalue $\bar{v}_{\text{g0}}$ can be determined as

$$\bar{v}_{\text{g0}}=\sqrt{\frac{1}{\bar{T}_{\text{ig}}}-1} \tag{14}$$

which is shown by the solid line in Fig. 1. The propagation velocity $\bar{v}_{\text{g0}}$ decreases with an increase in ignition temperature $\bar{T}_{\text{ig}}$. When the ignition temperature is equal to the adiabatic combustion temperature, i.e., when $\bar{T}_{\text{ig}} = 1$, propagation is not possible, and $\bar{v}_{\text{g0}}$ decreases to zero.

Heat loss always exists in real dust-explosion scenarios, e.g., heat loss to pipe walls and inevitable radiative heat loss from hot particles. Adding a linearized heat-loss term, Eq. (10) becomes

$$\bar{v}_{\text{g0}}\frac{d\bar{T}}{d\bar{x}}=\frac{d^2\bar{T}}{d\bar{x}^2}+H\left(\bar{T}-\bar{T}_{\text{ig}}\right)\bar{C}_{\text{p}}-\bar{K}\bar{T} \tag{15}$$

where $\bar{K}$ is the heat-loss coefficient. The relationship between $\bar{v}_{\text{g0}}$ and $\bar{T}_{\text{ig}}$ can now be expressed by the following implicit equation (cf. [6]):

$$\bar{T}_{\text{ig}}=\frac{\bar{v}_{\text{g0}}}{2\sqrt{\bar{v}_{\text{g0}}^2+4\bar{K}}}\,\frac{2-\bar{v}_{\text{g0}}\sqrt{\bar{v}_{\text{g0}}^2+4\bar{K}}+\bar{v}_{\text{g0}}^2}{1+(1-\bar{K})\bar{v}_{\text{g0}}^2} \tag{16}$$

which is plotted by the dashed line in Fig. 1. When the ignition temperature is low, heat loss has only a minor influence on propagation velocity. The heat-loss effect becomes more pronounced at larger ignition temperatures; propagation is impossible when the ignition temperature exceeds a critical value.

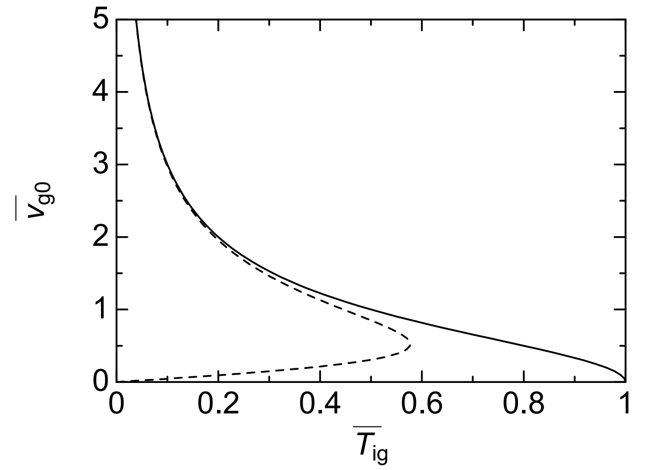


Figure 1: Propagation velocity and ignition temperature. Solid line without heat loss ($\overline{K} = 0$) and dashed line with heat loss ($\overline{K} = 0.05$).

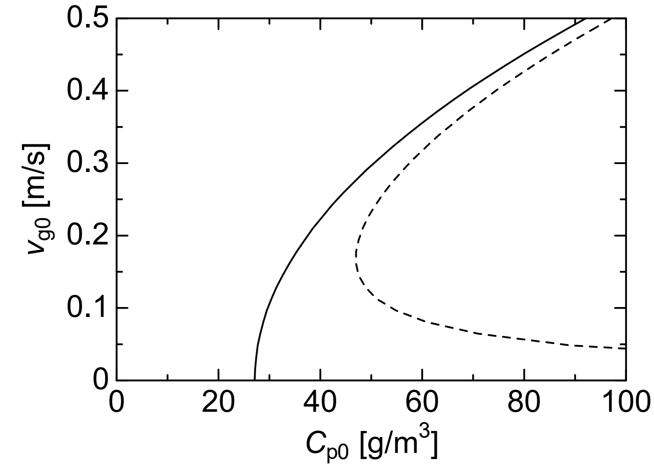


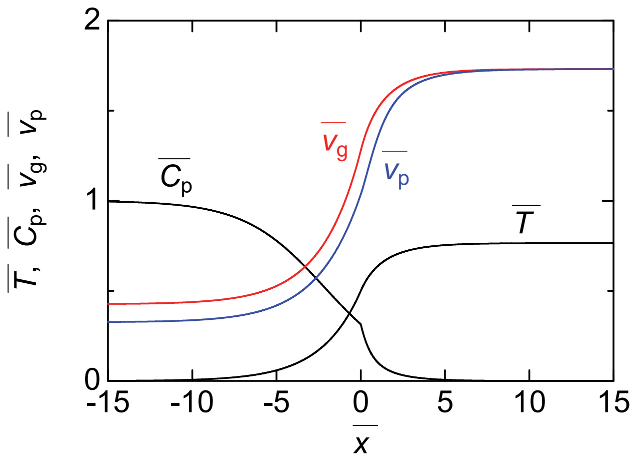
Figure 2: Propagation velocity and dust-cloud concentration. Solid line without heat loss ($\overline{K} = 0$) and dashed line with heat loss ($\overline{K} = 0.05$). Calculated with $Q = 3.1 \times 10^7$ J/kg, $\rho_{g0} = 1.2$ kg/m^3, $c_{g0} = 1000$ J/kg K, $\lambda_{g0} = 0.025$ W/m K, $T_{ig} = 1000$ K, $T_0 = 300$ K, $k_0 = 5000$ 1/s.

Noting that the definition of dimensionless temperature contains C_{p0}, the initial dust-cloud concentration, the results shown in Fig. 1 can be converted to a dimensional relationship between propagation velocity and dust-cloud concentration as shown in Fig. 2. Without heat loss, the lower limit of dust-cloud concentration corresponds to the condition where $T_{ad} = T_{ig}$. With heat loss, on the other hand, extinction occurs at a higher dust-cloud concentration. The propagation velocity at the extinction limit is then finite, about 60% of the value without heat loss. It is expected that the rate constant k_0, being the reciprocal of the characteristic time of reaction, depends on the particle diameter of dust cloud, and so does the dimensional propagation velocity, v_{g0}.

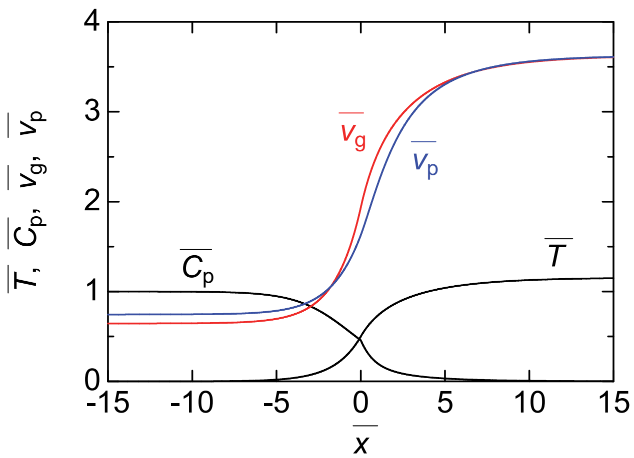
Basic equations (7)-(11) were numerically solved for cases of variable gas-phase density with gravity. Figure 3a shows the distributions of gas-phase/dust-cloud velocities, dust-cloud concentration, and temperature for $\bar{g} = -0.2$. The negative value of $\bar{g}$ means that the direction of gravity is the same as the propagation direction, corresponding to downward propagation. Because of the gravitational fall of particles, the dust-cloud velocity $\bar{v}_{\mathrm{p}}$ toward the reaction front is less than the gas-phase velocity $\bar{v}_{\mathrm{g}}$; gravity reduces the dust-cloud flux to the reaction front. The burned temperature is hence less than the adiabatic value, i.e., $\bar{T} < 1$.

Figure 3b shows the result for $\bar{g} = 0.2$, a case of upward propagation. The dust-cloud velocity is faster than gas-phase velocity in the unburned condition ($\bar{x} \to -\infty$). The gas-phase velocity increases along with temperature, followed by a delayed increase of dust-cloud velocity; the delay is caused by the relaxation time ($\bar{\tau}_{\mathrm{r}}$ in Eq. 8).

Figure 4 plots the propagation velocity as a function of gravity level. The dust-cloud flux to the reaction front increases with $\bar{g}$, thereby increasing the propagation velocity. When $\bar{g} < 0$, i.e., for downward propagation, there is a critical gravity level below which propagation is not possible.



(a) $\bar{g} = -0.2$ (downward propagation)



(b) $\bar{g} = 0.2$ (upward propagation)

Figure 3: Combustion wave structures. $\bar{T}_{\mathrm{ig}} = 0.5$, $\sigma = 5$, $C_{\tau} = 0.5$.

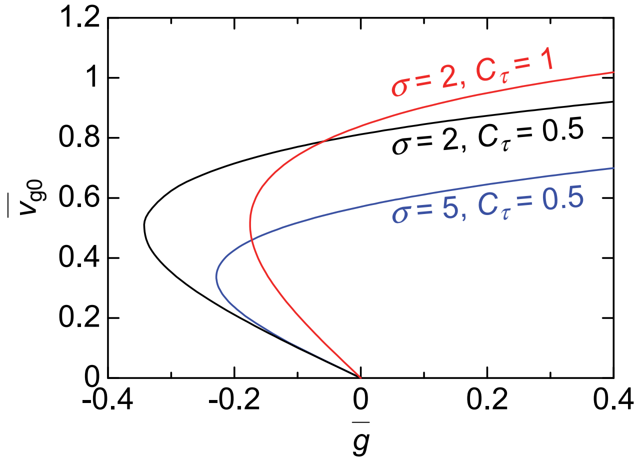


Figure 4: Influence of gravity on propagation velocity. $\bar{T}_{\mathrm{ig}} = 0.5$.

4 Conclusions

A continuum dust-explosion model was developed to simulate steady, one-dimensional propagation. Cases of constant density without gravity were first considered, and analytical expressions were obtained to express the propagation velocity as a function of ignition temperature with and without considering heat loss. Basic equations were then numerically solved for cases of variable gas-phase density with gravity. It was found that the propagation velocity increases with an increase in gravity level, and there is a critical gravity level for downward propagation. Propagation is not possible when the absolute value of gravity level exceeds the critical value.

Acknowledgments

This study was supported in part by JSPS KAKENHI Grant Numbers 18H03822, 19H01807, 21H04593, 21K14379, and the front-loading research project of Japan Aerospace Exploration Agency (JAXA).

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