# Formation and Regulation of Unsteady Detonation Mach Stem in A Confined Space

Shuzhen Niu<sup>1</sup>, Pengfei Yang<sup>2</sup>, Honghui Teng<sup>1</sup>

 School of Aerospace Engineering, Beijing Institute of Technology, Beijing 100081, China;
SKLTCS, CAPT, BIC-ESAT, Department of Mechanics and Engineering Science, College of Engineering, Peking University, Beijing 100871, China

# 1 Introduction

In recent years, detonation-based engines, respectively known as pulse detonation engine (PDE), rotating detonation engine (RDE) and oblique detonation engine (ODE), have attracted increasing attentions owning to reduced mechanical structure and high thermal efficiency. Using an oblique detonation wave (ODW), the ODE is suitable for hypersonic propulsion system. The initiation mechanisms and stable conditions of an ODW are required to realize ODE in engineering practice. The ODW has become a classical problem studied analytically since the 1960s, and there have been significant progress of oblique detonation beginning with theoretical analysis and numerical simulations. The experimental studies have been achieved over the last few years<sup>[1-3]</sup> due to the improvement of lab facilities. The previous studies mostly focus on these aspects, i.e. the initiation types, detonation surface instabilities and waves structures of ODW induced by a wedge under a uniform and steady inflow<sup>[4-6]</sup>. For the air-breathing engines, the entrance inflow of a combustor is mostly unsteady owning to the incomplete mixing and the maneuvering flight of aircraft <sup>[7]</sup>, which is crucial to the application of an ODE. By introducing variable wedge angles or inflow density disturbances, unsteady ODWs are found to be very complex and exhibit some new wave structures and complex dynamics<sup>[8,9]</sup>. Considering the geometric complexity of a combustor, the effects of inflow Mach number variation on the wave systems and the regulation law of ODW Mach reflection in a confined combustor need more explores. Different from the previous variable wedge angle and disturbed inflow density, we mainly focus on the effects of the sudden variation of inflow Mach number Ma on the formation of detonation Mach stem. Furthermore, a regulation law of suppressing unstable detonation is proposed and the related critical conditions are also analyzed in this study.

## 2 Numerical methods and physical model

To investigate the effects of varying inlet-velocity on the detonation wave, the schematic of a combustor is shown in Fig.1. In this study, the supersonic stream flows from left to right of the computational domain. The left dot line denotes the entrance of the combustor, and the right one is set to be outlet. The solid lines are assumed to be a wall which is given a non-slip boundary condition. *L* is the width of the whole computation domain, and  $L_c$  is defined as the distance of zero point and upper wall corner along *x*-direction, which is an important variable for the ODW flied structures.  $\theta_1$  is the angle of the wedge, and  $\theta_2$  is the angle of the second upper wall. *H* denotes the height of inlet entrance. In this paper, the inlet-velocity is given to be  $M_1$  firstly, and then a Mach stem formed before the corner, which is derived from the interaction of detonation wave and upper solid wall. Subsequently, the inflow Mach number  $M_2$ , which takes the place of  $M_1$ , is set to be the entrance velocity. By these, the formation and regulation of an unsteady Mach stem could be observed.



Figure 1: The schematic of the domain for combustor.

In many previous studies, the Euler equations with a two-step kinetic model are used to be the governing equation which can solve the flow field structure of ODW in a confined space. The effectiveness of this two-step model has been verified by previous studies<sup>[10]</sup>. There are two dimensionless chemical reaction progress variables for the two-step model:  $\xi(1\rightarrow 0)$  and  $\lambda(0\rightarrow 1)$ , which describe the induced reaction progress and the exothermic reaction progress for the heat release of chemical reaction, respectively. These two indexes can nicely describe the process of detonation combustion. The transport equations could be expressed by:

$$\frac{\partial(\rho\xi)}{\partial t} + \frac{\partial(\rho u\xi)}{\partial x} + \frac{\partial(\rho v\xi)}{\partial y} = H(1-\xi)\rho k_I \exp\left[E_I\left(\frac{1}{T_S} - \frac{1}{T}\right)\right],\tag{1}$$

$$\frac{\partial(\rho\lambda)}{\partial t} + \frac{\partial(\rho\lambda)}{\partial x} + \frac{\partial(\rho\nu\lambda)}{\partial y} = [1 - H(1 - \xi)]\rho(1 - \lambda)k_R \exp\left[-\frac{E_R}{T}\right],\tag{2}$$

among them, the Heaviside step function  $H(1-\zeta)$  is given by

$$H(1-\xi) = \begin{cases} 1, & \xi \le 1, \\ 0, & \xi > 1. \end{cases}$$
(3)

in which,  $E_I$  is the activation energy of induction, and  $E_R$  is the activation energy of heat release. To complete the model, two essential parameters, the chemical reaction rate constant  $k_I$  (induction zone) and  $k_R$  (exothermic zone), are introduced.  $k_R$  is set to be 1.0. And  $k_I = -u_{vn}$ , in which  $u_{vn}$  is the flow velocity behind the leading shock wave in the Chapman-Jouguet (CJ) detonation wave in a shock-fixed coordinate system. In addition, variables include pressure, density, temperature, *x*-direction velocity, and *y*-direction velocity, denoted by p,  $\rho$ , T, u, v, respectively, which are normalized by referenced free stream state (the subscript index 0) as follows:

$$p = \frac{\tilde{p}}{p_0}, \qquad \rho = \frac{\tilde{\rho}}{\rho_0}, \qquad T = \frac{\tilde{T}}{T_0}, \qquad u = \frac{\tilde{u}}{\sqrt{RT_0}}, \qquad v = \frac{\tilde{v}}{\sqrt{RT_0}}, \tag{4}$$

#### **3** Results and discussion

#### 3.1 Basic structure

The main parameters are set to be Q = 25,  $\gamma = 1.2$ ,  $E_I = 4.0T_S$ ,  $E_R = 1.0T_S$ , where Q denotes the amount of chemical heat release, which controls the CJ detonation velocity, and  $T_S$  denotes the post-

shock temperature. These chosen parameters in this study do not correspond to any detailed reactants, but the results are of the nature of universal application. And this characteristic can provide more general findings. Besides, the activation energies include  $E_{I}$  and  $E_{R}$ , which are chosen according to the hydrogenair mixture.

The geometry parameters, as is shown in Fig.1, are set to be H = 140, L = 320,  $\theta_1 = 25^\circ$ ,  $\theta_2 = 45^\circ$ .  $L_c$  is the variables which can affect the flow structure in this domain. Figure 2 shows the local temperature field contour of the flow field in  $L_c = 162$  and  $L_c = 163$ , and the black lines denote the position of wave front. It is clearly visible that a stable Mach stem is obtained with a small  $L_c$  under the interaction between an ODW and the expansion wave, as shown in Fig.2(a). When we change  $L_c$  to be 163, an unstable flow structure appears in Fig.2(b), in which the Mach stem moves upstream over time. This phenomenon demonstrates there is a critical  $L_c$  that makes the flow field between stable and unstable structures.



Figure 2: Temperature field for stable with  $L_c = 162(a)$ , unstable with  $L_c = 163(b)$ . The black lines represent the position of leading shock wave front.

In fact, the unstable structure unavoidable is extremely detrimental to the thrust performance of the flight, which can cause a serious drop in the propulsion efficiency. As a result, the detonation wave is pushed out the combustion combustor, leading the failure of detonation. In this paper, we focus on the unstable structure and study the effects of changes in inflow *Ma*, analyzing whether the changed *Ma* can play a certain regulatory role on the unstable Mach stem.

#### 3.2 Critical position for transformation

To investigate the effect of changing Ma on the unstable Mach stem structure, the flow field shown in Fig.2(b) is as the basic structure. Increasing Ma 7.0 to 8.0 based on Position 1, the Mach stem moves downstream rapidly, representing a stable structure formed behind the upper corner wall, which is shown in Fig.3(a). However, when we change Ma 7.0 to 8.0 based on position 2, a reversed result appeared in the computational domain. Based on Position 2, the incoming Ma is continuously increased to 8.5, or even to 9.0. The results show that the upstream movement of Mach stem is weakened and eventually move downstream. These phenomena demonstrate that increasing the incoming Mach number in time can significantly affect the unsteady detonation Mach stem, leading a stable flow field structure, otherwise a larger Ma should be provided.

In general, the transformation of flow field depends on the incoming Ma strongly to a certain extent, and the height and position of Mach stem are the main factors for the transformation. The unstable structure can be adjusted by varying Ma, which could improve the flight performance. While how to control properly the velocity is worthy the deeper thought.



Figure 3: Position of the Mach stem for different cases.

#### 3.3 Evolution of detonation wave structure

To study in detail how the increasing velocity act on the movement of unstable Mach stem, the processes for two typical cases: Ma 7.0 to 8.0 and Ma 7.0 to 9.0, based on position 2, are discussed. For Ma 7.0 to 8.0, as mentioned above, the Mach stem cannot stay in the computational domain, denoting an unstable process. While for Ma 7.0 to 9.0 the Mach stem moves downstream, and a recirculationzone forms after the upper corner wall, which represents a stable structure.



Figure 4: Pressure field for evolution of Ma 7.0 to 8.0 based on position 2 The black line denotes the subsonic zone.

Figure 4 shows the pressure field for evolution of Ma 7.0 to 8.0 based on the flow field Position 2, in which the flow time is reset to be zero simultaneously. As we can see, the contact surface of inflow velocity has already gone through the Mach stem at t = 20.26, but the Mach stem stay the same as t = 0, illustrating this the incoming flow cannot directly act on the movement of Mach stem. At t = 60.53, a series of pressure wave formed under the interaction of reflected shock wave and the downstream subsonic zone, and the Mach stem is pushed towards upstream. It should be noted that until t = 120.70, the Mach stem moves upstream obviously and becomes stronger, and the subsonic zone expands to a larger one. While there is a little difference for Ma 7.0 to 9.0, which is shown in Fig.5. The contact surface of inflow velocity has reached the Mach stem before t = 18.28, and the Mach stem moves downstream obviously, which can be seen from the subsonic zone of expansion wave front.



Figure 5: Pressure field for evolution of *Ma* 7.0 to 9.0 based on position 2; The black line denotes the subsonic zone.

# 4 Conclusions

Using the two-step induction-reaction model, two-dimensional oblique detonation waves in a confined space were simulated. By introducing the sudden variation of inflow velocity, the formation and regulation of an unstable detonation Mach stem is achieved in this study. To display the evolution of the detonation structures, a basic ODW flow fields is obtained firstly, featured by an unstable Mach stem that gradually grows stronger and moves upstream. Simulated results shows that a slight increase in inflow Mach number can suppress the upstream movement of Mach stem at the early stage of detonation instability. As the Mach stem moves upstream, it is increasingly hard to restabilize the ODW. More costs and efforts of inflow Mach number are required, as shown in Fig. 3. Besides, the unstable mechanisms of detonation wave are also discussed. Since the interactions of reflected shock wave and the downstream subsonic zone, a series of pressure waves are originated from the interaction point, as shown in Fig.4. These waves move upstream and push the leading detonation front, and finally results in an unsteady flow.

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