# Scaling Laws for Velocity Dynamics of the Ultra-Lean Hydrogen-Air Flames Expanding in Horizontal Cylindrical Hele-Shaw Cell

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## 1 Introduction

Two specific features of the hydrogen-air premixed gaseous combustion under the Earth gravity conditions in concentration range between 4 and 9 vol. % $H_2$ are well known. First feature — a hydrogen-air combustion incompleteness — was experimentally discovered by von Humboldt and Gay-Lussac in 1805 [1]. Only in 1914 Coward [2] introduced notion of "inflammability concentration limits" and on its basis gave a contemporary interpretation of the empirical data of von Humboldt and Gay-Lussac. Coward formulated also the second specific feature — a difference between the concentration limits for upward and downward flame propagation by reviewing the earlier data of Mallard [3], Bunte [4], Either [5], and his own data [2]. In mentioned concentration range, which can be referred hereafter as an ultra-lean hydrogen-air combustion, the upward propagating flames exist only.

Ronney revealed in microgravity experiments (drop tower — [6], aircraft — [7]) an existence of the separate, long-lived flame balls, predicted by Zel'dovich in 1944 [8]. Summarizing the results of Space Shuttle research mission [9] Ronney described qualitatively — how the deflagration flames with continuous, locally plane reaction front converted into a system of the discrete, locally spherical flame balls due to sequential decrease of initial hydrogen concentration in hydrogen-air mixture. In [10, 11] it was proposed to regard this phenomenon as a third specific feature of the ultra-lean hydrogen-air combustion and the following research questions were posed — How many distinct flame ball types exist and freely propagate in quiescent ultra-lean hydrogen-air gas mixtures? What is a specific mechanism for the Flame-Ball-to-Deflagration-Transition (FBDT)?

Realising the inherent limitations of the STS-83 and STS-94 Space Shuttle missions and the shortcomings of the used invasive flame visualization method Ronny proposed that 1) "longer duration of

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microgravity experiments is needed to determine the ultimate fate of flame balls in mixtures which do not exhibit cell splitting” and 2) “it would be advantageous to eliminate the ‘coloring’ agent $\mathrm{CF_3Br}$”, whose involvement changes structure of the flames and their concentration limits.

To overcome the mentioned unavoidable limitations of the STS-83 and STS-94 missions in [12] it was proposed to use simultaneously three experimental features — 1) a closed horizontal Hele-Shaw cell as a surrogate of zero gravity in the Earth lab conditions, 2) a central ignition in cylindrical Hele-Shaw cell to facilitate studies of the ontogenetic features of quasi-two-dimensional evolution of flames and their basic constituents in microgravity conditions, for case with “reduced” dimension in comparison with three-dimensional case, studied by Ronney, 3) a visualization of the flame trails, formed by condensing the water vapour (reaction product), in reflected visible light.

Experiments [10, 13] on free, quasi-two-dimensional, cylindrical expansion of the ultra-lean flames in initially quiescent hydrogen-air mixtures in closed, horizontal Hele-Shaw cell confirm the key qualitative phenomenological observations in microgravity conditions, described in [9] for free three-dimensional Flame-Ball-to-Deflagration Transition (FBDT). It was also revealed the direct experimental evidence that mechanism of Flame-Ball-to-Deflagration Transition (FBDT) is defined by two critical (in terms of hydrogen-air stoichiometry variation) morphological phenomena — the primary bifurcation of the pre-flame kernel and the higher (secondary, tertiary and so on) order bifurcations of the Drifting Flames Balls (DFB). In [10] research focus was on the morphological features of the FBDT and their quantification — concentration ranges for 1) the basic macroscopic morphotypes (ray-like, dendritic and quasi-continuous) of the ultra-lean flames and 2) the fundamental microscopic flame constituents (“elementary building blocks”) — self-quenching, self-sustained and self-branching Drifting Flame Balls (DFB) — which define an overall macroscopic shape and evolution of the ultra-lean flames in horizontal Hele-Shaw cell.

Goal of this work — to describe quantitively the kinematic features of the ultra-lean hydrogen-air flames, expanding in horizontal Hele-Shaw cell. This report is restricted to quantification only the macroscopic, averaged over ensemble of the drifting flame balls flame propagation velocity.

## 2 Experimental data

Details of the experimental setup, measurement procedures and video capturing of the flames are described in [10, 13]. For post-processing and analysis purposes we used the original videos obtained in 2019 for the following experimental conditions: diameter of cylindrical horizontal Hele-Shaw cell — 15 cm, thickness of channel — 5 mm, variation of hydrogen concentration $[\mathrm{H_2}] = x$ in range 4–12 vol. % $\mathrm{H_2}$ at normal pressure and temperature.

## 3 Data processing

To collect data characterizing the combustion of hydrogen-air mixtures in horizontal Hele-Shaw cells at various concentrations of hydrogen in air, we will use frame-by-frame difference projections, examples of which are shown in Figure 1. To construct these projections, the frame immediately preceding the moment of ignition is excluded from the frames displaying the combustion process of the hydrogen-air mixture. As a result of this procedure, only those pixels that represent the trajectory of the flame front will have a color other than black. The sequential combination of such images in the differential mode forms contrasting black-and-white stripes, which are clearly visible against the background of the frame at the moment before the ignition of the hydrogen-air mixture.

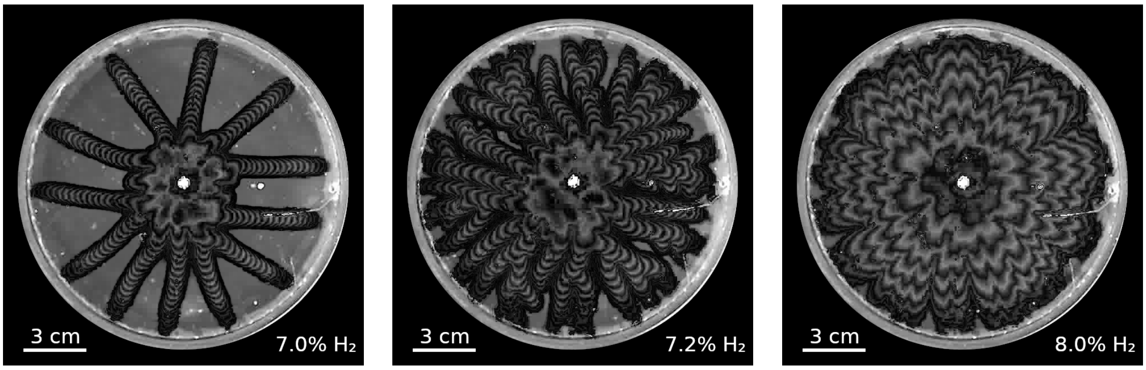

Figure 1: Difference frame-by-frame projections of the ultra-lean hydrogen-air flames in horizontal Hele-Shaw cells at different hydrogen concentrations $x = 7.0$, 7.2, 8.0%.

For approximation of a dependence of the path length $s$ traversed by the flame front on the time $t$ elapsed since the ignition of the mixture we proposed an exponential model:

$$s_i = b_0(1 - \exp(b_1 - b_2 t_i)) + \varepsilon_i \quad \text{for} \quad i = 1, 2, \ldots, n, \tag{1}$$

where $b = (b_0, b_1, b_2)$ is the parameter vector estimated from the sample data $\{s_i\}$ and $\{t_i\}$ with sample size $n$; $\varepsilon_i$ — deviations minimized by the nonlinear least squares method $\sum_{i=1}^{n} \varepsilon_i^2 \to \min$.

Figure 2 shows examples how exponential model equation (1) fits the sample data by obtained using the difference frame projections from Figure 1. The residual standard error for approximating sample data by model (1) is about 0.32 cm.

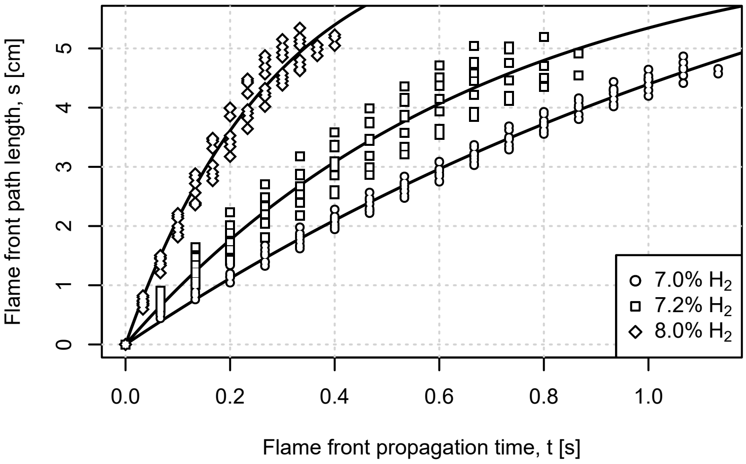

Figure 2: Averaged flame front path position x(t) versus time for hydrogen-air flames in the horizontal Hele-Shaw cell at hydrogen concentrations $x = 7.0, 7.2, 8.0$ vol. % $H_2$: points — samples, line — exponential fit according to equation (1).

Differentiation of the function $x(t)$, approximated by exponential scaling law (1), allows us to estimate the time dependence for the averaged (over a whole ensemble of the drifting flame balls, leading the combustion reaction) velocity of the flame front in the horizontal Hele-Shaw cell:

$$\frac{ds}{dt} = v_0 \exp(b_1 - b_2 t), \tag{2}$$

where $v_0 = b_0 b_2$ is the initial front visible velocity of the ultra-lean hydrogen-air flame in the horizontal Hele-Shaw cell. Temporal scaling law (2) defines the dependence of the averaged flame velocity upon time during ontogenetic evolution of the individual flame at fixed hydrogen-air stoichiometry — from its origination due to spark ignition to its arriving to boundary of flammable mixture, restricted by wall of the Hele-Shaw cell.

Comparative analysis of the approximations in Figure 2 shows that with an increase in the concentration of hydrogen in air $x$, both the initial $v_0$ and transient $v$ velocities of the flame front, averaged over ensemble of the edge drifting flame balls, increase. Figure 3 shows an estimate of the dependence of the initial velocity of the flame front $\{v_{0j}\}$ on the hydrogen concentration $\{x_j\}$ in mixture.

For hydrogen-air mixtures with concentrations $x \geq 7$ vol. % $H_2$, the dependence can be approximated by a linear function with a positive slope. At lower concentrations $x < 7$ vol. % $H_2$ there exist only the self-extinguishing and self-sustained drifting flame balls (as it was pointed out in [10]). Behavior of the self-extinguishing drifting flame balls was totally transient and stochastic. Taking these facts into account, for a uniform (over all initial hydrogen concentrations under study) approximation of the dependence $v_0(x)$, we proposed a log-exp model:

$$v_{0j} = \ln(c_0 + \exp(c_1 + c_2 x_j)) + \epsilon_j, \quad \text{for} \quad j = 1, 2, \ldots, m, \tag{3}$$

where $c = (c_0, c_1, c_2)$ is the parameter vector estimated from the sample data $\{v_{0j}\}$ and $\{x_j\}$ with sample size $m$; $\epsilon_j$ — deviations minimized by the nonlinear least squares method $\sum_{j=1}^{m} \epsilon_j^2 \to \min$.

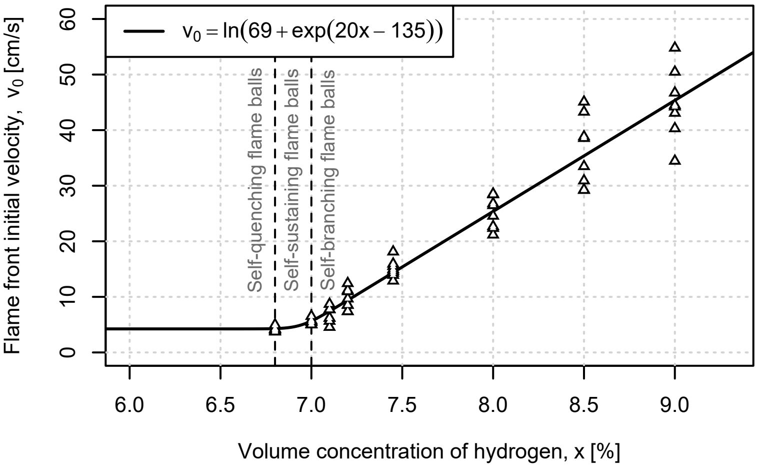

Figure 3: Approximation (3) of the initial velocity $v_0$ for the ultra-lean hydrogen-air flame front at various hydrogen concentrations $x$ in the horizontal Hele-Shaw cell.

Scaling law (3) defines the dependence of the maximal flame velocity $v_0(x)$ upon initial hydrogen concentration in hydrogen-air gas mixtures under study.

The confidence intervals for the parameters of the model (3) according to the sample data shown in Figure 3: $\mathbf{I}_{0.95}(c_1) = (18.1, 21.3)$ cm/s. The residual standard error for approximating sample data by model (3) is about 3.3 cm/s, which allows us to conclude that the estimates of the averaged parameters of the combustion process of the hydrogen-air mixture are adequate.

The approximating function of the model (3) describes the crossover from the horizontal asymptote $v_0 \approx 4.23$ cm/s at $x < x_c$ to the oblique asymptote $v_0 = c_1 + c_2 x$ at $x \geq x_c$, where $x_c = (\ln(c_0) - c_1)/c_2$. Parameter $c_1 \approx 20$ cm/s corresponds to an increase in the initial velocity of the flame front $v_0$ with an increase in the concentration of hydrogen in air $x$ by one vol. % $H_2$. The ratio $x_c = (\ln(c_0) - c_1)/c_2$ gives us an estimate of the critical concentration of hydrogen in air $x_c \approx 7$ vol. % $H_2$, which corresponds to the transition from a non-branching to a self-branching drifting flame balls.

## 4 Conclusion

1. To study dynamics of the quasi-two-dimensional ultra-lean hydrogen-air flames, which are freely expanding in horizontal, closed cylindrical Hele-Shaw cell, the original experimental videos were post-processed using frame-by-frame projection method.

2. For quantification of the two revealed flame dynamics characteristics — dependence of averaged velocities of the flames upon time and dependence of initial maximal velocity of flame upon initial hydrogen-air mixture stoichiometry two scaling laws have been proposed — temporal and stoichiometric.

3. First scaling law — exponential correlation — fits the averaged flame path dependence upon time. It was obtained that proposed temporal scaling law uniformly approximates flame path dependence in hydrogen-air mixtures with initial hydrogen concentration, which exceeds the critical value $x \geq x_c \approx 7$ vol. % $\mathrm{H}_2$ for 5 mm thickness of Hele-Shaw cell slot. In these hydrogen-air mixtures the self-branching drifting flame balls exist. Here averaged flame velocity follows exponential law in time — maximal velocity $v_0(x)$ is obtained at initial time moment, later velocity value decays exponentially in time.

4. For approximation of the dependence of the maximal initial velocities $v_0(x)$ upon initial hydrogen concentration we propose a second scaling law — the crossover function (log-exp model (3)). Both for the ray-like and the dendritic flame morphotypes the maximal initial averaged velocity linearly depends upon initial hydrogen concentration for all hydrogen-air mixtures with $x \geq 7$ vol. % $\mathrm{H}_2$. This fact — a single scaling law $v_0(x)$ for the two topologically different ultra-lean flame morphotypes — can be interpreted as an evidence of self-similarity (independence upon hydrogen concentration scale) nature of the Flame-Ball-to-Deflagration-Transition phenomenon.

5. Along with the studies of an averaged flame velocity of the ultra-lean flames in Hele-Shaw cell, which characterizes flame dynamics at macroscopic level and presented in this report, the studies of flame dynamics at microscopic level of the individual drifting flame balls shall be performed.

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