Unified Characteristic Relationships of Hydrogen-Oxygen-Argon Detonation Dynamics in Narrow Channels

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1 Introduction

Detonations in narrow channels usually propagate in the presence of boundary layer losses, exhibiting significant velocity deficits and narrower propagation limits. Such response of detonation dynamics to wall losses, however, strongly depends on the characteristic channel dimensions and mixture compositions, which to some extent govern the magnitude of the boundary-layer-induced losses. Since the pioneering work of Fay [1], who first proposed the boundary-layer-induced flow divergence mechanism for modeling the detonation losses subject to boundary layer effects, a great deal of efforts have been subsequently made in extending and also applying the classical model of Fay in predicting the detonation dynamics measured from experiments, e.g., see Refs. [2–4]. In these works, the predictions were interpreted in terms of the velocity deficits in relation to initial pressures or the characteristic channel width, and good agreement has been achieved between experiments and the theoretical model for the argon-diluted weakly unstable detonations. On the other hand, as the characteristic relationship characterizing the detonation velocity deficits with respect to losses typically relies on the geometry and mixture conditions, an interesting question thus arises that whether there is a unified relation for capturing the dynamics of detonations in narrow channels, irrespective of the channel width and mixture kinetic sensitivity. Although Nakayama et al. [5] have experimentally demonstrated the potential universality of detonation speed-curvature relationships for curved detonations in three different mixtures, the recent work of Xiao & Radulescu [6] showed that such universality is limited by the uncertainty of the normalization scale obtained from experiments. As such, the choice of appropriate characteristic length scales is significant for the unification. Therefore, the present communication aims to seek a specific length scale for unifying the detonation dynamics in narrow channels of varied widths, from both the theoretical and experimental perspectives.

2 The Generalized ZND Model with Flow Divergence

Despite the intrinsic unsteadiness and multi-dimensional nature of gaseous cellular detonations, i.e., comprising the incident shock, Mach stem, transverse waves and shear layers, the very recent works of Xiao & Radulescu [4] and Xiao & Weng [7] have demonstrated the very good predictability of hydrogen-oxygen-argon detonation dynamics by the quasi-1D steady Zel'dovich-von Neumann-Doering (ZND) model in the presence of flow divergence. In this regard, we continue to adopt such well-performed generalized ZND model for theoretically computing the dynamics of hydrogen-oxygen-argon detonations in narrow channels. In the shock-attached frame of reference, the extended ZND model can be expressed as [3, 8]

$$\frac{\mathrm{d}p}{\mathrm{d}t} = -\rho u^2 \frac{\dot{\sigma}_{re} - \dot{\sigma}_A}{\eta} \tag{1a}$$

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -\rho \frac{\dot{\sigma}_{re} - M^2 \dot{\sigma}_A}{\eta} \tag{1b}$$

$$\frac{\mathrm{d}u}{\mathrm{d}t} = u \frac{\dot{\sigma}_{re} - \dot{\sigma}_A}{\eta} \tag{1c}$$

$$\frac{\mathrm{d}y_i}{\mathrm{d}t} = \frac{W_i \dot{\omega}_i}{\rho} \quad (i = 1, \cdots, N_s) \tag{1d}$$

$$\frac{\mathrm{d}x'}{\mathrm{d}t} = u \tag{1e}$$

with

$$\eta = 1 - M^2, \qquad \dot{\sigma}_{re} = \sum_{i=1}^{N_s} \left(\frac{W}{W_i} - \frac{h_i}{c_p T} \right) \frac{\mathrm{d}y_i}{\mathrm{d}t}, \quad \dot{\sigma}_A = \frac{u}{A_{tot}} \frac{\mathrm{d}A_{tot}}{\mathrm{d}x'} \tag{1f}$$

where $\dot{\sigma}_{re}$ is the thermicity of ideal gases, while $\dot{\sigma}_A$ is the rate of flow divergence. According to Fay's boundary-layer mechanism [1], for detonations in narrow channels (where the channel height h is much larger than the channel width w), the boundary-layer-induced flow divergence can be modeled as

$$\frac{\mathrm{d}ln\left(A_{tot}\right)}{\mathrm{d}x'} = \frac{2}{w + 2\delta^*(x')} \frac{\mathrm{d}\delta^*(x')}{\mathrm{d}x'} \tag{2a}$$

with the boundary layer displacement thickness $\delta^*(x')$ given by [7, 9]

$$\delta^*\left(x'\right) = K_M \sqrt{\nu_s \int_0^{x'} \frac{1}{u(x')} dx'} \tag{2b}$$

where ν_s is the post-shock kinetic viscosity, and $u(x')$ is the local instant flow velocity in the shock-attached frame of reference. K_M is the Mirels' constant, which has been calculated to be approximately 4.5 for hydrogen-oxygen-argon detonations [4]. One can refer to the recent works of Zangene et al. [9] and Xiao & Weng [7] for more details in obtaining the revised version of Mirels' boundary layer theory [10]. By following the ZND computation framework of Xiao & Radulescu [4], we could then obtain the response of detonation dynamics to varying initial conditions and geometry dimensions.

3 Results and Discussion

By varying the mixture kinetic sensitivity or the characteristic channel width, detonations in narrow channels exhibit notably different responses in terms of velocity deficits and propagation limits. For example, Fig. 1 shows the effect of channel width and also the argon dilutions on hydrogen-oxygen-argon detonation velocity deficits as a function of initial pressures. Note that D_{CJ} in the axis denotes the ideal Chapman-Jouguet (CJ) detonation speed without any deficits. Clearly, both the decrease of the characteristic channel width in Fig. 1a and the increase of argon dilutions in reducing the kinetic

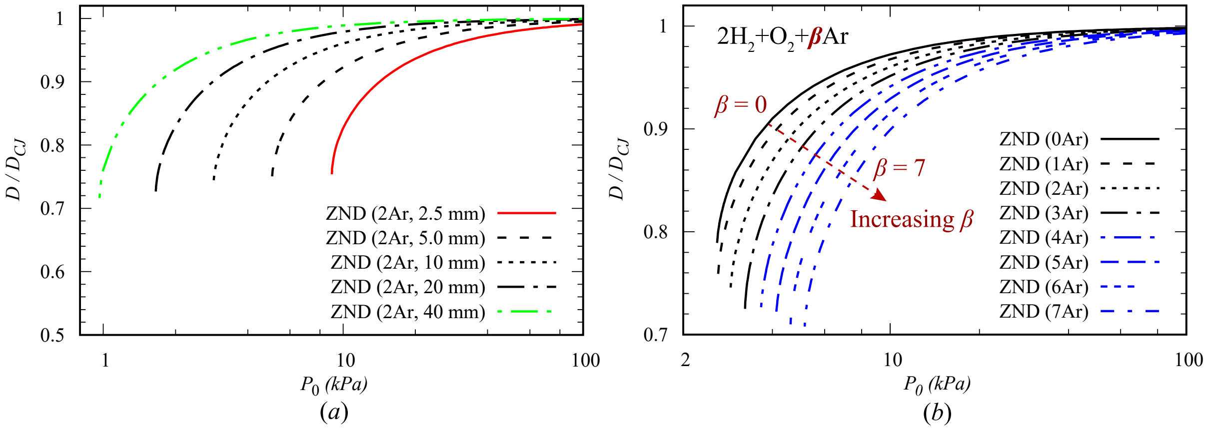

Figure 1: Detonation speeds (D/D_{CJ}) as a function of initial pressures: (a) the effects of channel width (w) on $2H_2/O_2/2Ar$ detonation dynamics; (b) the effects of argon dilutions on hydrogen-oxygen detonations in a narrow channel of $w = 10$ mm.

sensitivity of mixtures in Fig. 1b could lead to more significant velocity deficits and higher propagation limit pressures. Of noteworthy is that the characteristic channel width w is directly related with the magnitude of boundary layer losses, as indicated by Eq. (2a). A narrower channel thus results in larger flow divergence due to boundary layers inside the detonation reaction zones, thereby giving rise to much more reduced propagation speeds.

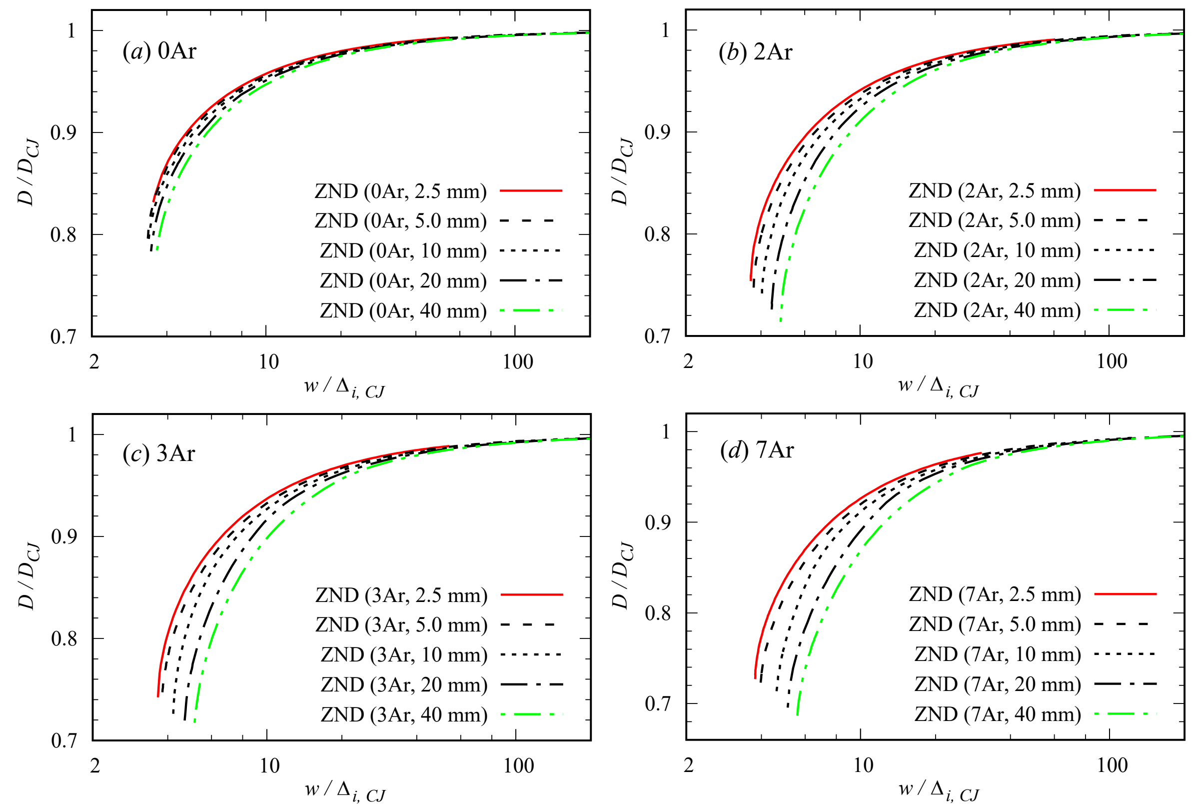

Figure 2: The generalized ZND model calculated $D/D_{CJ} - w/\Delta_{i,CJ}$ relationships for stoichiometric hydrogen-oxygen detonations with varied argon dilutions in channels of different width w ranging from 2.5 mm to 40 mm.

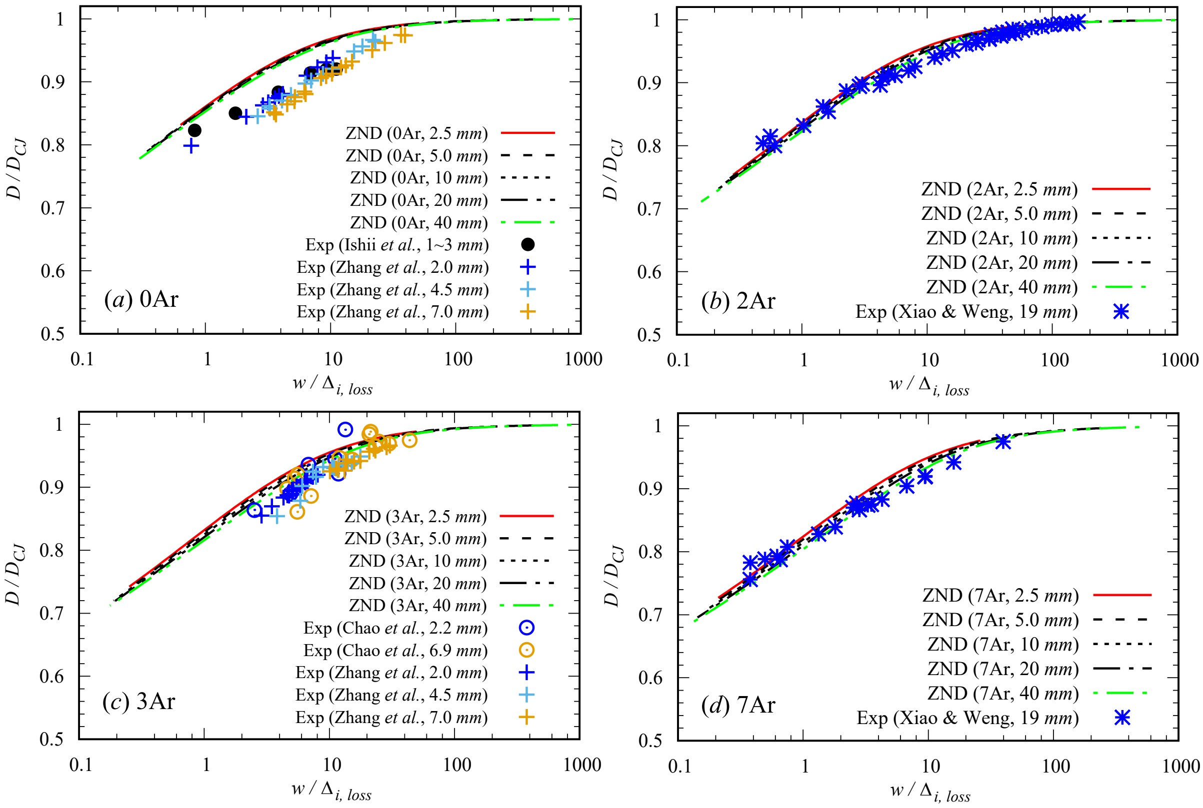

Figure 3: The theoretical and experimental $D/D_{CJ} - w/\Delta_{i,loss}$ relationships for stoichiometric hydrogen-oxygen detonations with varied argon dilutions. Note that the generalized ZND model predictions were made for $H_2/O_2/Ar$ detonations in channels of different width w ranging from 2.5 mm to 40 mm. The experimental data were adapted from the works of Ishii et al. [16], Zhang et al. [17], Xiao & Weng [7] and Chao et al. [3], respectively.

In the framework of detonation shock dynamics (DSD) [11], the detonation front propagates in a curved manner diverging the flows inside the trailing reaction zone, thereby leading to reduced propagation speeds. As such, detonations in a specific mixture are expected to follow a unique velocity-curvature relationship, i.e., the $D(\kappa)$ relation, which has been explored extensively in the well-posed curved detonation experiments by Nakayama et al. [5] and Radulescu et al. [4, 6, 12]. Whereas for detonations in narrow channels, as the boundary-layer-induced curvature is difficult to measure from the experiments, a large number of works (e.g., see Refs. [2, 13]) have proposed to adopt the boundary-layer-induced flow divergence interpreted in terms of the characteristic geometry length scale (e.g., the channel width or the tube diameter) normalized by the kinetic length scale, such as the detonation cell size or the induction zone length. In this regard, the present study also adopts the same methodology in analyses of the ZND model calculated hydrogen-oxygen-argon detonation dynamics in narrow channels. Figure 2 shows the characteristic $D/D_{CJ} - w/\Delta_{i,CJ}$ relationships for argon-diluted stoichiometric hydrogen-oxygen detonations in narrow channels of different widths w. Note that here $\Delta_{i,CJ}$ is the induction length of the ideal CJ detonation, which is defined as the distance between the leading shock and the peak thermicity, calculated by multiplying the ignition time with the post-shock velocity (in the shock-attached frame of reference). The detailed San Diego reaction mechanism [14] was utilized for describing the reaction kinetics involved in all the computations of the study. Evidently, while varying the channel width w, the $D/D_{CJ} - w/\Delta_{i,CJ}$ curves of hydrogen-oxygen-argon detonations in Fig. 2 appear to exhibit slight differences instead of perfectly collapsing together.

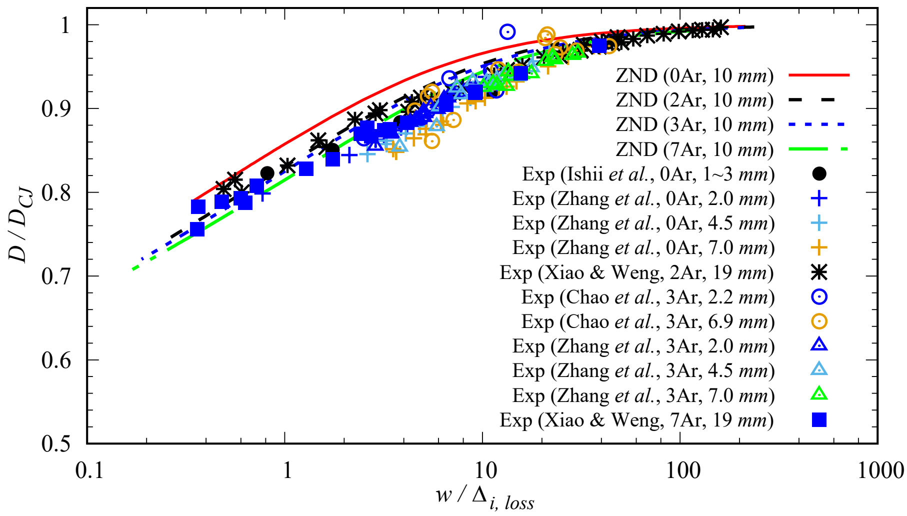

Figure 4: The unified $D/D_{CJ} - w/\Delta_{i,loss}$ relationships for hydrogen-oxygen-argon detonations in narrow channels from both the theoretical and experimental perspectives.

On the other hand, the recent works of Xiao et al. [7, 15] have demonstrated the lengthening of the reaction length scales due to losses that typically exist in real gaseous detonations. As such, we further consider the presence of losses when calculating the realistic induction length $\Delta_{i,loss}$ by incorporating the corresponding velocity deficits of detonations. Figure 3 presents the ZND model predicted $D/D_{CJ} - w/\Delta_{i,loss}$ relationships in comparison with the relevant experiments of detonations in narrow channels. It can be seen that, with the more realistic induction length $\Delta_{i,loss}$ whose calculation incorporates the losses, the speed-flow divergence relationships appear to collapse excellently together while being independent of the channel width w. Also, relatively good agreement between the experiments and the predictions can be observed in well obeying the unique characteristic $D/D_{CJ} - w/\Delta_{i,loss}$ relation for every single mixture. Finally, in order to further check the universality of $D/D_{CJ} - w/\Delta_{i,loss}$ relationships, we collected all the relevant narrow channel experiments and also the ZND model predictions of hydrogen-oxygen-argon detonations together in Fig. 4. Clearly, all the experiments and theoretical calculations of $D/D_{CJ} - w/\Delta_{i,loss}$ appear to collapse very well together, suggesting no dependence on argon dilutions and channel widths. It thus well demonstrates the realistic induction length $\Delta_{i,loss}$ in unifying the characteristic speed-flow divergence relationships of hydrogen-oxygen-argon detonations in narrow channels.

4 Conclusion

The present work adopted the extended ZND model with the boundary-layer-induced flow divergence for computing the dynamics of hydrogen-oxygen-argon detonations in narrow channels. Effects of the characteristic channel width and also the argon dilutions on detonation dynamics have been demonstrated. Although the typical induction length $\Delta_{i,CJ}$ fails to collapse the $D/D_{CJ} - w/\Delta_{i,CJ}$ relationships, the more realistic induction length $\Delta_{i,loss}$ evaluated by considering the presence of losses was found to be capable of well unifying both the experimentally obtained and also the theoretically calculated speed-flow divergence relationships of hydrogen-oxygen detonations in narrow channels, being independent of argon dilutions and channel dimensions. Future efforts should be devoted to other characteristic length scales of detonations and also other more complicated hydrocarbon fuels for establishing the unified laws in describing the macro-scale dynamics of real gaseous detonations subject to losses.

5 Acknowledgement

This work was supported by the fellowship of China Postdoctoral Science Foundation (Grant No. 2021M691590), Jiangsu Postdoctoral Research Funding Program (Grant No. 2021K461C), the National Natural Science Foundation of China (Grant No. 12102195), and the Natural Science Foundation of Jiangsu Province (Grant No. BK20210343). Q. Xiao would like to personally thank Matei Radulescu for the supervision during the PhD study as well as the continuous support and discussions.

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