Combustion Instability Prediction Using Minimal Experimental or Computational Data

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1 Introduction

Mathematical treatment of combustion instability [1] based on Rayleigh's criterion [2] shows that predicting heat release oscillations with respect to acoustic pressure oscillations is critical for predicting combustor stability. The prediction problem is rendered challenging mainly because of the difficulty in predicting the fluctuating component of heat release without extensive testing. A common approach in instability prediction is to first obtain the flame frequency response function (FRF) as the open-loop transfer function between flame heat release oscillations and acoustic oscillations and then use it in combination with an acoustic description of the system dynamics to obtain instability frequencies and growth rates [3-5]. Unfortunately, the FRF approach has several drawbacks: (1) analytical models for obtaining transfer functions are difficult for complex configurations, (2) detailed flame experiments for obtaining transfer functions are very costly in terms of time and effort and (3) mathematical treatment of combustion instabilities is computationally demanding due to non-linear coupling of different phenomena and could lead to potential discrepancies between simulated and experimental results. To overcome problems associated with FRF based methods, a simplified description of heat release oscillations is developed that could greatly reduce the amount of experimental or computational data required for predicting combustion instability resulting from heat release perturbations. Using phase resolved CH* chemiluminescence data from a laboratory scale dump combustor undergoing vortex dominated marginal amplitude instability at 150 Hz, first it is shown that the heat release oscillations can be adequately modeled as sinusoidal perturbations with two spatially varying parameters: (1) the amplitude of the oscillations and (2) the phase offset of the oscillations with respect to a reference pressure signal. A method that requires a substantially small dataset to estimate the parameters is then developed. Using the simplifying observation that the amplitude of heat release oscillation at any given location will be proportional to the average heat release at that location, it is shown that the two parameters can be estimated by simply knowing (1) the average heat release distribution and (2) the heat release distribution corresponding to the pressure oscillation maxima. Using the estimated values of the parameters, spatio-temporal variations in heat release oscillations can be reconstructed, which

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in conjunction with pressure oscillation information could be used to obtain local and global Rayleigh indices. Linear stability of the combustor resulting from a perturbation of the stable heat release rate distribution can then be predicted.

2 Experimental Setup



Fig. 1 Schematic of Dump Combustor



Fig.2 (a) Phase locked instantaneous CH* Chemiluminescence (b) Phase resolved fluctuating component of CH* Chemiluminescence intensity

Fig. 1 shows a schematic of the laboratory scale dump combustor burning a premixed charge of air and ethylene. An inflow velocity of 45 m/s and an equivalence ratio of 0.67 is used for exciting a naturally unstable vortex dominated mode of the combustor at 150 Hz frequency. A pair of flush-mounted Kistler 211B5 water-cooled pressure transducers are used to measure pressure oscillations in the combustor. The negative-to-positive zero crossing of the bandpass filtered dynamic pressure signal taken near the nozzle is used as the reference phase (i.e. $\theta = 0$). Fig. 2a shows a select sample of instantaneous phase locked CH* chemiluminescence images. Eight different pressure oscillation phases, θ (i.e. ω t), are selected for investigation of which only 4 are shown. Fig. 2b shows the normalized local fluctuation of

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chemiluminescence intensity obtained by subtracting the cycle averaged CH* Chemiluminescence intensity from each of the corresponding phase averaged CH* Chemiluminescence intensities. A set of 200 images were obtained and averaged for each θ .



3 Model Development

Fig. 3 (a) Empirically calibrated amplitude (b) Empirically calibrated phase (c) Model error (d) Pressure and heat release oscillations

Assuming that the temporal variations of q'/q'_{max} can be described using a simple sinusoidal basis function, a harmonic fit of the unsteady heat release distribution is first obtained. The fit is completely characterized by the amplitude A(z), phase offset $\psi(z)$ with respect to the reference pressure signal as described in Eq. 1

$$q'(z,\theta)/q'_{max} = A(z)\sin(\theta + \Psi(z))$$
(1)

Both A and ψ are solely spatial functions and can be calibrated using experimentally obtained results. A(z) is given by Eq. 2 where the notation max_{θ} represents the maximum value for all $\theta: 0 \le \theta \le 2\pi$

$$A(z) = \frac{\max_{\theta}(q'(z,\theta)/q'_{max}) - \min_{\theta}(q'(z,\theta)/q'_{max})}{2}$$
(2)

The model local phase $\psi(z)$ is obtained indirectly using an algorithm that fits a sine wave to the temporal variation in chemiluminescence intensity at each location while cycling through all possible values of $\psi(z)$ between 0 and π till the phase value that minimizes the sum of squares of the differences between model and empirical values, $\sigma(z)$, as shown in Eq. 3 is obtained. N denotes the number of phases for which chemiluminescence data is available.

$$\sigma(z) = \left(\frac{1}{N} \sum_{i=1}^{N} [A(z)\sin(\theta_i + \Psi(z)) - q'(z,\theta_i)/q'_{max}]^2\right)^{\frac{1}{2}}$$
(3)

Fig. 3a and 3b shows visualizations of A_{cal} and ψ_{cal} while Fig. 3c shows that of σ_{cal} . A_{cal} , Ψ_{cal} and σ_{cal} represent values of A, Ψ and σ obtained by calibrating against experimentally obtained data. It can be readily seen from Fig. 3c that σ_{cal} is low over much of the combustion chamber which indicates high model accuracy. It supports the simplifying assumption that the problem of predicting heat release oscillations in a combustor is equivalent to the problem of predicting the two parameters of the harmonic fit viz. amplitude and phase. Higher model error $\sigma(z)$ is concentrated in the region where the shear layer fluctuates. Fig. 3d shows that in some locations, q' is non-sinusoidal and perhaps multi-modal in regions like the shear layer where large discrepancies between experimental and modelled data is observed. The inadequacy of a unimodal sinusoidal basis function in capturing oscillations in certain regions where the oscillations could be multimodal and / or non-sinusoidal might be responsible for the discrepancy.



4 Estimation of Parameters

Fig. 4 (a) Estimated amplitude (b) Estimated phase (c) Model error

While A_{cal} and Ψ cal are obtained by calibrating against experimentally obtained data, a way of estimating A and Ψ without requiring an elaborate set of phase resolved heat release data is attempted. The estimated values are represented as Aest, ψ cal and σ est. A simple manipulation of Eq. 1 shows that $\psi(z)$ can be readily obtained using only A and q' at $\theta = \pi/2$ (i.e. peak combustor pressure oscillation phase) as described in Eq. 4. Although $\psi(z)$ can obtained using A(z) and q'(z, θ) at any θ , the choice of $\theta = \pi/2$ is useful since in stability analysis regions of elevated q' at combustor pressure maxima indicate regions that will experience the coupling between pressure and heat release oscillations most strongly during the onset of instability.

$$\Psi(z) = \arccos\left[\frac{q'(z,\frac{\pi}{2})/q'_{max}}{A(z)}\right]$$
(4)

To obtain $\psi(z)$ using Eq. 4, prior knowledge of A(z) and heat release at pressure oscillation maxima will be needed. A physically intuitive estimate results from the claim that $A_{est}(z) \propto \overline{q} / max_z(\overline{q})$ which is based on the plausible hypothesis that a region with higher average heat release might have higher energy budget for feeding into the oscillatory component at the onset of instability. A comparison between A_{est} (Fig. 4a) and A_{cal} (Fig. 3a) show that while there are some discrepancies primarily at the nozzle, the two are largely similar. Just like σ_{cal} , σ_{est} is low over much of the combustion chamber indicating high model accuracy and supports the simplifying assumptions.

Global Rayleigh index calculations, using combustor pressure oscillations and heat release oscillations, as evidenced through CH* chemiluminescence, show excellent agreement with the same calculated using calibrated and estimated parameters. The comparisons are shown in Table 1. The results indicate that modeling the heat release oscillations using estimated parameters can produce accurate results when used in the stability analysis of a combustor at the onset of instability while greatly reducing the amount of experimentally or computationally generated data that is usually required for predicting instability by traditional methods.



Fig. 5. Rayleigh index maps calculated using (a) experimentally obtained results (b) model with calibrated parameters and (c) model with estimated parameters.

4 Conclusions

A simplified, heuristic model of heat release fluctuation was established using experimental results wherein the fluctuations were assumed to be sinusoidal. The simplifying assumptions allowed for a complete description of the heat release fluctuation over an instability cycle with just the average heat release distribution and heat release distribution at peak pressure oscillation phase. Reconstruction of the normalized local heat release fluctuation using the model showed excellent agreement with the experimental results.

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The practical usefulness of such a model, in the context of combustion instability, is to accurately estimate the Rayleigh index and the combustor stability characteristics. Global Rayleigh indices calculated using the models were of the same sign as that calculated using experimental data. Global RI calculated using calibrated parameters was within 6% of the value obtained using experimentally measured data while that calculated using estimated parameters was within 16%. The results indicate that even a simple heat release fluctuation model can be very effective in estimating the stability of the combustor. Furthermore, the local Rayleigh index map produced using the model was qualitatively very similar to the original experimental results, indicating that such models could be effective in building tools for stability analysis.

The observation of the CH* chemiluminescence data reveals that there is a strong correlation between the vortex dynamics and the fluctuating heat release rate. In particular, the fluctuation is seen to follow the lifecycle of the periodic vortices very closely. Thus, it may be possible to model the fluctuating heat release at the peak pressure oscillation phase using only our understanding of vortex dynamics and vortex-combustion behavior. Since the average heat release is easily obtained using RANS solvers this could open up the possibility of predicting combustor stability with virtually no experimental data.

References

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