Thermodynamic Analysis of Unsteady Propulsion Systems

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1 Introduction

Unsteady detonative propulsion devices are being studied for their potential to increase thermodynamic efficiency over steady deflagrative devices. The basis of any study concerning these propulsion systems generally starts with a thermodynamic analysis. The current work is based on the analysis by Paxson and Kaemming [1]. In it, they show how the control volume approach applies to an unsteady, constant volume combustor and that an unsteady combustor has fill and blowdown phases that must be accounted for. This accounting ensures that energy is conserved in the analysis. Currently, Paxson and Kaemming's analysis only applies to constant volume combustion since it requires a spatially uniform state within the combustor. The challenge is extending the analysis to study detonative combustion which requires a model that can handle spatial non-uniformity.

Another goal of the current work is to develop a better understanding of how the unsteady, quasi one-dimensional Euler equations are used in the analysis of unsteady propulsion devices. An often overlooked term relevant to the ideal expansion of combustion products during a blowdown process is also introduced. By carefully applying the correct form of the mass, momentum, and energy equations to the propulsion device and the processes occurring within, new insights are developed in the thermodynamic analysis of unsteady propulsion devices. These insights lead to new questions and research topics for future work.

2 Unsteady Quasi One-Dimensional Euler Equations

The integral forms of the unsteady, quasi one-dimensional Euler equations are

$$\frac{d}{dt}\int_{x_{in}(t)}^{x_{out}(t)} \rho A dx = [\rho A(u-u_b)]_{in} - [\rho A(u-u_b)]_{out}$$

$$\frac{d}{dt}\int_{x_{in}(t)}^{x_{out}(t)} \rho u A dx = [\rho u A(u-u_b)+PA]_{in} - [\rho u A(u-u_b)+PA]_{out} + \int_{x_{in}(t)}^{x_{out}(t)} P\left(\frac{\partial A}{\partial x}\right)dx$$

$$\frac{d}{dt}\int_{x_{in}(t)}^{x_{out}(t)} \rho e_t A dx = [\rho e_t(u-ub)A+PuA]_{in} - [\rho e_t(u-ub)A+PuA]_{out} - \int_{x_{in}(t)}^{x_{out}(t)} P\left(\frac{\partial A}{\partial t}\right)dx.$$

This form of the equations allows for moving inflow and outflow boundaries. The boundary velocity is given by u_b. The moving boundaries are useful when examining the flow of reactants into the combustor and the flow of products out of the combustor during the fill process shown later. Additionally, the energy equation has an extra work term associated with time-varying area. This term arises if A/A^* must vary to ideally expand the flow or if multiple unsteady streamtubes are interacting with each other. Further information on the unsteady quasi one-dimensional Euler equations with time-dependent area variations may be found in the textbook by Warsi [2].

3 Conservation of Energy - The Refill Process

The unsteady propulsion cycle being considered is an idealized pulse detonation engine cycle. A steady inflow process is assumed with an unsteady combustion and an unsteady blowdown process. The unsteady combustion and blowdown require the combustor to undergo the following processes cyclically: fill, combustion, and blowdown followed by fill again. The proper application of the conservation equations to the refilling process is essential to ensuring energy conservation when examining the cycle as a series of intermediate steps.

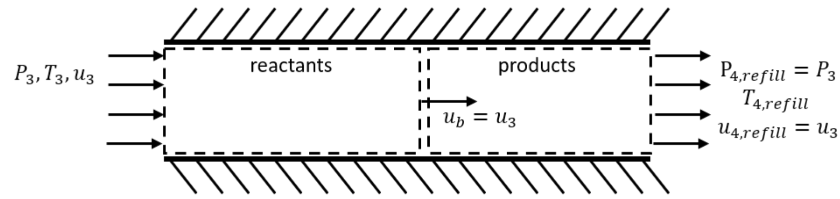

Figure 1: Control volume analysis of the unsteady refilling process.

Specifically, the moving control surface boundary between the reactants and products means that for reactants entering the combustor, the energy flux, $\dot{m}e_t$, is required to correctly ensure energy conservation. For a steady flow, the enthalpy flux, $\dot{m}h_0 = \dot{m}(c_vT + u^2/2 + P/\rho)$ is used. This important distinction is due to the moving boundary requirement when applying a control volume analysis of an unsteady flow process. This causes the PuA terms in the energy equation to cancel out. Note that if the control volume is drawn around the entire propulsive device, there is no moving boundary and the enthalpy flux should be used at the boundaries. However; when analyzing the processes occurring inside an unsteady propulsion system, it is important to use the correct flux to ensure energy conservation.

4 Application to Detonative Combustion

The analysis done by Paxson and Kaemming was only for a constant volume combustion process [1]. The challenge with applying their model to a detonation-based combustion device is that an unsteady detonation wave is not only temporally nonuniform but spatially nonuniform as well. Whereas a constant volume combustion process can be approximated as occurring everywhere simultaneously, a detonation wave consists of the detonation followed by an expansion.

The simplest unsteady, detonative combustor that can be studied is a one-dimensional detonation wave traveling in a tube. This is also known as a Pulse Detonation Engine. The advantage to studying this type of detonative combustor is that analytical solutions for the detonation and expansion waves are known [3]. For calorically perfect gases with constant molecular weights, the Chapman-Jouguet Mach number is

$$M_{CJ} = \sqrt{(\gamma+1)Q + 1 + \sqrt{[(\gamma+1)Q+1]^2 - 1}}$$

where $Q = q/c_p/T_3$. The expansion wave profile behind the detonation wave is given by

$$\frac{a}{a_4} = 1 - \frac{\gamma - 1}{\gamma + 1}\left(1 - \frac{x}{a_4 t}\right)$$

where a_4 is the speed of sound in the products that have been decelerated to zero velocity behind the detonation. Using the speed of sound; the pressure, temperature, and density through the expansion wave may be found using the isentropic relations.

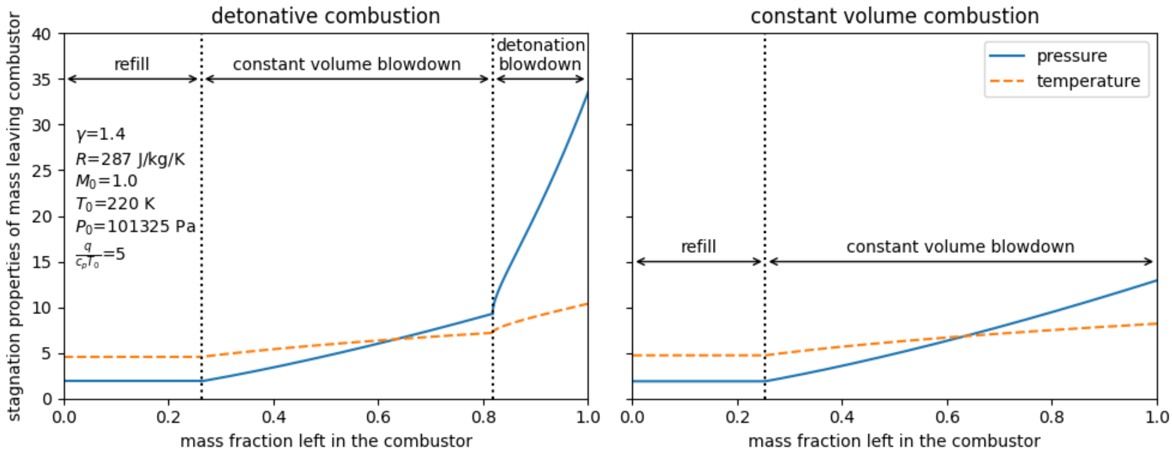

Figure 2: Stagnation property variation of mass leaving the combustor.

It is possible to numerically integrate the fluxes out of the combustor due to the motion of the detonation wave and the expansion wave. The conservation of mass equation is used to calculate the amount of mass leaving the combustor due to the detonation and expansion:

$$m_{t=L/a_4} - m_c = \int_{L/u_{cj}}^{L/a_4} \frac{2\rho_4 a_4 A_4}{\gamma - 1}\left[\frac{\gamma - 1}{\gamma + 1}\left(1 - \frac{L}{a_4 t}\right)\right]\left[1 - \frac{\gamma - 1}{\gamma + 1}\left(1 - \frac{L}{a_4 t}\right)\right]^{\frac{2}{\gamma - 1}} dt$$

This integral is evaluated from the instant the detonation wave is at the exit of the combustor until the tail end of the expansion wave leaves the combustor. After this, there is a uniform state in the combustor and the constant volume blowdown relations may be used to determine properties in the combustor. This is illustrated in Fig. 2 where the stagnation pressure and temperature of the mass leaving the combustor is plotted for a detonative and constant volume combustion process. Note that for the detonation blowdown phase, the stagnation properties are larger than for an equivalent constant volume combustion process. However, after the detonation and expansion leave the combustor, the blowdown and refill proceeds the same as the constant volume. This increase in stagnation properties during the detonation blowdown represents the extra work potential of detonative combustion over constant volume combustion.

5 Ideal Expansion of Unsteady Flows

Ideally expanding a flow generally means to expand the flow down to the ambient pressure. For the ram cycle considered in this work, this expansion is accomplished using a converging-diverging nozzle. The equation to determine the nozzle expansion ratio, A_8/A_9, is derived from the steady quasi one-dimensional Euler equations and is given by

$$\frac{A_8}{A_9} = \frac{\left[1 - \left(\frac{P_9}{P_{t,4}}\right)^{\frac{\gamma-1}{\gamma}}\right]^{\frac{1}{2}} \left(\frac{P_9}{P_{t,4}}\right)^{\frac{1}{\gamma}}}{\left(\frac{\gamma-1}{2}\right)^{\frac{1}{2}} \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1}}}$$

where $P_9 = P_0$ is a constant but $P_{t,4}$ varies with time. This means the nozzle expansion ratio also varies with time. This moving boundary means that the previous equation is incorrect since it doesn't take into account the unsteady effects due to the changing nozzle ratio. The additional work term in the equation depends on $\partial A/\partial t$. Since the flow boundaries must change to ideally expand the flow, work may be done of the flow or the flow may be doing work on the boundaries. The effect of this work term is to change the exit velocity of the flow leaving the nozzle.

The main implication of this additional work term is that it is not possible to ideally expand an unsteady flow without work transfer to or from the fluid streamtube. For a single stream, this would mean a moving nozzle which is impractical on the time scales of most unsteady propulsion devices. Another possibility is a plug or aerospike nozzle concept where the work transfer is accomplished through the boundary between the streamtube and the atmosphere. An additional solution is to have multiple streams that transfer work between each other in the limit that they interact with each other but do not mix.

6 Discussion and Conclusions

The goal of this work is to carefully apply the correct form of the conservation laws to ensure that the thermodynamic analysis of unsteady propulsion systems conserves energy. An often overlooked aspect of unsteady flows is when to use the energy flux or enthalpy flux to determine the energy entering or exiting a control volume. For unsteady flows, the filling of the combustor with reactants and the removal of products is most easily done by having two control volumes representing the reactants and products. This means there is a moving control volume boundary between the reactants and products that cancels out the pressure work terms. This cancellation is the reason for using the energy flux over the enthalpy flux.

By recognizing that a detonation is spatially non-uniform, it is shown that a simple model representing an unsteady detonation wave in a tube can be integrated to correctly recover the energy flux out of the combustor. The model examined was for a Pulse Detonation Engine; however, the method applies to any unsteady combustor that is also spatially non-uniform. This extends the methodology of Paxson and Kaemming to combustion models more complicated than a spatially uniform, constant volume process.

Lastly, ideal expansion is shown to require time-dependent area variation of the streamtube leaving the combustor. This means that the quasi-steady approximation is incorrect even in the limit of a slowly varying flow. For a single stream, a moving nozzle is impractical. The question that arises from this analysis is that if the nozzle is fixed, what is the ideal fixed area ratio that maximizes the thrust? Another question is how does a plug nozzle interact with the flow and can the atmosphere be used to ideally expand an unsteady flow with work transfer? It is also possible to consider multiple streams that interact with each other in the limit of not mixing but still transferring work between the streams. This interaction would require solving the unsteady Euler equations with time-dependent area variation. The analysis could be done using either a traditional CFD code or with the method of characteristics. For an ideal, isentropic expansion, the exit pressure and temperature would not change since the states are related isentropically and work does not change that. The effect of the additional work term would be in determining the exit velocity which is needed for thrust calculations.

References

[1] D. E. Paxson and T. Kaemming, "Influence of unsteadiness on the analysis of pressure gain combustion devices," *Journal of Propulsion and Power*, vol. 30, no. 2, pp. 377–383, 2014.

[2] Z. U. Warsi, *Fluid dynamics: theoretical and computational approaches*. CRC press, 2005.

[3] S. Browne, J. Ziegler, and J. Shepherd, "Numerical solution methods for shock and detonation jump conditions," *GALCIT report FM2006*, vol. 6, p. 90, 2008.