# Testing a Reduced Model of Flame Spread over a Thin Solid in a Narrow Channel

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## 1 Introduction

Flame spread characteristics near the extinction limit are often different from those far from extinction. It is therefore important from the fire safety viewpoint to understand flame spread characteristics near the extinction limit. A tiny flame is often observed under a near-limit condition, possibly leading to a different flame-spread mechanism, thereby necessitating a new modelling approach.

An example of near-limit phenomenon is flame spread in a narrow channel. A number of experiments have been conducted [e.g., 1–3], among which the experiments by Zik and Moses [1] have been a subject of later modelling studies [4]. Zik and Moses burned a thin paper sheet in a narrow channel and observed the formation of interesting finger-like patterns. Most models of the Zik-Moses experiments adopted reduction techniques to reduce the original 3-D (or 2-D) governing equations into 2-D (or 1-D). Further, it is often assumed that the gas and the solid phases are in thermal equilibrium. These assumptions are expected not to cause serious errors because of the narrowness of the domain in consideration. However, it has been rarely tested whether the reduced models yield similar results to the original ones.

The objective of this study is to test the validity of a reduced 1-D model. A 2-D model is first developed, which is then reduced to 1-D. Predictions of the 1-D model are compared with those of the 2-D model under various conditions to test the validity of the 1-D model from the viewpoint of distributions of temperature and species mass fractions as well as the flame spread rate.

## 2 Numerical Model

This section describes the present numerical model, which is to simulate previous narrow-channel experiments [1-3]. In the previous experiments, a thin solid such as a cellulosic sheet was burned in a narrow

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gap between two parallel plates. The Zik-Moses experiments [1] are often referred to as smouldering spread because visible flames were not observed. However, blue flames can be observed [5] in dark environments under similar conditions to Ref. [1]. The Zik-Moses experiments are therefore treated as gas-phase flame spread in this study. The height of narrow-channel in the previous experiments is typically a few millimetres. An oxidizer flow is usually supplied in the opposite direction to flame spread because extinction occurs without the forced oxidizer flow under near-limit conditions.



Figure 1. A schematic diagram of the 2-D numerical model.

Figure 1 schematically shows the 2-D model. The following are major assumptions adopted in this study: (1) Thermal decomposition of the fuel solid produces a char and a combustible gas by Reaction 1, fuel solid  $\rightarrow v_{1,g}$  fuel gas +  $(1 - v_{1,g})$  char. (2) The fuel gas reacts with oxygen in the gas phase to produce gaseous products by Reaction 2,  $v_{2,F}$  fuel gas + oxygen  $\rightarrow (v_{2,F} + 1)$  gaseous products. (3) All the material properties including the gas-phase density are constant. (4) The oxidizer flow is uniform in the gas phase. (5) Heat loss to the upper plate is considered using a heat transfer coefficient. (6) The solid phase, which consists of the fuel solid and the char, is thermally thin, and its thickness is constant.

Based on these assumptions, the energy and mass conservation equations for the solid phase that serve as boundary conditions for the gas phase are derived.

$$\frac{\partial \rho_{\mathrm{F,S}}}{\partial t} = -\omega_1 \tag{1}$$

$$c_{\rm s}\frac{\partial\rho_{\rm s}T_{\rm s}}{\partial t} = \lambda_{\rm s}\frac{\partial^2 T_{\rm s}}{\partial x^2} + \frac{2q_{\rm g,s}}{d_{\rm s}} + Q_1\omega_1, \qquad \rho_{\rm s} = \rho_{\rm F,s,0} - \nu_{\rm 1,g}(\rho_{\rm F,s,0} - \rho_{\rm F,s})$$
(2)

where  $q_{g,s} = \lambda_g (\partial T_g / \partial z)_{z=0}$  is the heat flux from the gas phase, and  $\omega_1 = B_1 \rho_{F,s} e^{-E_1/RT_s}$  is the rate of reaction 1 ( $\rho$  is the density, c is the specific heat,  $\lambda$  is the thermal conductivity, d is the thickness, Q is the heat of reaction,  $\nu$  is the stoichiometric coefficient, B is the pre-exponential factor, E is the activation energy, R is the universal gas constant, and subscripts F and s denote the fuel and the solid phase, respectively).

The 2-D energy and species (i = F, 0) conservation equations in the gas phase are as follows:

$$\rho_{\rm g} c_{\rm g} \frac{\partial T_{\rm g}}{\partial t} + \rho_{\rm g} c_{\rm g} u \frac{\partial T_{\rm g}}{\partial x} = \lambda_{\rm g} \left( \frac{\partial^2 T_{\rm g}}{\partial x^2} + \frac{\partial^2 T_{\rm g}}{\partial z^2} \right) + Q_2 \omega_2 \tag{3}$$

$$\rho_{\rm g} \frac{\partial Y_i}{\partial t} + \rho_{\rm g} u \frac{\partial Y_i}{\partial x} = \rho_{\rm g} D \left( \frac{\partial^2 Y_i}{\partial x^2} + \frac{\partial^2 Y_i}{\partial z^2} \right) - \nu_{2,i} \omega_2, \qquad (i = \mathrm{F}, \mathrm{O})$$
(4)

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where  $\omega_2 = B_2 \rho_g^2 Y_F Y_0 e^{-E_2/RT_g}$  is the rate of reaction 2 (*u* is the oxidizer flow speed, *D* is the diffusivity, and subscripts g denotes the gas phase, respectively). Boundary conditions are summarized in Fig. 1. The value of the Nusselt number to account for heat loss at the upper plate is fixed at 4.0, in the 2-D simulations. When solving the 2-D system, an ignition source at 2000 K is placed near the solid surface for a time period of 0.5 s as shown in Fig. 1.

1-D governing equations are derived by integrating the 2-D equations (3) and (4) in the vertical (z) direction.

$$\rho_{\rm g} c_{\rm g} \frac{\partial T_{\rm g}}{\partial t} + \rho_{\rm g} c_{\rm g} u \frac{\partial T_{\rm g}}{\partial x} = \lambda_{\rm g} \frac{\partial^2 T_{\rm g}}{\partial x^2} - \frac{2q_{\rm g,s}}{d_{\rm g}} + Q_2 \omega_2 - \frac{2h(T_{\rm g} - T_0)}{d_{\rm g}}$$
(5)

$$\rho_{\rm g} \frac{\partial Y_{\rm F}}{\partial t} + \rho_{\rm g} u \frac{\partial Y_{\rm F}}{\partial x} = \rho_{\rm g} D \frac{\partial^2 Y_{\rm F}}{\partial x^2} + \frac{d_{\rm s}}{d_{\rm g}} \nu_{1,\rm g} \omega_1 - \nu_{2,\rm F} \omega_2 \tag{6}$$

$$\rho_{\rm g} \frac{\partial Y_{\rm O}}{\partial t} + \rho_{\rm g} u \frac{\partial Y_{\rm O}}{\partial x} = \rho_{\rm g} D \frac{\partial^2 Y_{\rm O}}{\partial x^2} - \omega_2 \tag{7}$$

where  $h = Nu \lambda_g/2d_g$  is the heat transfer coefficient (*Nu* is the Nusselt number), and  $T_0$  is the ambient temperature.

Upon the integration, it is necessary to model heat transfer term between the gas and the solid phases,  $q_{g,s}$ . The following model is introduced in this study:

$$q_{\rm g,s} = h_{\rm g,s} \left( T_{\rm g} - T_{\rm s} \right) = \frac{\lambda_{\rm g}}{\delta_{\rm g}} \left( T_{\rm g} - T_{\rm s} \right) \tag{8}$$

where  $h_{g,s}$  is the heat transfer coefficient that is modelled as  $h_{g,s} = \lambda_g/\delta_g$ ;  $\delta_g$  is a model parameter that expresses the thickness of the gas-phase thermal boundary layer. The value of  $\delta_g$  should depend on the experimental parameters, u and  $d_g$ .  $\delta_g$  is nevertheless assumed to be constant in this study, and its dependence on u and  $d_g$  will be studied in a later work.

Eqs. (1)–(4) are the governing equations of the 2-D model, while Eqs. (1), (2), (5)–(8) constitute the 1-D model. These governing equations are respectively solved by an explicit finite difference method. The size of computational domain in the x direction is fixed at  $4 \times 10^{-2}$  m. The time step for the 2-D model is  $10^{-6}$  s, while it is  $10^{-7}$  s for the 1-D model. The values of material properties are taken from the literature [3, 5, 6] and shown in Table 1.

Table 1: The values of physical and chemical properties.

The values of the pre-exponential factors,  $B_1$  and  $B_2$  are estimated in this study. It is important to accurately compute the flame temperature because of the exponential dependence of the reaction rate on it. As the value of specific heat is fixed in this study, the heat of combustion,  $Q_2$ , is adjusted such that the adiabatic flame temperature defined by  $T_f = T_0 + c_g(v_{2,F} + 1)Q_2$  becomes 3300 K, a representative adiabatic flame temperature in pure oxygen.

The gas-phase thickness,  $d_g$ , and the oxidizer flow velocity, u, are parameters that are varied to test their effects. The 1-D and 2-D calculation results will be compared in the next section in terms of the distributions of gas-phase temperature, species mass fractions, as well as the flame spread rate.

## **3** Results and Discussion

Figures 2 and 3 show results obtained by the 2-D model for u = 0.04 m/s and 0.08 m/s, respectively, at  $d_g = 4 \times 10^{-3}$  m. With the increase in u, the maximum values of  $T_g$ ,  $Y_F$ , and  $Y_0$  increased owing to the enhanced oxygen supply that enhanced gas-phase oxidation and hence solid pyrolysis. On the other hand, the value of  $Y_F$  in the region behind the flame decreased with the increase in u because the increased oxygen supply led to a less oxygen-deficient condition. The flame standoff distance decreased with the increase in u.



(a) 300 1687
(b) 0 0.757
(c) 0 1

Figure 2. Numerical results.  $d_{\rm g} = 4 \times 10^{-3} \,\mathrm{m}$ ,  $u = 0.04 \,\mathrm{m/s}$ ,  $t = 6 \,\mathrm{s}$ ,  $x = 0.02 \,\mathrm{m} - 0.03 \,\mathrm{m}$ . (a) Temperature,  $T_{\rm g}$  (K), (b) Fuel gas mass fraction,  $Y_{\rm F}$  (–), (c) Oxygen mass fraction,  $Y_{\rm O}$  (–).

Figure 3. Numerical results.  $d_{\rm g} = 4 \times 10^{-3}$  m, u = 0.08 m/s , t = 5 s , x = 0.025 m - 0.035 m . (a) Temperature,  $T_{\rm g}$  (K), (b) Fuel gas mass fraction,  $Y_{\rm F}$  (–), (c) Oxygen mass fraction,  $Y_{\rm O}$  (–).



Figure 4. Distributions of gas-phase temperature,  $T_g$ , fuel gas mass fraction,  $Y_F$ , and oxygen mass fraction,  $Y_0$  predicted by the 2-D model at  $d_g = 4 \times 10^{-3}$  m.

Figure 4 shows the distributions of gas-phase temperature and species mass fractions predicted by the 2-D model for u = 0.04 and 0.08 m/s at  $d_g = 4 \times 10^{-3}$  m. The average values in the z direction are plotted as a function of  $x - x_f$ , where  $x_f$  is the x coordinate at which the reaction rate,  $\omega_2$ , is maximum. When u =

0.04 m/s, the temperature distribution is nearly symmetric with respect to  $x = x_f$ . When u = 0.08 m/s, on the other hand, the slope for  $x < x_f$  is somewhat lower than that for  $x > x_f$ . This is because the flame is tilted as shown in Figure 3. This trend indicates that reduction to the 1-D system is more favourable at lower oxygen flow speeds.

The gas-phase distributions shown in Figure 4(a) resemble those of typical diffusion flames in their profiles. There is little oxygen on the fuel side,  $x < x_f$ , and the value of  $Y_F$  is nearly zero on the oxidizer side,  $x > x_f$ . The oxidation reaction takes place in a thin reaction region. On the other hand, an appreciable amount of fuel gas exists in  $x > x_f$ .

Figure 5 compares distributions of  $T_g$ ,  $Y_F$ , and  $Y_O$  between the 2-D and the 1-D models. The value of model parameter  $\delta_g$  was chosen to be  $0.5 \times 10^{-3}$  m for  $d_g = 4 \times 10^{-3}$  m. Furthermore, the value of the Nusselt number needs discussion here. The Nusselt number has been introduced to account for heat loss at the upper plate,  $z = d_g/2$ . While the flame has a 2-D structure as exemplified in Figures 2 and 3, the heat loss term using the Nusselt number is the only mechanism in the 1-D model to consider the 2-D effect; it was found that use of the same Nusselt number as the 2-D model was not sufficient to reproduce a similar gas-phase temperature distribution to the 2-D model. The value of the Nusselt number in 1-D calculations was therefore increased to 70 to simulate the 2-D effect.





Figure 5. Comparison between the 2-D and the 1-D models for u = 0.04 m/s and  $d_g = 0.004$  m at t = 6 s.

Figure 6. Flame spread rate and oxidizer flow speed,  $u. \bullet: d_g = 4 \times 10^{-3} \text{ m (2-D)}, \circ: d_g = 4 \times 10^{-3} \text{ m}$ (1-D),  $\blacktriangle: d_g = 10 \times 10^{-3} \text{ m (2-D)}, \Delta: d_g = 10 \times 10^{-3} \text{ m (1-D)}.$ 

In Figure 5,  $T_g$  obtained by the 2-D model has a broader distribution and a lower peak temperature than those obtained by the 1-D model. This is because the  $T_g$  value of the 2-D model shown in the figure is the average over the z direction. On the other hand, the distributions of  $Y_F$  and  $Y_O$  are nearly one-dimensional (see Figure 2), and the 2-D and the 1-D models yielded almost identical profiles, indicating that reduction to the 1-D model was reasonable at least under this condition, although the values of model parameters must be properly chosen. Use of effective thermal properties may be a strategy to improve the model accuracy.

Figure 6 shows predicted flame spread rate as a function of oxidizer flow speed, u, for  $d_g = 4 \times 10^{-3}$  and  $10 \times 10^{-3}$  m. The value of  $\delta_g$  was set to  $1.25 \times 10^{-3}$  m for  $d_g = 10 \times 10^{-3}$  m to achieve the best agreement between the 2-D and the 1-D models. The flame spread rates shown are the average over the flame front location between x = 0.015 and 0.032 m, where quasi-steady spread was confirmed. Overall, the flame spread rate first increases with an increase in u and then decreases, which is typical for opposed-flow flame spread. The flame spread rate increases with the increase in  $d_g$ ; the narrower the channel is, the more enhanced the heat loss to the upper plate becomes. The 1-D model tends to reproduce the 2-D spread

rate when the oxidizer flow speed is relatively small. However, its deviation from the 2-D model is apparent when u is increased; the 1-D model tends to overestimate the spread rate. There are two reasons for the deviation. The major reason is the two-dimensional nature of the flame at large u as exemplified in Figure 3. The flame is inclined when u is large, and the 1-D model cannot capture the two-dimensional effect per se. Another reason is that the flame standoff distance from the solid surface decreases with an increase in u(see Figures 2 and 3). On the other hand, the value of  $\delta_g$ , which is a model parameter that corresponds to the thickness of thermal boundary layer, is fixed in this study for a given gas phase thickness,  $d_g$ . If  $\delta_g$  is decreased, heat transfer from the gas phase to the solid phase is enhanced via Eq. (8), which tends to decrease the flame temperature. The flame spread rate is then expected to decrease, approaching the value predicted by the 2-D model.

Summarizing the discussion thus far, the 1-D model, which was derived by reducing the original 2-D model, can reproduce the results obtained by the 2-D model only when the values of model parameters ( $\delta_g$  and Nu in the present model) are fine-tuned. As the 1-D model is computationally much less expensive than the 2-D model, a guideline as to how the values of model parameters can be determined would be desired. Further research is necessary to establish such a guideline.

# 4 Conclusions

The objective of this study is to test the validity of reduction techniques that have been often used to derive reduced-dimensional models of flame spread in a narrow space. A 2-D model of flame spread over a thin solid in a narrow channel was first developed and then reduced to obtain a 1-D model. Comparison between the results obtained by the 2-D and the 1-D models indicated that the 1-D model could reproduce the flame spread rate as well as temperature and species distributions if the values of model parameters were properly determined. Although fine-tuning the model parameters is not an easy task, methodology to specify their values needs to be established considering that the reduced-dimensional model is computationally much less expensive than the original model.

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