

# Investigation on Damage Norm and Criterion of the Shock Wave in Underwater Explosion

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**Abstract** In this study, the applicability of various forms of power parameters including peak pressure, energy flux density, and impulse of the shock wave in underwater explosion as damage norm and criterion was first examined, and universal damage norm and criterion of the shock wave in underwater explosion were proposed. Based on the explosion similarity law, the limits of peak pressure, energy flux density, and impulse of the shock wave in underwater explosion as the damage norm and criterion were analyzed, and a general form of the damage power parameter was proposed as:  $W^n/R$ , in which  $W$  denotes the explosive charge,  $R$  denotes the explosive distance, and  $n$  is an undetermined coefficient. Through dimensional analysis, the functional relationship between  $W^n/R$  and structure damage was derived.  $W^n/R$  was proposed as a general form of damage norm and criterion of the shock wave in underwater explosion. Next, using infinite element analysis software AUTODYN, the effects of the shock wave in underwater explosion on two targets—circular plate and cylinder—were simulated, and the iso-damage curves in which different damage power parameters were used as the damage norm and criterion were plotted and compared. The results show that the proposed general form as the damage norm and criterion of the shock wave in underwater explosion is scientifically reasonable, universal, and practical to use and can also be regarded as a combined damage norm and criterion of peak pressure and impulse.

Keywords: Armament science and technology; underwater explosion; shock wave; damage norm and criterion; iso-damage curve

## 1 Introduction

Shock wave in underwater explosion is a core factor that damages target, and the damage norm and criterion of the target serves as the important basis of warhead power design and weapon damage evaluation[1]. The damage norms based on shock wave parameters such as peak pressure, impulse, and energy flux density[2] are now commonly used. Early scholars tended to use the peak pressure or energy flux density of the shock wave when it arrives at the vessel to describe impact environment and target damage degree, i.e., target suffers almost the same damage degrees under nearly same peak pressures of shock wave or energy flux densities[3][4]. For example, scholars from the former Soviet Union have divided vessel damage degree into three levels according to peak pressure of the shock wave, and scholars from the countries in North Atlantic Treaty Organization (NATO) have divided vessel damage degree into twelve levels based on the energy flux density[3~7]. Keil *et al.* [3]used the geometrical position of explosion and energy flux density of shock wave as the variables and first proposed the concept of impact factor based on the explosion geometrical position. The related results confirmed the functional relationship between the

plastic deformation of ship hull and the impact factors. Afterwards, a great deal of research[8~14] was conducted on application range and precision of the impact factor and acquired different forms of impact factor through modification. Assessment of damages on underwater target and underwater weapon damage effectiveness using impact factor has become a common practice in the past several decades.

However, whether a single shock wave parameter or the shock factor that was modified based on energy flux energy has certain theoretical defect cannot be neglected in underwater damage assessment. Energy flux density can be calculated by integrating the peak pressure with respect to time. When the two applied loads have same energy flux density, the variation curves of pressure with time are not necessarily the same, thereby resulting in different damages on the structure. The damage degree of the target not only depends on the combined action of load peak pressure and impulse[15], but also is greatly affected by the load waveform. The carrier waveform imposes non-negligible effect on structural response[16]. Since the Second World War, the damage assessment method based on P-I graph has been extensively applied for evaluating the damages on buildings, structures and

living creatures under the action of shock wave[17]. Using the load peak pressure and impulse, both the upper and lower limits of the damage degree were calculated for exploring the assessment method of the damage degree of the target under P–I combined load. For different materials (such as steel [18–22], aluminum[23], galss[24], concrete[25~30]) and different structures (such as plate[18][21], cylindrical shell[20][22], beam[19][23][30], slab[26][28][29]) under different boundary conditions(such as simply-supported [18][19], clamped[22][23]), the assessment models of structural damage degrees under different loads based on P–I graph method were derived.

An accurate and reasonable damage norm and criterion should comprehensively measure the relationship between the load pressure and duration. Therefore, in the first section of this study, based on the P–I graph theory, dimension analysis theory and the analysis of explosion similarity law[34], a general expression of the shock wave parameter in underwater explosion was derived as  $W^n/R$ , in which the  $n$  ranges for the parameters of peak pressure and impulse. Next, through dimensional analysis, the functional relationship between the general form of the shock wave parameter  $W^n/R$  and structural damage degree was established. It was also proved that the value of  $n$  as a coefficient of characterizing the parameters of shock wave, pressure and impulse, has one-to-one correspondence with the target's structural damage degree. In the third section, a general expression of damage norm and criterion,  $C = W^n/R$ , is proposed. According to numerical simulation results, using  $W^n/R$ , the parameter for characterizing the power of the shock wave in underwater explosion, as the damage norm and criterion, is reasonable and universal.

## 2 Methods

### 2.1 Characterization of damage power of the shock wave in underwater explosion

Damage norm can be described as a specific form of the characteristic parameter (or the derived quantity) of the selected damage power, which is equivalent to an independent variable of the function. Based on the definition of damage law and damage norm, the damage criterion can be defined as the specific function value of the damage law, i.e., the value or the value range of the independent variable at a certain target

damage probability[32].

Currently, the damage norm of the shock wave in underwater explosion generally adopts the specific expressions of peak pressure, energy flux density, and impulse of the shock wave. Based on the dimensional analysis theory and explosion similarity law, the peak pressure, energy flux density, and impulse of the shock wave when the condensed explosive explodes in water can be written as[31][33]:

$$P_m = k_1 \left( \frac{W^{1/3}}{R} \right)^\alpha \quad (1)$$

$$I(t/W^{1/3}) = k_2 W^{1/3} \left( \frac{W^{1/3}}{R} \right)^\beta = k_2 \left( \frac{W^{(1+\beta)/3\beta}}{R} \right)^\beta \quad (2)$$

$$E_f(t/W^{1/3}) = k_3 W^{1/3} \left( \frac{W^{1/3}}{R} \right)^\gamma = k_3 \left( \frac{W^{(1+\gamma)/3\gamma}}{R} \right)^\gamma \quad (3)$$

where  $W$  denotes the explosive charge, with a unit of kg, and  $R$  denotes the explosive distance, with a unit of m. Table 1 lists the shock wave parameters of some typical explosives[31].

Tab.1 Shock wave similar parameters of some explosives[31]

Explosives	$\alpha$	$\beta$	$\gamma$
TNT	1.13	0.89	2.05
Loose Tetryl	1.15	0.98	2.10
Pentolite	1.13	1.05	2.12
HBX-1	1.14	0.86	2.04
PENT	1.19	0.90	2.09
NUCLEAR	1.13	0.91	2.04

According to Eqs. (1)–(3), for a certain type of explosive, if the calculated results of  $W^{1/3}/R$ ,  $W^{(1+\beta)/3\beta}/R$ , and  $W^{(1+\gamma)/3\gamma}/R$  under different shock wave loads are identical, the power parameters under different loads are also the same. Therefore, the power parameters of the shock wave, including peak pressure, impulse, and energy flux density, can be characterized by  $C_p$ ,  $C_I$ , and  $C_E$ , respectively. The following expressions can be derived:

$$C_p = \frac{W^{N_p}}{R} \quad (N_p = 1/3) \quad (4)$$

$$C_I = \frac{W^{N_I}}{R} \quad (N_I = (1 + \beta)/3\beta) \quad (5)$$

$$C_E = \frac{W^{N_E}}{R} \quad (N_E = (1 + \gamma)/3\gamma) \quad (6)$$

The general expression of the power parameters of the shock wave in underwater explosion can be written as:

$$C_N = \frac{W^N}{R} \quad (7)$$

The current damage norm of the shock wave in underwater wave can be regarded as a particular case of Eq. (7). Using the expression of the peak pressure norm  $C_p$ , the value of the coefficient  $N_p$  is 1/3. As regard to the expression of impulse norm  $C_I$ , the value of the coefficient  $N_I$  fluctuates around 0.7. Using the expression of the shock factor norm  $C_E$ , the coefficient of  $N_E$  also changes for different types of explosives; however, for the convenience of application,  $N_E$  is generally set as 0.5. Table 2 lists the  $N_p$ ,  $N_I$ , and  $N_E$  values for different explosives.

Tab.2  $N_p$ ,  $N_I$ ,  $N_E$  of some explosives

Explosives	$N_p$	$N_I$	$N_E$
TNT	0.33	0.71	0.496
Loose Tetryl	0.33	0.67	0.492
Pentolite	0.33	0.65	0.491
HBX-1	0.33	0.72	0.497
PENT	0.33	0.70	0.493
NUCLEAR	0.33	0.70	0.497

When performing assessment on target damage degree using any a form of damage norm ( $C_p$ ,  $C_I$ , and  $C_E$ ), target damage degrees are identical if the  $C_N$  values are same under different operating conditions. However, the assessment criterion based on  $C_p$  norm neglects temporal characteristics of the shock wave pressure, which can be approximately treated as quasi-state loading, the assessment criterion based on  $C_I$  does not take into account the dynamic effect of instantaneous high pressure of the shock wave, and the assessment based on  $C_E$  norm neglects the waveform characteristics of pressure-time curve and pressure and time, finally leading to certain errors from the actual damage results[34][35].

Base on that, we proposed to use  $C_N = W^N/R$  as a general expression of damage power of the shock wave in underwater explosion. In the above general expression, the coefficient N is affected by the pulse load waveform, pressure, time, target structure characteristics, and deformation severity degree, but the value should not exceed the peak pressure  $N_p$  and the impulse  $N_I$ . Since the  $N_I$  values are different under different explosive charging conditions, the  $N$  value also changes. For example, the value of n for TNT ranges from 1/3 to 0.708. Finally, the expression of damage power parameters of the shock wave in underwater explosion can be written as:

$$C = \frac{W^n}{R} \quad (N_p \leq n \leq N_I) \quad (8)$$

## 2.2 Analysis of the similarity of structural response under the shock wave

When the target structure, boundary constraint, explosive type, and water environment remain unchanged, the intensity of the shock wave in underwater explosion can be varied by changing explosive charge and explosion distance, and therefore, the damage degree of the target structure is only determined by the explosive charge  $W$  and the explosion distance  $R$ . When the structural response  $X$  is used for characterizing the damage degree of the target structure,  $X$  can be expressed as:

$$X = f(W, R) \quad (9)$$

Since the damages caused by underwater explosion have quite complex and difficult physical processes, strong correlations of  $W$  and  $R$  with  $X$  can hardly be analytically explained by the related theories. Dimensional analysis can reveal constitutive relationships among various physical quantities to a certain degree. When the target structure is damaged by the shock wave, the physical parameters that affect the characterization parameter  $X$  of damage degree are explosive charge  $W$ , explosive charging density  $\rho_e$ , chemical energy of unit mass of explosive  $Q_v$ , the expansion index of the detonation products  $\gamma$ , the explosion distance  $R$ , initial density of water  $\rho_0$ , initial hydrostatic pressure  $p_0$ , the index of water equation of state  $n$ , the characteristic dimension of target structure  $L$ , the density of the structure material  $\rho_s$ , the elastic modulus  $E$ , Poisson's rate  $\nu$ , and the dynamic yield limit of the material  $\sigma_d$ . Therefore,  $X$  can be expressed as:

$$X = f(W, \rho_e, Q_v, \gamma, R, \rho_0, p_0, n, L, \rho_s, E, \nu, \sigma_d) \quad (10)$$

Assuming that  $W$ ,  $\rho_e$ , and  $Q_v$  are the fundamental quantities, Eq. (10) can be rewritten as the following dimensionless function:

$$X = f\left(\gamma, \frac{R}{(W/\rho_e)^{1/3}}, \frac{\rho_0}{\rho_e}, \frac{p_0}{\rho_e Q_v}, n, \frac{L}{(W/\rho_e)^{1/3}}, \frac{\rho_s}{\rho_e}, \frac{E}{\rho_e Q_v}, \nu, \frac{\sigma_d}{\rho_e Q_v}\right) \quad (11)$$

In Eq. (11), only  $W$  and  $R$  are variables and Eq. (11) can be written as:

$$\left(\gamma, \frac{\rho_0}{\rho_e}, \frac{p_0}{\rho_e Q_v}, n, \frac{\rho_s}{\rho_e}, \frac{E}{\rho_e Q_v}, \nu\right) = const \quad (12)$$

Since the hardening characteristic of the strain rate of the material, the dynamic yield limit  $\sigma_d$  is not a constant, the term  $\sigma_d/(\rho_e Q_v)$  in Eq. (12) is not only

affected by the material's constitutive relation. However, for a given target structure, fixed explosive charge  $W$  and explosion distance  $R$ , the material's constitutive relation can be uniquely determined, and therefore the damage degree of the target structure can only be determined. Eq. (12) can be further simplified as:

$$\frac{X}{(W/\rho_e)^{1/3}} = f\left(\frac{R}{(W/\rho_e)^{1/3}}, \frac{L}{(W/\rho_e)^{1/3}}\right) \quad (13)$$

where  $r_e = (W/\rho_e)^{1/3}$  denotes the characteristic dimension of explosive charge. Eq. (13) suggests that the damage degree of the target  $X$  is mainly connected to the scaled distances  $R/r_e$  and  $L/r_e$ . In other words, the ratio of the characteristic size of the target structure  $L$  to the characteristic size of explosive charge  $r_e$  also affects the degree of final structural deformation. Eq. (13) can further be rewritten as:

$$X = \frac{W^{1/3}}{\rho_e^{1/3}} f\left(\rho_e^{1/3} \left(\frac{w^{1/3}}{R}\right)^{-1}, \rho_e^{1/3} \left(\frac{w^{1/3}}{L}\right)^{-1}\right) \quad (14)$$

According to Eq. (14), when the characteristic dimension of the target far exceeds or is far below the explosive charging radius, the shock wave can be approximately regarded as the spherical shock wave of a point explosive source or the plane wave, and therefore the effect of  $r_e/L$  on the characterization parameter  $X$  of the damage degrees of target structure can be ignored. When the characteristic length of the target is close to characteristic explosive charging size, the target structure suffers from different modes of damages under the shock waves of different shape characteristics, and  $X$  is strongly correlated to  $r_e/L$  under different modes of damages, as described previously[36]. Since the characteristic lengths of common underwater targets are generally far greater than the explosive charging radius, the effect of the characteristic length of the target structure is always neglected in the present model design. Accordingly, Eq. (14) can be approximated as:

$$X = l_1 W^{1/3} f\left(\frac{w^{1/3}}{R}\right) \quad (15)$$

By comparing Eq. (15) with Eqs. (2) and (3), the functional expression of the damage degree  $X$  of the target structure was found to be similar to the expression of power of the shock wave, and the damage degree is directly related to the power of the shock wave. Therefore, the damage degree  $X$  of the target structure can be characterized by the following expression:

$$X = kW^{1/3} \left(\frac{W^{1/3}}{R}\right)^\varphi = k \left(\frac{W^{(1+\varphi)/3\varphi}}{R}\right)^\varphi \quad (16)$$

According to the derivations in Sections 1 and 2, the expression  $C = W^n/R$  not only can represent the power of the explosive shock wave, but also characterizes the damage degree of the object structure. It is feasible to use the  $C = W^n/R$  expression as the damage norm of the shock wave in underwater explosion, in which the  $n$  value determines the form of damage norm under certain damage degree. The value of  $n$  is connected to the material properties, structure, and boundary constraint conditions. For different materials with different structures, the  $n$  value differs greatly under different boundary constraint conditions. Further, the values of  $n$  are different under different damage degrees keeping other conditions the same. When the parameters of  $C$  under different explosive charging amounts and explosion distances are same, the characterization parameters of damage degrees, i.e., the values of  $X$  are identical.

### 2.3 Calculation of damage norm and criterion of the shock wave

According to the derivations in the above sections, a novel damage norm and criterion was proposed in this study for assessing the damage degrees of the target structure under the shock wave in water explosion. The proposed damage norm and criterion can be expressed as:

$$X_i: C_i = \frac{W^{n_i}}{R} \quad (1/3 < n_i < N_i) \quad (17)$$

in which  $X_i$  is the indicator of the damage on the target structure (such as the deflection and elastic/plastic strain), and  $C_i$  denotes the damage criterion corresponding to  $X_i$ .

The damage norm and criterion can be calculated below. As shown in Fig. 17, after the selection of the indicator of target damage degree  $X$ , if there exist several damage degrees  $X_1 < X_2 < X_3 \dots < X_i$ , any  $X_i$  corresponds to a  $(n_i, C_i)$ . Different associations of variables in Eq. (17), denoted as  $(W, R)$ , can damage the target structure to different extents. If the target structure suffers from the damage with the degree of  $X_1$  under the action of the explosive shock wave in the operating conditions  $(W_1, R_1)$  and  $(W_2, R_2)$ , the following expression can be derived:

$$C_1 = \frac{W_1^{n_1}}{R_1} = \frac{W_2^{n_1}}{R_2} \quad (1/3 < n_1 < N_i) \quad (18)$$

According to Eq. (18), the following expression can be derived:

$$lgC_1 = n_1 lgW_1 - lgR_1 \quad (19)$$

$$lgC_1 = n_1 lgW_2 - lgR_2 \quad (20)$$

The parameter of damage norm, denoted as  $n_1$ , can be calculated as:

$$n_1 = \frac{lgR_1 - lgR_2}{lgW_1 - lgW_2} \quad (21)$$

### 3 Numerical study

#### 3.1 Numerical model

Using the finite element software AUTODYN, dynamic responses of different typical targets under the shock wave in underwater explosion were simulated, during which the finite element models of a circular plate and a cylinder were established. Fluid-solid coupling algorithm was used in the present simulation, and the explosive load was mapped to water area with the use of remap method. Figs. 1 and 2 display the finally established numerical simulation.

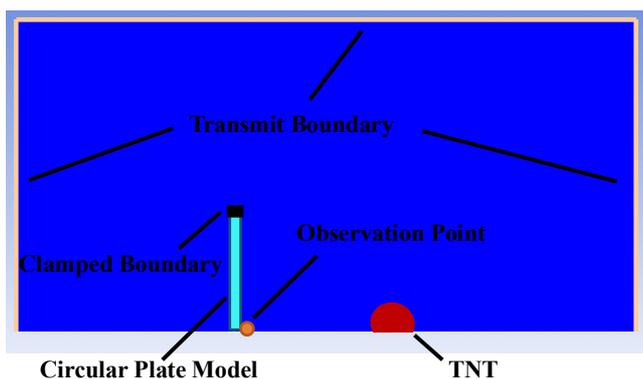


Fig.1 Finite element model of circular plate

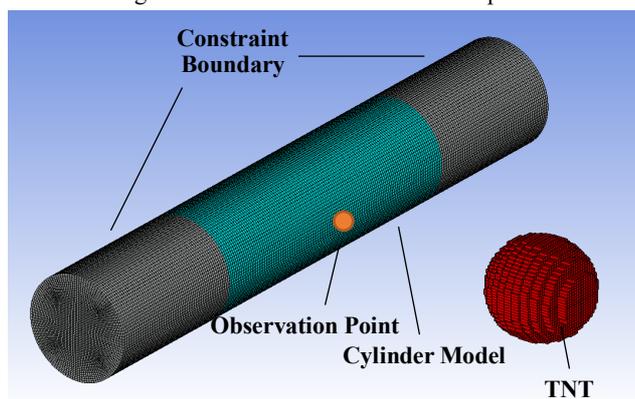


Fig.2 Finite element model of cylinder

##### 3.1.1 Geometrical parameters

Geometrical parameters of circular plate: a two-dimensional axially-symmetrical model was established, in which the range of water area was 2000 mm × 1000 mm, the mesh size of water area was set as 1 mm, and the radius and thickness of the circular plate

were 300 mm and 3 mm, respectively, and the mesh size of the circular plate was 1 mm.

Geometrical parameters of cylinder: a three-dimensional planar-symmetry model was established, in which the size of the water area was 1000×1000×2400mm<sup>3</sup>, and the diameter, length, and thickness of the cylinder were 200, 300, and 4 mm, respectively. Fine meshes were generated. The size of the meshes of water area around the cylinder and the cylinder was 3 mm. The cylinder was filled with air.

##### 3.1.2 Boundary conditions

In the established numerical model of circular plate, the edge of the circular plate was set as *Clamped Boundary* for simulating the clamping state, while the boundary of water area in the model was set as *Transmit Boundary* for simulating the infinite water area. In the established numerical model of cylinder, the cylinder was connected to fixed accessories to simulate the clamped boundary condition, and the boundary of water in the model was also set as *Transmit Boundary*.

##### 3.1.3 Numerical model matrix

According to the application range of the similarity law of underwater explosion (7–900 times greater than the explosive charging radius), explosive charge was set as 50, 100, 200, 400, and 800 g. Under each explosive charging condition, the explosion distance was set as 7, 10, 13, 16, 18, and 20 times greater than the explosive charging radius.

##### 3.1.4 Material properties

The material properties were derived from the AUTODYN material library and literature[37], as shown in Table 3.

Tab.3 Material properties adopted in simulation

Material <sup>[5]</sup>	EOS Equation	Strength Model	Failure Model
TNT-2	JWL	/	/
Water	Polynomial	/	/
Steel Q235	SHOCK	JOHNSON COOK	JOHNSON COOK

### 3.2 Numerical results

The deformation of targets during underwater explosion was determined by the combined action of the impact of after flow and the collapse or fluctuation of bubbles rather than only the shock wave. Load generation phase, the action phase on the target, and the deformation phase of the target occurred at different moments, and therefore the damage on the target only under the action of shock wave can be obtained by analyzing the historical curves of related physical

parameters with respect to time. Notably, the effects of different damage factors on target damages were coupled and difficult to distinguish. Therefore, the operating condition corresponding to the coupled damage effect was not taken into account in analyzing the simulation results

Permanent deformation of the target is an important damage representation for the assessment of underwater explosion damage and can also serve as the basis in the design of the damage effect of underwater ordnance. Through numerical calculation, the variation rules of the deflections at the geometrical centers of both circular plate and cylinder under different operating conditions were concluded for characterizing the damage to the target. Figs. 3–5 display the numerical simulation results of typical models. As shown in Figs. 3 and 4, the geometrical centers and boundaries of both the cylindrical plate and the cylinder were seriously damaged. Under the action of the shock wave, shear force and tensile force were generated and acted on the edge of the structure, thereby causing great deformation at the edge. The geometrical center of the structure exactly pointed towards the explosion center and was subjected to the concentration of stress, and therefore maximum deformation appeared at the center. As shown in Fig. 5, the deformation of the structure can be divided into three phases: simultaneous increase in the elastic deformation and plastic deformation to peak, gradual decrease in the elastic deformation, and no significant change of the deflection. After the permanent deformation of the structure center became stable, the corresponding deflections of the centers were recorded, as the results are listed in Tables 4 and 5.

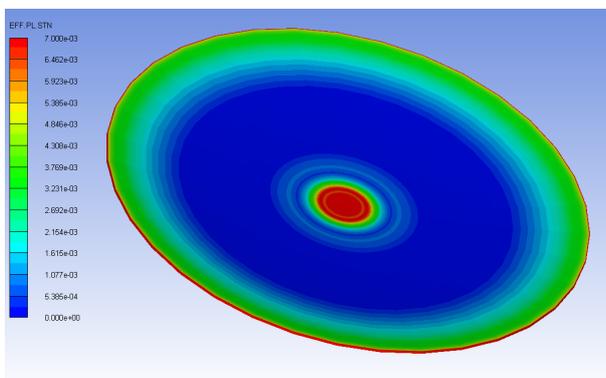


Fig.3 Numerical calculation result of circular plate

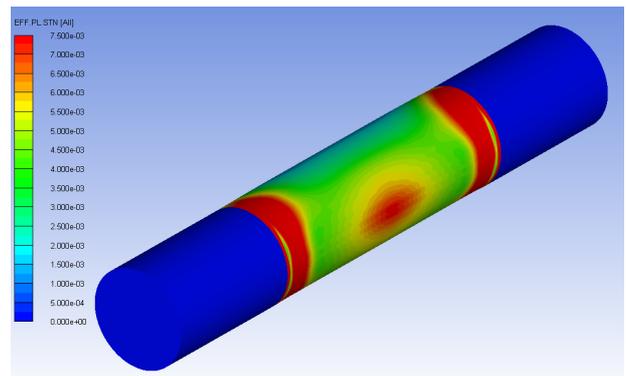


Fig.4 Numerical calculation result of cylinder

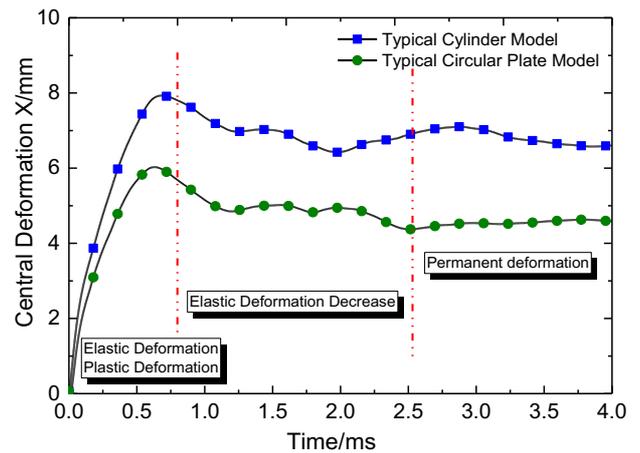


Fig.5 Deformation-time curve of target geometric center

Based on the variation rules of the center deflection as described in Tables 4 and 5, different operating conditions corresponding to iso-damage degrees were explored by means of linear interpolation and data fitting. Five typical deflections at the geometrical centers of the circular plate and the cylinder were selected for characterization and the validation of the results, finally affording the explosion distances under iso-damage matrices of the circular plate and the cylinder when the explosive charge was set at different values, as listed in Tables 6 and 7.

Tab.4 Deformation calculation result of circular plate

Deflection (X/mm)		Charge weight (w/g)				
		50	100	200	400	800
Charge distance (n times of charge radius)	7	31.98	36.02	38.98	42.52	45.97
	10	22.50	24.61	25.53	27.06	30.02
	13	15.90	16.62	17.13	18.85	21.31
	16	11.11	11.70	12.23	14.03	16.59
	18	8.97	9.46	10.22	11.76	12.37
20	7.25	7.82	8.60	10.19	10.45	

Tab.5 Deformation calculation result of cylinder

Deflection (X/mm)		Charge weight (w/g)				
		50	100	200	400	800

Charge distance (n times of charge radius)	7	43.66	-	-	-	-
	10	18.68	20.01	21.04	24.63	28.98
	13	9.11	9.99	11.32	12.06	14.48
	16	5.00	6.05	6.40	8.11	9.54
	18	3.66	4.56	4.92	6.14	7.42
	20	2.86	3.52	4.03	4.78	5.95

Tab.6 Iso-damage matrix of circular plate

Charge distance (R/mm)	Charge weight (w/g)					
	50	100	200	400	800	
Central	30	146.5	205.9	271.6	362.4	496.2
Deflection	25	176.5	240.7	312.0	413.9	566.7
(X/mm)	20	213.5	284.2	364.0	482.7	658.5
	15	261.6	342.0	436.8	586.3	789.8
	10	330.7	428.7	559.7	760.3	1023.6

Tab.7 Iso-damage matrix of cylinder

Charge distance (R/mm)	Charge weight (w/g)					
	50	100	200	400	800	
Central	30	158.5	204.7	260.1	359.4	482.6
Deflection	25	172.1	222.4	284.7	386.1	518.1
(X/mm)	20	188.9	244.6	315.5	420.0	564.0
	15	211.3	274.3	356.6	467.0	628.9
	10	244.5	319.6	418.9	543.5	740.4

## 4 Analysis and discussion

### 4.1 Calculation of damage norm and criterion of the shock wave

The results in Tables 6 and 7 were plotted to iso-deflection curves of the explosion distance with the explosive charge under logarithmic coordinate system, as shown in Figs. 6 and 7. Overall, the iso-deflection curves exhibited favorable linearity and the slopes of the curves differed slightly, further confirming an objective expression  $C = W^n/R$  or  $lgR = nlgW - lgC$  for accurately judging the damage degree of the target.

The simulation results in Tables 6 and 7 were calculated according to the calculation method as described in Section 2.3, and the damage norms and criteria of different targets under the action of underwater shock wave were concluded, as listed in Table 8. Apparently, the damage norm parameter  $n$  exhibited certain difference among different targets, which was connected to target structure, test environment, damage results, and test conditions. Therefore, different damage degrees of the target, as described in Table 8, corresponding to different damage norms and criteria, i.e., the proposed method can more

accurately assess the damage degree of the target.

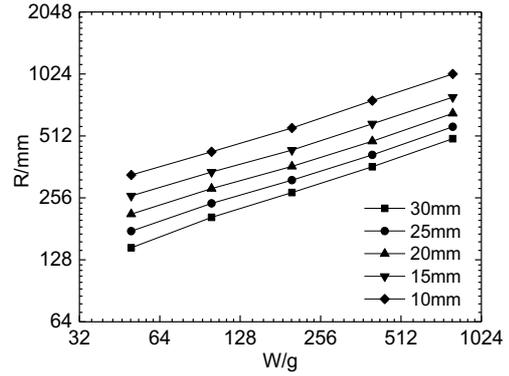


Fig.6 Iso-damage curve of circular plate

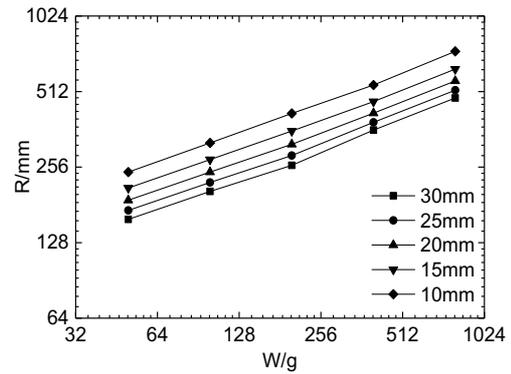


Fig.7 Iso-damage curve of cylinder

Tab.8 Damage norm and criterion of plate and cylinder

		Circular Plate	Cylinder
Central deflection (X/mm)	30	$C_{30} = W^{0.435}/R = 1.83$	$C_{30} = W^{0.402}/R = 1.93$
	25	$C_{25} = W^{0.416}/R = 1.63$	$C_{25} = W^{0.398}/R = 1.80$
	20	$C_{20} = W^{0.402}/R = 1.41$	$C_{20} = W^{0.394}/R = 1.65$
	15	$C_{15} = W^{0.397}/R = 1.18$	$C_{15} = W^{0.392}/R = 1.48$
	10	$C_{10} = W^{0.406}/R = 0.91$	$C_{10} = W^{0.392}/R = 1.27$

### 4.2 Comparison between different damage norm and criterion

The damage norms and criteria for the circular plate and the cylinder under the action of underwater shock wave, as proposed in previous Section, were then compared with the damage norm and criterion based on a single shock wave parameter. The iso-damage curves of the circular plate and the cylinder were plotted based on the results in Tables 6 and 7. The iso-damage curves of the circular plate based on the damage norm of peak pressure, energy flux density, and impulse are shown in Figs. 8, 9, and 10, respectively, whereas the iso-damage curves of the cylinder based on the damage norm of peak pressure, energy flux density, and impulse are shown in Figs. 11, 12, and 13, respectively, in which the vertical coordinate represents the deflection deformation and the transverse coordinate represents

the power characterization parameter of the shock wave  $C_p$ ,  $C_I$ , or  $C_E$ .

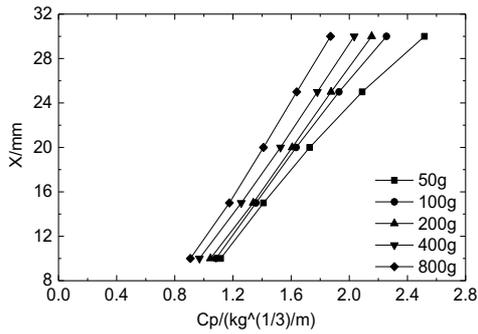


Fig.8 Iso-damage curve of circular plate under peak pressure

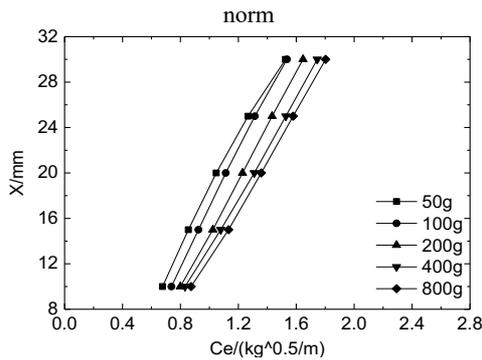


Fig.9 Iso-damage curve of circular plate under energy flux density norm

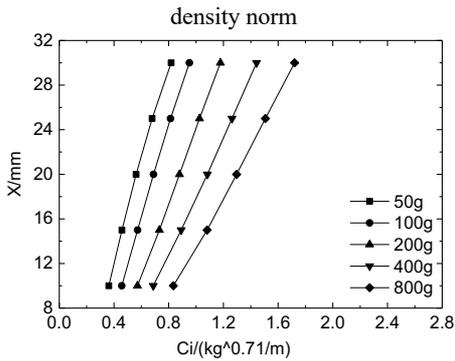


Fig.10 Iso-damage curve of circular plate under impulse norm

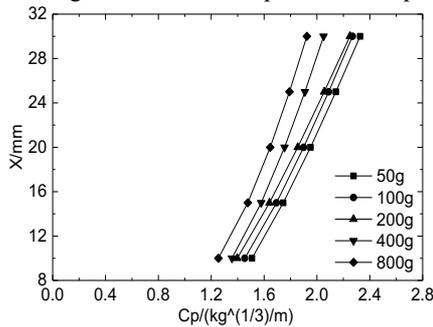


Fig.11 Iso-damage curve of cylinder under peak pressure norm

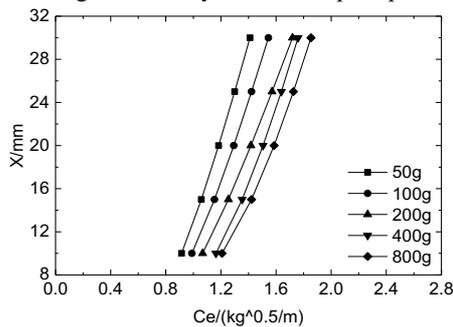


Fig.12 Iso-damage curve of cylinder under energy flux density norm

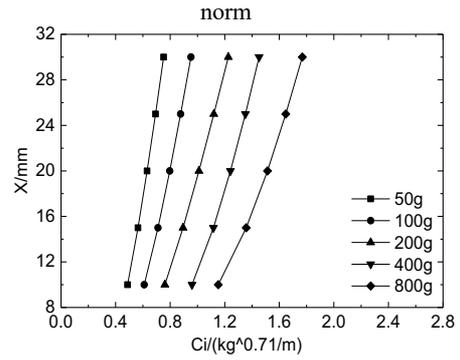


Fig.13 Iso-damage curve of cylinder under impulse norm

When underwater shock wave imposed certain damage  $X$  on the target structure, the corresponding damage criterion should be unique, i.e., the calculated results of  $C_p$ ,  $C_I$ , or  $C_E$  under different iso-damage operating conditions should equal to each other. As shown in Figs. 8–13, the data points corresponding to same damage degrees were scattered. At the same peak pressure, the damage degrees were different, because of the difference in impulse or energy flux density. Under same impulse or energy flux density, the damage degrees also differed at different peak pressures. Accordingly, using a single power characterization parameter of the shock wave as the damage norm and criterion cannot accurately represent and assess the damage degree of the structure under the action of underwater shock wave.

The damage norm and criterion as listed in Table 8 was plotted into iso-damage curves shown in Figs. 14 and 15. In contrast with Figs. 8–13, the proposed damage norm and criterion in this study exhibited lower data dispersion degree under different damage degrees. Accordingly, it can be concluded that the proposed damage norm and criterion was more accurate for accessing the target damage than the other norms, with less error. In order to quantify the difference among different damage assessment norms, mean square errors in the assessment of the damage degrees of different target structures using different damage assessment norms were calculated, as listed in Table 9.

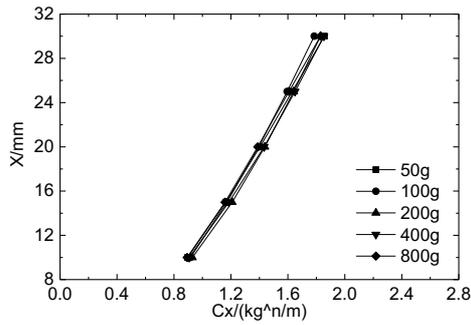


Fig.14 Iso-damage curve of circular plate under damage norm in this paper

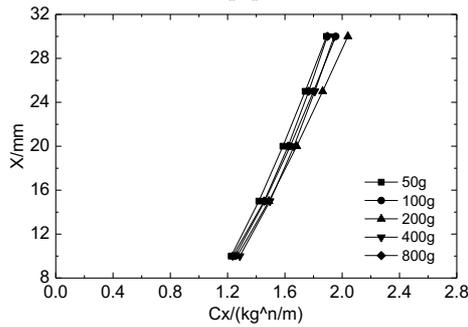


Fig.15 Iso-damage curve of cylinder under damage norm in this paper

Tab.9 Standard deviation of different damage norm

Standard deviation	Peak pressure norm	Shock factor norm	Impulse norm	norm In this paper
Circular Plate	0.13	0.10	0.26	0.02
Cylinder	0.11	0.14	0.31	0.04

### 4.3 Plotting and analysis of P-I graphs

As stated above, the proposed characterization parameters of damage power of underwater shock wave and the related damage norm and criterion can be essentially regarded as the combined norm of peak pressure and impulse. Fig. 16 displays typical P-I graphs consisting of iso-damage curves (or referred to as iso-damage-mode curves, the vertical asymptotic line of pulse, and quasi-static asymptotic line[38]. The damage degrees at any point in the iso-damage line were totally identical; however, the loads that caused the damages were not necessarily same. According to the difference in characteristics, the load can be classified as pulse load, dynamic load, and quasi-static load[39]. There exists no obvious boundary among different loads, but two asymptotic lines specified both upper and lower limits of peak pressure and impulse of the suffered loads, respectively.

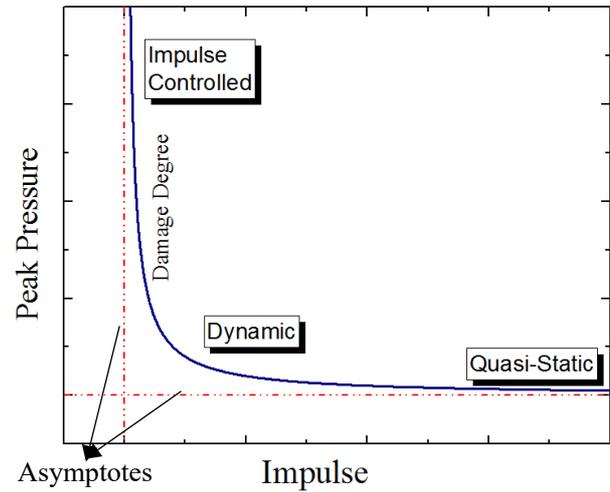


Fig.16 Typical P-I diagrams

According to the damage norm and criterion in Table 8, P-I graphs of the circular plate and the cylinder under the action of underwater shock wave were plotted, as shown in Figs. 17 and 18. For a direct and clear representation, the curves at two typical conditions ( $X = 10 \text{ mm}$  and  $X = 30 \text{ mm}$ ), corresponding to light damage degree and heavy damage degree, were plotted. It can be easily observed that the iso-damage curve plotted based on the proposed damage norm and criterion in this study exhibited identical variation tendency with the P-I graphs in Fig. 16, exhibiting the characteristics of inverse proportional function and had to asymptotic lines. It can be confirmed that the proposed damage norm and criterion is the combined norm of peak pressure and impulse. Moreover, by labeling the calculated values of peak pressure and impulse as listed in Tables 6 and 7, the damage data points were coincident with the iso-damage curves, further proving the accuracy of the data.

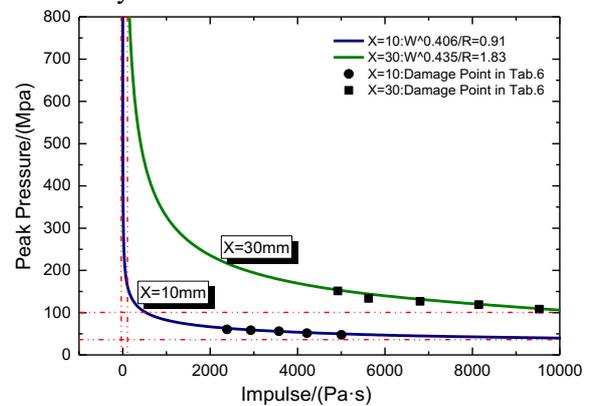


Fig.17 P-I diagrams of circular plate under shock waves in water

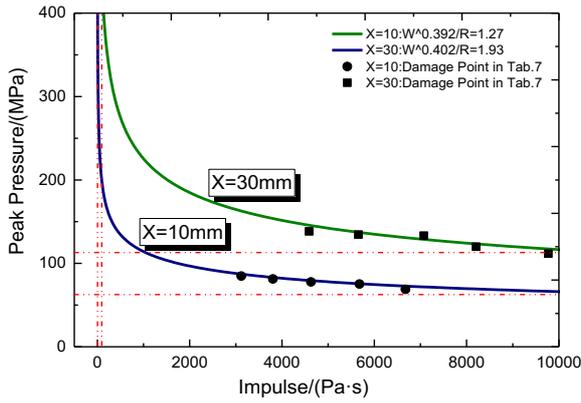


Fig.18 P-I diagrams of cylinder under shock waves in water

However, the marked damage points in Figs. 17 and 18 were concentrated on the iso-damage curves, which were determined by the similarity law of underwater explosion and the experimental conditions. According to the similarity law of the explosion in underwater explosion, the application range of the explosion distance was approximately 7–900 times greater than the explosive charging radius, and the corresponding peak pressure ranged from 0.68 MPa to 170 MPa. Therefore, the vertical coordinates of the damage points were in the range 0–200 MPa. Moreover, the operating conditions under the shock wave with great peak pressure and small impulse was close to that of contact explosion, while the operating condition under the shock wave with small peak pressure and great impulse was close to large-dosage explosion at a great distance. The proposed damage norm and criterion was not applicable to the first case. With regard to the latter case, both test and numerical calculations were difficult. Therefore, the proposed damage norm and criterion has certain application range, which completely relies on the test and exhibits objective constraints.

Finally, based on the P–I graphs in Fig. 18, the damage modes of the typical targets were plotted, as shown in Fig. 19. Different damage modes and degrees are marked in different colors. According to Fig. 18, the damage modes can be classified as Damage Model I, Damage Model II, and Damage Model III, corresponding to slightly plastic deformation, moderately plastic deformation, and heavily plastic deformation or even failure, respectively. When the calculated data points of peak pressure and impulse under different operating conditions fell into a plot in a certain color, the target suffered from the corresponding damage mode or the damage degree. The above

conclusion will be further validated by combining the test and numerical calculation results for verification.

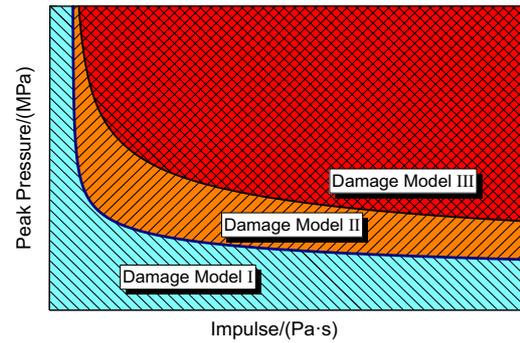


Fig.19 Damage mode based on P-I diagrams

## 5 Conclusions

The conclusions of this study are as follows.

- 1) In underwater explosion, the power parameters including peak pressure, energy flux density, and impulse cannot interpedently used as the damage norm and criterion. The reasons are described below. At the same peak pressure, the damage degrees are different because of different energy flux densities or impulses. Under the same energy flux density or impulse, the degree of damage also differs at different peak pressures.
- 2) A general form of power parameter  $W^n/R$  as the damage norm and criterion exhibits scientific reasonability and favorable universality and feasibility.
- 3)  $W^n/R$  can be viewed as a combined damage norm and criterion of peak pressure and impulse, in which  $n$ , ranging from  $1/3$  to  $N_i$ , varies among different targets under different damages.

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