Small Size Rotating Detonation Engine: Scaling and Minimum Mass Flow Rate

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1 Introduction

The concept of the rotating detonation engine (RDE) was first investigated in the mid 60s and early 70s [1–3]. Its application to a viable propulsion device has seen a resurgence in the last 10 years, since about 2006 with the experimental work from Bykovskii et al. [4, 5], Wolanski et al. [6–8], and numerous recent simulation-based works (see, for example, [9, 10]). This resurging interest has sparked considerable enthusiasm from student teams to develop an RDE engine for a small-scale vehicle, with a diameter under 10 cm, to be used in a competition setting where the target altitude is 10,000 ft (for example, the Intercollegiate Rocket Engineering Competition). Given the testing environment available to such student projects, an additional requirement was imposed that the mass flow rate from the engine be low enough to be tested at one of two facilities: a closed, fixed volume dump tank, or an open-air testing site allowing only vehicles equipped with G-class engines, i.e., with a total impulse between 80–160 Ns. In addition, competition requirements discourage the use of pure oxygen as an oxidizer, encouraging instead the use of nitrous oxide, N_2O. These experimental and operational constraints have led to an effort to develop an RDE with the smallest possible thrust. Assuming that specific impulse, $I_{\mathrm{sp}} = \mathrm{Thrust}/\dot{m}g_0$ is relatively insensitive, to first order, to the thrust, then mass flow rate is a direct measurement of thrust. This paper thus aims to predict the smallest possible mass flow rate for successful operation of an RDE using stoichiometric H_2/O_2 and H_2/N_2O mixtures.

The existence of a lower mass flow rate and thrust limit has a practical application in RDE design. If this lower bound exists, then it defines part of the range of applicability of this engine concept. The existence of a lower mass flow rate limit has been observed in [11] and is discussed in [12] though it is approached by considering area blockage from overpressure. In this approach, we use the simplest model possible to find a range of viable operational characteristics based on geometric scaling.

2 Analysis of Physical Limits to RDE Operation

Three main geometric limits dictate successful RDE operation. (1) A minimum number of detonation cells must fit within the annulus thickness, such that $h \approx 2.5 - 3\lambda$ [4, 12]. (2) The operational parameters must result in at least one detonation present in the annulus, i.e., $\omega > 1$ where ω is the wavenumber [6, 8]. (3) The aspect ratio of the detonable region must result in a stable wave, i.e. $L_{\rm cr} \gtrsim Kh$ where K should be a reasonably large value on the order of 5–10. To gain perspective on these criteria, we examine Fig. 1 showing a cross-section of the annular combustion chamber. The zone over which the detonation forms is represented in red. Injection is from the left and combustion product outflow is on the right. There likely is a standoff between the injection ports and the left-hand side of the detonation where the jets emanating from the injection plane widen and a detonable medium is set-up. The detonation occurs over a critical region of width $L_{\rm cr} \approx 12 \pm 5\lambda$ [4, 12] and occupying the entire height of the combustion chamber, h. The

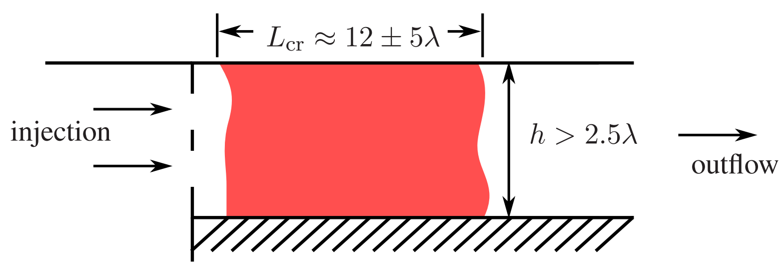


Figure 1: Annular combustion chamber cross-section showing the detonation (traveling into the page) region in red. The injection ports are on the left and combustion product outflow is on the right.

obtention of a steady detonation process is expected to require that the critical length be on a larger scale than the turbulent, standoff region created by the injection process. If we assume, as a starting point that the injection disturbance is on the scale of the annulus height, we can conclude that the critical length must be several times larger than the annulus height leading to our third criterion $L_{\rm cr} \gtrsim Kh$. Using the detonation cell size scaling of the critical length, we have $12 \pm 5\lambda \geq Kh$. A value of $K = 5$ results in

$$\frac{h}{\lambda} \leq 2.4 \pm 1 \tag{1}$$

in agreement with the geometric scaling of [4]. It is interesting to note that if $L_{\rm cr}/\lambda$ is indeed close to a constant, it is then impossible to have more than about 3 cells across the combustion chamber height while at the same time providing a detonation surface larger than 5 times the channel height to overcome the turbulence and other disturbances generated by the injector geometry. It therefore appears that criteria (1) and (3) are, in fact, the same requirement for successful RDE operation if we accept that $L_{\rm cr}/h = 5$ is a reasonable aspect ratio.

The second physical limit for RDE operation is dictated by the requirement that the geometry accommodate at least one detonation wave, i.e., that the wavenumber $\omega > 1$. To derive an expression for the wavenumber, we "unwrap" the annular combustion chamber into a rectangular section, as shown in Fig. 2. The injection is again from the left and the detonation waves travel here from bottom to top. We finally "trim" the region to keep only the space between two following detonations. In the case that only a single detonation ($\omega = 1$) is present in the combustion chamber, the bottom detonation is seeing the back of its own reaction zone. The distance between two detonation waves is thus $\pi D/\omega$ and each detonation propagates over a width of $L_{\rm cr}$. The injection occurs continuously from the left-hand side injection surface. The zone of fresh mixture available to the next detonation grows continuously from a width of zero just behind the top detonation to a width of $L_{\rm cr}$ as the next detonation arrives. The unmixed contact surface between the combustion products and the fresh mixture is represented in Fig. 2 by the dotted line at an angle θ. This angle is also directly related to the ratio of the injection velocity, $V_{\rm inj} = \dot{m}/\rho A_{\rm chamber}$ where $A_{\rm chamber} = \pi D h$, to the detonation propagation velocity, U_d. Equating the two definitions of the angle, we recover an expression for

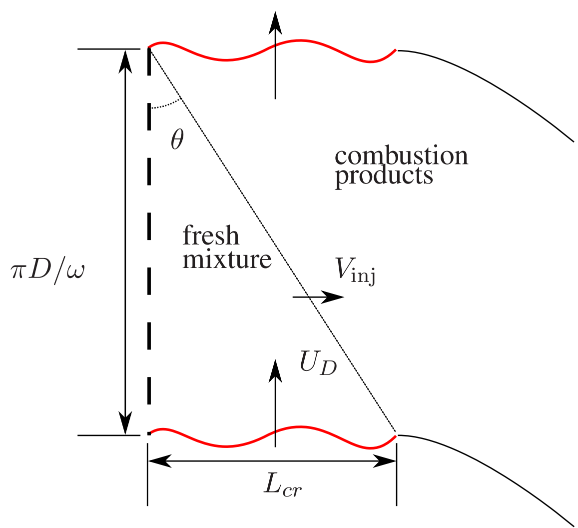


Figure 2: Unwrapped combustion chamber section showing two detonations (in red) following each other with injection ports on the left side.

the wavenumber

$$\omega = \frac{\dot{m}/\rho}{L_{\rm cr} U_d h} > 1, \tag{2}$$

where we have already imposed that at least one detonation wave be allowed in the geometry to respect our second physical limit. We note here that this result is identical to that obtained by [6, 8] except for an extraneous factor of 2 stemming from an algebraic mistake in the definition of h introduced in [6]. The critical length is essentially a self-adjusting parameter, i.e. that if more mass flows through the combustion chamber, a critical value is reached after which an extra detonation becomes the stable solution. We substitute the $L_{\rm cr} = C_L \lambda$ scaling and note that the detonation cell size is known to reduce with increasing pressure, roughly inversely proportional with pressure $\lambda = \lambda_{\rm ref} P_{\rm ref}/P$. The reference detonation cell size, $\lambda_{\rm ref}$ is shown in table 1 for the two mixtures of interest at the reference pressure $P_{\rm ref} = 1$ atm.

Table 1: Cell size of the considered mixture at $P_{\rm ref} = 1$ atm.

Mixture	Cell size (mm)
$2H_2$+O_2	1.6 [13]
H_2+N_2O	1.5 [14]

The final expression for the wavenumber is

$$\omega = \frac{\dot{m} R_{\mathrm{sp}} T}{C_L \lambda_{\mathrm{ref}} P_{\mathrm{ref}} U_d h} > 1, \tag{3}$$

where $C_L = 12 \pm 5$ is the proportionality constant in the scaling of L_{cr} and T is the temperature of the fresh mixture ahead of the detonation. The pre-detonation temperature, the detonation velocity and the cell size will all be impacted by the mixing of the jets at the injection plane with the combustion products as this mixing would result in a dilution of the detonable mixture with an effectively hot, inert diluent.

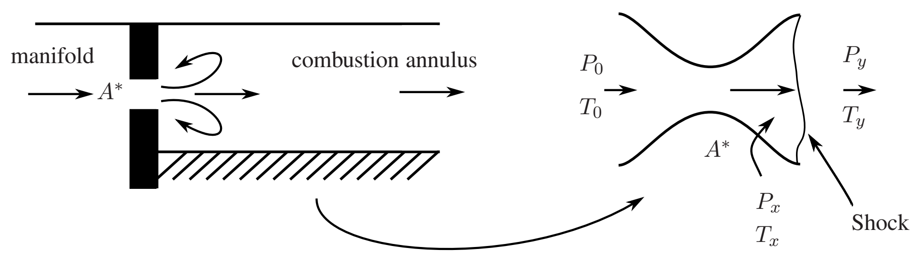


Figure 3: Schematic of a premixed RDE showing the simplification of the injection plane to a choked isentropic nozzle with equal throat area followed by a normal shock.

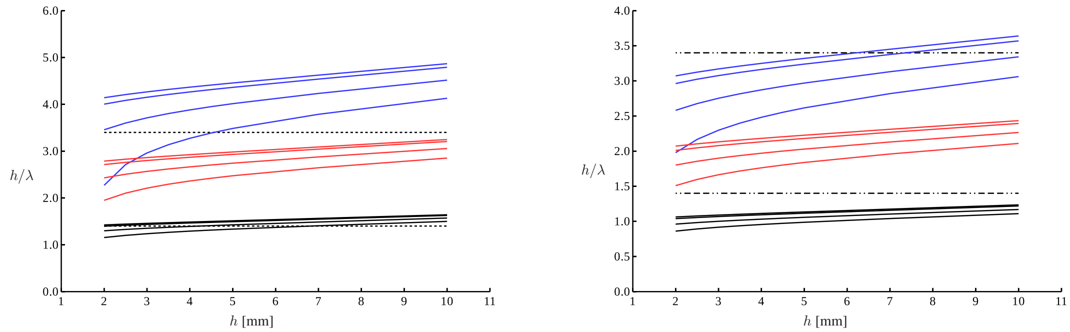
For the purpose of this estimation, we assume the simplest model with no mixing between reactants and products. To calculate the pre-detonation state and obtain the pressure, density, temperature, and injection velocity, we represent the injection area as an isentropic nozzle fed by a reservoir at a total temperature of $T_0 = 300$ K and a total pressure P_0. A normal shock is located at the exit of this fictitious nozzle. The post-shock conditions are the combustion chamber, pre-detonation conditions. This simplification is sketched in Fig. 3. The required injection area for choking of the nozzle is calculated by

$$\frac{\dot{m}}{A^*} = \frac{\sqrt{\gamma} P_0}{\sqrt{R T_0}} \frac{1}{\left(1 + \frac{\gamma - 1}{2}\right)^{(\gamma+1)/[2(\gamma-1)]}}. \tag{4}$$

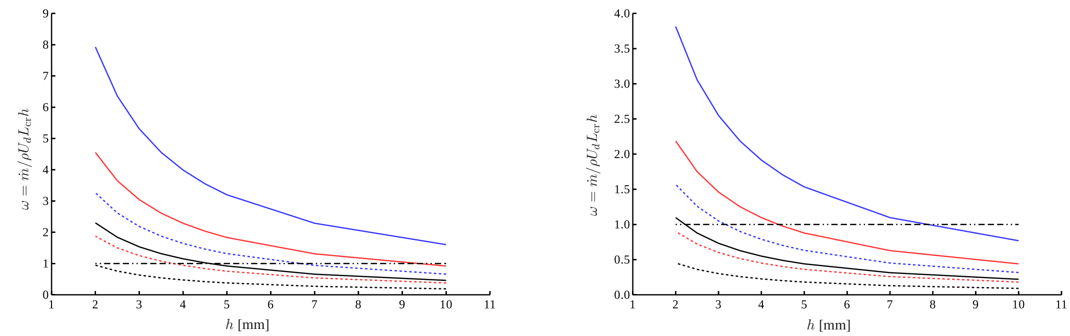
The remainder of the state is computed using isentropic flow and normal shock relations. This particular assumption of flow conditions through injection is made as it reproduces the experimental conditions measured in premixed, gas-phase RDEs by Wolanski [6].

3 Results

Since we restricted ourselves to room temperature tank storage, the problem has three free parameters, the injection mass-flow rate, $\dot{m}$, the injection stagnation pressure, P_0, and the combustion chamber annulus height, h. To determine whether an RDE will operate at a given set of conditions, we must verify two criteria, namely that $h/\lambda = 2.4 \pm 1$ and $\omega > 1$. The results, for both mixtures considered, are shown in Fig. 4 for mass flow rates of $\dot{m} = 0.05$, 0.1, and 0.15 kg/s, for stagnation pressures of 35, 100, 400, and 745 psi (2.4, 6.8, 27.2, and 51 atm), and for annulus thicknesses ranging from 2 to 10 mm. The exterior diameter of the annulus was fixed at $D_{\mathrm{outer}} = 76.5$ mm. Given that the detonation velocity will suffer a decrement due to curvature and the low number of cells across the channel, the detonation velocity was fixed at $U_d = 2$ km/s, a representative value, for all calculations.



(a) $2H_2+O_2$ (b) H_2+N_2O



(c) $2H_2+O_2$ (d) H_2+N_2O

Figure 4: Scaled chamber height, h/λ (top row) and wavenumber, ω (bottom row), for $2H_2+O_2$ (left) and H_2+N_2O (right) mixtures with varying mass flow rates of $\dot{m}$ = 0.05 kg/s (black), 0.1 kg/s (red), and 0.15 kg/s (blue). h/λ increases with upstream total pressure, P_0. Results are shown here for P_0 = 35, 100, 400, and 745 psi. The wavenumber ω has an upper (solid) and lower (dashed) bound based on the extrema of $L_{\rm cr} = 12 \pm 5\lambda$. The horizontal lines are the physical limits of operation, $h/\lambda = 2.4 \pm 1$ and $\omega > 1$.

One first observation that was made is that the wavenumber is, for this range of parameters, mostly independent of P_0 with variations less than 10% for all cases. To simplify the representation, we have therefore only plotted ω for $P_0 = 100$ psi as a reference. Given the uncertainty on $L_{\rm cr}$, the lower (dashed lines in Fig. 4c and 4d) and upper (solid lines) bounds of the wavenumber are plotted.

The range of allowable mass flow rates is wider for the H_2/O_2 mixture than for H_2/N_2O. Examining the latter case for the lowest value of mass flow rate, we see that h/λ is consistently below 1.5 regardless of stagnation pressure and annulus height. For $\dot{m} = 0.1$ kg/s, h/λ is low, but in the acceptable range. For $h < 4$ mm, there are possible solutions where $\omega > 1$. A lower limit of mass flow rate is thus found, for H_2/N_2O between $0.05 < \dot{m}_{\rm min} < 0.1$ kg/s. For H_2/O_2, h/λ values are marginal for this same low mass flow rate, but wavenumber values are acceptable for $h <$ 2–4 mm. A minimum mass flow rate is expected for H_2/O_2 of $\dot{m}_{\rm min} \lesssim 0.05$ kg/s.

An interesting comparison can be made with the data of [11] presented in [12]. A lower limit, for H_2/enriched air is found to be $\dot{m}_{\rm min} \approx 250 A_{\rm chamber}$ for stoichiometric operation. For an engine with annulus size $D = 74.5$ mm, $h = 2$ mm, this corresponds to $\dot{m}_{\rm min} = 0.1$ kg/s, only a factor of 2 greater than the results of the current analysis, despite our numerous approximations and the extra nitrogen dilution.

4 Conclusion

A simple model, based on geometric constraints for successful RDE operation was computed using an isentropic nozzle and normal shock flow field to find the pre-detonation conditions. A minimum mass flow rate for the successful operation of an RDE with an outer annulus diameter of 76.5 mm was found to be $0.05 < \dot{m}_{\min} < 0.1$ kg/s for stoichiometric H_2/N_2O and $\dot{m}_{\min} \lesssim 0.05$ kg/s for stoichiometric H_2/O_2. These results are of the same order of magnitude as minimum mass flow rate conditions in operating an RDE reported in the literature. Testing of an RDE design based on the current calculations is ongoing.

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