Effect of Boundary Conditions on Thermo-Acoustic Instability of Flames Propagating in Tubes

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1 Introduction

Flames propagating in tube have been a subject of investigation for more than a century due to their practical significance. Guenoche [1] summarized the behavior of flames propagating in tubes with different boundary conditions. They noted that flame showed vibration when it is ignited at open end of an open-closed or open-open tube for certain mixture conditions. Later open-closed tube was employed to study thermo-acoustic instability [2,3]. When the flame is ignited at the closed end of closed-open tube [4] or closed-closed tube [5], tulip flames are observed. Flames propagating in both ends closed tube can also lead to deflagration to detonation transition [6]. Role of acoustic instability on flame front instability is often questioned. Pressure [7] and velocity [8] coupling mechanisms are proposed to explain the interplay of flame front and acoustics leading to thermo-acoustic instability. In this work, we present a simple theoretical analysis to explain the role and occurrence of thermo-acoustic instability in tubes subject to various boundary conditions. The goal is to explain why thermo-acoustic instability happens only under certain boundary conditions.

2 Analytical method

The one-dimensional acoustics is considered in a tube filled with gases at two different temperatures and density representing unburnt and burnt gas. One-dimensionality of acoustics is usually valid because radial or circumferential velocity fluctuations are much smaller compared to axial velocity fluctuations [3] and axial velocity fluctuations at different radial locations are similar [5] in experiments. The wall loss and radiation loss from the open ends of tube are not considered to keep the analysis simpler; also, their inclusion doesn't make any qualitative difference in instability characteristics [9]. The spatio-temporal (x, t) pressure and velocity fluctuations can be written as

$$\delta p_{1,2} = \{A_{1,2}\cos(k_{1,2}x) + B_{1,2}\sin(k_{1,2}x)\}\, exp(i\omega t) \quad (1)$$

$$\delta u_{1,2} = -\frac{i}{c_{1,2}\rho_{1,2}}\{A_{1,2}\sin(k_{1,2}x) - B_{1,2}\cos(-k_{1,2}x)\}\exp(i\omega t) \quad (2)$$

Where $k_{1,2} = \frac{\omega}{c_{1,2}}$, ω is the circular frequency, $c_{1,2}$ is speed of sound, $\rho_{1,2}$ is density. The subscripts 1 and 2 refer to unburnt and burnt sections of the gas. The constants $A_{1,2}$ and $B_{1,2}$ can be eliminated using four boundary conditions. Two boundary conditions can be written by imposing no velocity fluctuation at the closed end and no pressure fluctuation at the open end of tube. This is shown for various tube configurations in Fig. 1. Two more boundary conditions are specified at flame front. Considering low Mach number assumption, there is no pressure jump across flame front. This can be written as

$$\delta p_1 = \delta p_2 \; at \; x = 0 \quad (3)$$

The velocity fluctuations across flame front can be related by the velocity coupling transfer function, Γ

$$\frac{\delta u_2(0)}{\delta u_1(0)} = (1+\Gamma) \quad (4)$$

Or pressure coupling transfer function, Z

$$\frac{\delta u_2(0) - \delta u_1(0)}{\delta p(0)/\rho_1 c_1} = Z \quad (5)$$

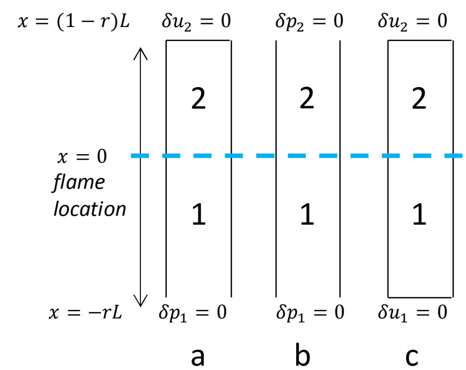

Figure 1. Tube configurations and boundary conditions

3 Results and discussion

3.1 Open-closed tube

We consider only downward propagating flames. In this condition, there are two possibilities. First, open end is at top where flame is ignited. This boundary condition has been tested extensively, both, experimentally [2,9,10] and theoretically [7,8] as vibrating flames are observed in this configuration. Second, closed end is at top where mixture is ignited and flame travels downwards toward open end. After eliminating, the constants using boundary conditions at the ends and across flame, eigen mode equation can be derived in this condition for velocity coupling as

$$\frac{\rho_2 c_2}{\rho_1 c_1}\cot\{rX\}\cot\left\{(1-r)\frac{c_1}{c_2}X\right\}(1+\Gamma) = 1 \quad (6)$$

Similarly, for pressure coupling the eigen mode equation can be written as

$$\frac{\rho_2 c_2}{\rho_1 c_1}(\cot\{rX\} - iZ)\cot\left\{(1-r)\frac{c_1}{c_2}X\right\} = 1 \quad (7)$$

Free eigen mode equation, assuming no coupling is identical for both, velocity and pressure coupling

$$\frac{\rho_2 c_2}{\rho_1 c_1}\cot\{rX_0\}\cot\left\{(1-r)\frac{c_1}{c_2}X_0\right\} = 1 \quad (8)$$

Stability analysis can be performed by substituting, $X = X_0 + \delta X$ in eqn. 6 and 7, where X_0 is calculated from eqn. 8. The non-dimensional growth rate can be derived for velocity coupling as

$$-Im(\delta X) = \frac{-Im(\Gamma)}{(1-r)\frac{c_1}{c_2}cosec\left((1-r)\frac{c_1}{c_2}X_0\right)sec\left((1-r)\frac{c_1}{c_2}X_0\right) + r\,cosec(rX_0)sec(rX_0)} \tag{9}$$

And, for pressure coupling as

$$-Im(\delta X) = \frac{Re(Z)}{(1-r)\frac{\rho_1 c_1^2}{\rho_2 c_2^2}\left(1+tan^2\left((1-r)\frac{c_1}{c_2}X_0\right)\right) + r\,cosec^2(rX_0)} \tag{10}$$

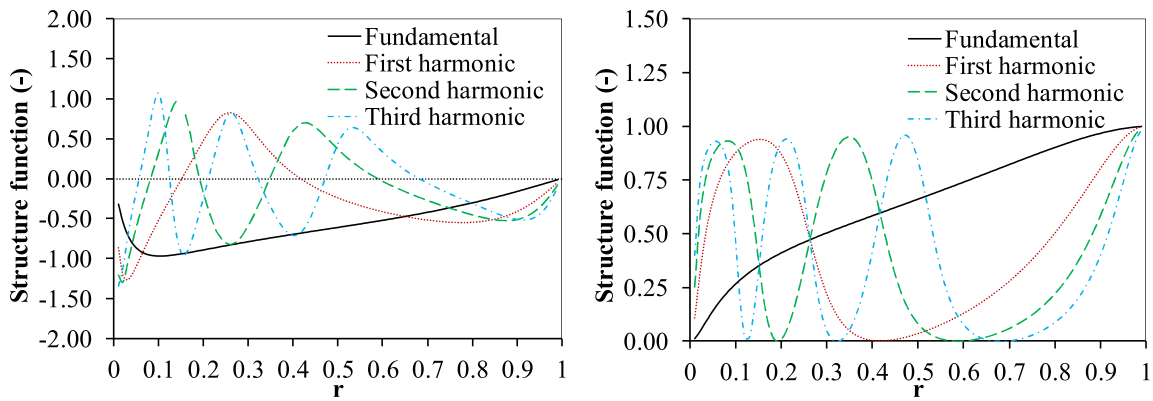

Figure 2. Acoustic structure function for top end closed tube for velocity coupling (left) pressure coupling (right).

Acoustic structure function is defined for velocity and pressure coupling as the coefficient of $Im(\Gamma)$ and $Re(Z)$ in eqn. 9 and 10. It is informative to learn about the acoustic structure functions. A positive acoustic structure function would mean instability because $Im(\Gamma)$ and $Re(Z)$ are always positive for downward propagating flames [7,8]. It is important to recall that velocity coupling produces a much stronger instability compared to pressure coupling [10]. Figure 2 shows the structure function for velocity coupling and pressure coupling respectively. The fundamental mode of this system is stable with regards to velocity coupling. Higher modes of tube can be unstable for certain locations, but they require larger energy to become unstable because higher modes also have higher losses. Experiments in upward propagating flame reveal the presence of tulip flame if the flame is ignited close to closed end and propagates towards open end. But, vibrating flames are not observed. Some small pressure oscillations can be due to pressure coupling which always produces a weak instability. We also performed experiments in downward propagating flames ignited at top closed end and observed no thermo-acoustic instability. This indicates that acoustics have negligible role in the flame motion and tulip flame formation occurring under this boundary condition.

3.2 Open-open tube

Similarly, in open-open tube, the eigen mode equations for velocity coupling is obtained as

$$\frac{\rho_2 c_2}{\rho_1 c_1}\cot\{rX\}\tan\left\{(1-r)\frac{c_1}{c_2}X\right\}(1+\Gamma) = 1 \tag{11}$$

And, for pressure coupling the eigen mode equation can be written as

$$\frac{\rho_2 c_2}{\rho_1 c_1}(\cot\{rX\} - iZ)\tan\left\{(1-r)\frac{c_1}{c_2}X\right\} = 1 \tag{12}$$

free eigen mode equation can be written independent of coupling mechanism as

$$\frac{\rho_2 c_2}{\rho_1 c_1}\cot\{rX_0\}\tan\left\{(1-r)\frac{c_1}{c_2}X_0\right\} = -1 \tag{13}$$

Stability analysis can be performed in a similar manner to find the growth rate for velocity coupling as

$$-Im(\delta X) = \frac{-Im(\Gamma)\frac{\rho_2 c_2}{\rho_1 c_1}\tan\left\{(1-r)\frac{c_1}{c_2}X_0\right\}}{(1-r)\frac{\rho_2}{\rho_1}\left(1+tan^2\left((1-r)\frac{c_1}{c_2}X_0\right)\right)+r\,(1+tan^2(rX_0)} \tag{14}$$

And for pressure coupling as

$$-Im(\delta X) = \frac{Re(Z)}{(1-r)\frac{\rho_1 c_1^2}{\rho_2 c_2^2}\left(cosec^2\left((1-r)\frac{c_1}{c_2}X_0\right)\right)+r\,cosec^2(rX_0)} \tag{15}$$

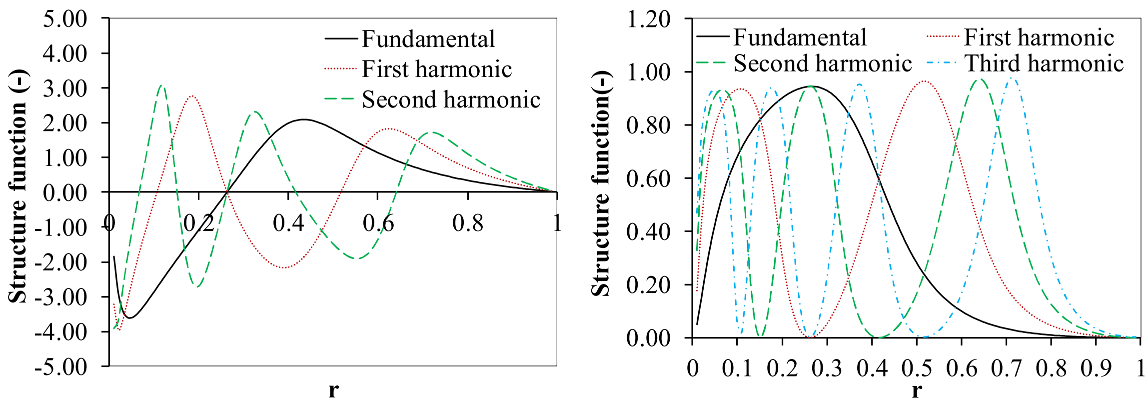

Figure 3. Acoustic structure function for both ends open tube for velocity coupling (left) pressure coupling (right).

Figure 3 shows the acoustic structure functions for open-open tube considering velocity and pressure coupling. The flame can become unstable in the upper two-thirds of tube but is stable in the lower one-third of the tube. This is similar to the experiments where flame is ignited at open end and travels towards closed end leading to instability. The difference however is in the bottom one-third of tube. Experiments in horizontal open tube reveal that the flame fluctuations are maximum around center and decay as flame approaches another open end [11]. Pressure fluctuations were not reported to indicate thermo-acoustic instability. Experiments are being undertaken for this boundary condition to reveal the nature of thermo-acoustic instability in this configuration and will be reported in a related future publication.

3.3 Closed-closed tube

Similarly, the eigen mode equation in closed-closed tube for velocity coupling is obtained as

$$\frac{\rho_2 c_2}{\rho_1 c_1}\tan\{rX\}\cot\left\{(1-r)\frac{c_1}{c_2}X\right\}(1+\Gamma) = -1 \tag{16}$$

Similarly, for pressure coupling the eigen mode equation can be written as

$$\frac{\rho_2 c_2}{\rho_1 c_1}(\tan\{rX\} + iZ)\cot\left\{(1-r)\frac{c_1}{c_2}X\right\} = -1 \tag{17}$$

Free eigen mode equation

$$\frac{\rho_2 c_2}{\rho_1 c_1}\tan\{rX_0\}\cot\left\{(1-r)\frac{c_1}{c_2}X_0\right\} = -1 \tag{18}$$

Stability analysis is performed to give the non-dimensional growth rate for velocity coupling as

$$-Im(\delta X) = \frac{-Im(\Gamma)\ tan(rX_0)}{(1-r)\frac{\rho_1 c_1^2}{\rho_2 c_2^2}\left(1 + tan^2\left((1-r)\frac{c_1}{c_2}X_0\right)\right) + r\ (1 + tan^2(rX_0)} \tag{19}$$

And, for pressure coupling as

$$-Im(\delta X) = \frac{Re(Z)}{(1-r)\frac{\rho_1 c_1^2}{\rho_2 c_2^2}\left(1 + tan^2\left((1-r)\frac{c_1}{c_2}X_0\right)\right) + r\ (1 + tan^2(rX_0)} \tag{20}$$

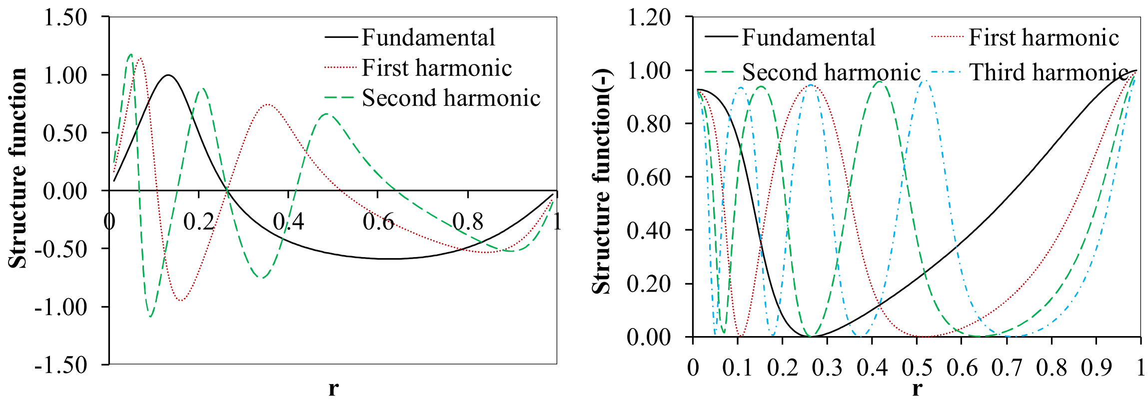

Figure 4. Acoustic structure function for both ends closed tube for velocity coupling (left) pressure coupling (right)

Figure 4 shows the acoustic structure functions for both ends closed tube for both velocity and pressure coupling mechanisms. The flame is stable for most part of the tube and thermo-acoustic instability happens only when flame nears the other end. As, the flame is predicted to be thermo-acoustically stable it seems the flame-acoustic interaction will have limited effect on deflagration to detonation transition which occurs in long closed tubes [6]. Flame dynamics in this condition is more complicated because combustion leads to increase in overall pressure due to expansion of gases leading to higher temperature [5]. Pressure oscillations are observed in closed tubes, but these pressure oscillations can be reasonably described by large flame area variations and without considering the flame-acoustic interactions [5]. Assuming gases as ideal, the pressure ratio is same as temperature ratio. However, this should not change the qualitative understanding of these results as the boundary conditions will remain unchanged even at increased pressure. If the flame can be treated as acoustically compact, the pressure equalization across flame front is also valid. The increased pressure, however, can lead to change in the transfer function due to change in burning velocity, flame structure or Markstein number. But, as long as the transfer function is positive or has a known sign, the inferences drawn from these results can be used to understand the thermo-acoustic instability of such systems.

4 Conclusions

Effect of boundary conditions on thermo-acoustic instability of flames propagating in tubes is studied analytically. It is found that fundamental mode is completely/mostly stable when flame is ignited near the closed end of an open-closed/closed-closed tube. Open-open tube shows instability when flame is in the upper two third of the tube but is stable when flame is in the lower one third of tube. Smaller pressure fluctuations cannot be ruled out due to pressure coupling as it is destabilizing irrespective of boundary conditions but is not expected to lead to strong thermo-acoustic instability. Direct comparison with experiments is in progress and will be presented in our upcoming publication.

Acknowledgements

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