# Oblique Detonation Interaction with a Wall for Large Angles of Attack

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## 1 Introduction

A detonation in a condensed-phase high explosive (HE) generates very large pressures (several tens of GPa) through the reaction zone. This results in the lateral deflection of any surrounding confiner (typically metal or plastic), and the generation of often complex gasdynamic wave interactions at the HE/confiner interface. The degree to which the detonation-confiner interaction affects the detonation structure and evolution depends to a large extent on the material impedance properties of the confiner relative to those of the HE. For example, if the detonation shock intersects a strong (typically high impedance) confiner sufficiently obliquely, a reflected shock into the HE results. Conversely for a similar situation with a weak (typically low impedance) confiner, a reflected PM fan results. For shallower detonation incident angles, a strong confiner will generate subsonic flow for a portion of the reaction zone at the HE-confiner interface. For steady flow, the detonation driving zone (DDZ) (the region between the detonation shock and sonic locus) is influenced by the material properties of the confiner. Conversely, for a weak confiner, the flow at the HE/confiner interface will be sonic in a frame traveling with the detonation shock-confiner intersection point. In this case, the properties of the confiner do not influence the DDZ [1], and thus the speed of the detonation.

Much remains to be understood about the reflection and interaction patterns that can develop due to detonation interaction with a confiner. In a recent study, Bdzil and Short [2] have examined asymptotically the flow structures that can develop when either a Chapman-Jouguet (CJ) instantaneous energy release detonation or a small-resolved heat release (SRHR) detonation impact obliquely on a rigid wall at small angles of incident ($\phi_e \ll 1$, fig. 1). For the CJ detonation, traditional Mach stem structures are found behind the lead CJ wave. For the SRHR model, more complex patterns emerge. The purpose of the current work is to study oblique detonation interaction with a rigid wall for large angles of attack both for a CJ detonation wave and for a fully spatially distributed reaction zone detonation.

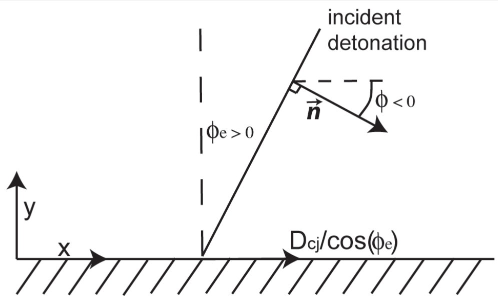


Figure 1: Detonation impacting obliquely on a rigid wall at an angle $\phi_e$. From [2].

## 2 Model

The detonation flow is governed by the 2D reactive Euler equations. These are written in conservative form as

$$\frac{\partial \mathbf{y}}{\partial t} + \frac{\partial \mathbf{f_r}}{\partial r} + \frac{\partial \mathbf{f_z}}{\partial z} = \mathbf{g}, \tag{1}$$

where,

$$\mathbf{y} = \Big(\rho, \rho u_r, \rho u_z, \rho E, \rho\lambda\Big)^{\mathsf{T}}, \quad \mathbf{f_r} = \Big(\rho u_r, \rho u_r^2 + p, \rho u_r u_z, u_r\,(\rho E + p), \rho u_r \lambda\Big)^{\mathsf{T}}, \tag{2}$$

$$\mathbf{f_z} = \Big(\rho u_z, \rho u_r u_z, \rho u_z^2 + p, u_z\,(\rho E + p), \rho u_z \lambda\Big)^{\mathsf{T}}, \quad \mathbf{g} = \Big(0, 0, 0, 0, \rho\Lambda\Big)^{\mathsf{T}}. \tag{3}$$

Here, $r$ and $z$ denote spatial coordinates parallel and perpendicular to the wall respectively (fig. 1), while $\rho$, $\mathbf{u}$, $E$ and $p$ are the density, material velocity vector, specific total energy and pressure, respectively. For the two-dimensional planar flow considered in the following, the velocity vector $\mathbf{u} = (u_r, u_z)^{\mathsf{T}}$. The reaction progress variable, $\lambda \in [0, 1]$, tracks the conversion of reactants to products. The specific total and internal energies are given by

$$E = e(\rho, p, \lambda) + \frac{1}{2}(u_r^2 + u_z^2), \quad e = \frac{p + A}{(\gamma - 1)\rho} - q\lambda, \tag{4}$$

where we have assumed a Tait (stiffened-gas) equation-of-state model for $e$. Also, $\gamma$ is the adiabatic exponent, $A$ the stiffened gas constant and $q$ the specific reaction enthalpy of the fuel species. In the strong shock limit employed here, which assumes the pressure in the ambient HE state is zero,

$$q = \frac{D_{CJ}^2}{2(\gamma^2 - 1)} \left(1 - \frac{A}{\rho_0 D_{CJ}^2}\right)^2, \tag{5}$$

where $D_{CJ}$ is the Chapman-Jouguet detonation speed and $\rho_0$ is the initial density of the HE. For the distributed reaction model, the reaction rate is given by

$$\Lambda = kp(1 - \lambda)^{1/2}, \tag{6}$$

where $k$ is a rate constant. For the CJ detonation model, $\lambda$ jumps instantaneously from 0 to 1.

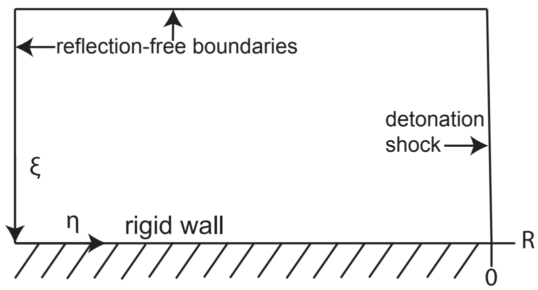


Figure 2: Transformation from laboratory variables to shock-attached coordinates

## 3 Shock-fitted coordinate transformation

For studying the oblique impact of a detonation on a wall, we adopt a shock-fit, shock-attached formulation for two-dimensional detonating flows introduced by Henrick [3] (see also Romick and Aslam [4]). The coordinates $r$ and $z$ are transformed according to

$$r(\xi,\eta,\tau)=\xi,\quad z(\xi,\eta,\tau)=z_s(\xi,\tau)+\eta,\quad t=\tau, \tag{7}$$

creating the rectilinear coordinate system in figure 2, where $z=z_s(\xi,\tau)$ describes the shock shape evolution, $\eta=0$ represents the shock locus in the transformed frame, $\xi=R$ is the location of the wall and $\xi=0$ represents the lateral extent of the domain. Under the transformation (7), the flow equations (1) become

$$\frac{\partial \mathbf{Y}}{\partial \tau}+\frac{\partial \mathbf{F}_\xi}{\partial \xi}+\frac{\partial \mathbf{F}_\eta}{\partial \eta}=\mathbf{G}, \tag{8}$$

where

$$\mathbf{Y}=|J|\mathbf{y},\quad \mathbf{F}_\xi=\quad \frac{\partial z}{\partial \eta}\left(\mathbf{f_r}-\frac{\partial r}{\partial \tau}\mathbf{y}\right)-\frac{\partial r}{\partial \eta}\left(\mathbf{f_z}-\frac{\partial z}{\partial \tau}\mathbf{y}\right) \tag{9}$$

$$\mathbf{F}_\eta=-\frac{\partial z}{\partial \xi}\left(\mathbf{f_r}-\frac{\partial r}{\partial \tau}\mathbf{y}\right)+\frac{\partial r}{\partial \xi}\left(\mathbf{f_z}-\frac{\partial z}{\partial \tau}\mathbf{y}\right),\quad \mathbf{G}=|J|\mathbf{g},\quad |J|=\frac{\partial r}{\partial \xi}\frac{\partial z}{\partial \eta}-\frac{\partial r}{\partial \eta}\frac{\partial z}{\partial \xi}, \tag{10}$$

and

$$\frac{\partial z}{\partial \tau}=\frac{\partial z_s}{\partial \tau},\ \frac{\partial z}{\partial \xi}=\frac{\partial z_s}{\partial \xi},\ \frac{\partial z}{\partial \eta}=1,\ \frac{\partial r}{\partial \tau}=0,\ \frac{\partial r}{\partial \xi}=1,\ \frac{\partial r}{\partial \eta}=0,\ |J|=1,\ \frac{\partial |J|}{\partial \tau}=0. \tag{11}$$

The transformation Jacobian requires the evaluation of $\partial z_s/\partial\tau$ and $\partial z_s/\partial\xi$. These are obtained through the shock surface evolution equations,

$$\frac{\partial z_s}{\partial \tau}=D_{ns}\sqrt{\left(\frac{\partial z_s}{\partial \xi}\right)^2+1},\quad \frac{\partial}{\partial \tau}\left(\frac{\partial z_s}{\partial \xi}\right)=\frac{\partial}{\partial \xi}\left(D_{ns}\sqrt{\left(\frac{\partial z_s}{\partial \xi}\right)^2+1}\right), \tag{12}$$

where $D_{ns}=D_{ns}(\xi,\tau)$ is the normal speed of the shock. Following Henrick [3] and Romick and Aslam [4], an evolution equation for $D_{ns}$ can be constructed through the standard shock jump Rankine-Hugoniot conditions, written in the form $\mathbf{y}_{i,s}=\mathbf{y}_{i,s}(D_{ns})$. It then follows that

$$\frac{\partial D_{ns}}{\partial \tau}=\frac{\partial}{\partial D_{ns}}\Big(\mathbf{y}_{i,s}(D_{ns})\Big)\left(\mathbf{G}-\frac{\partial \mathbf{F}_\xi}{\partial \xi}-\frac{\partial \mathbf{F}_\eta}{\partial \eta}\right)\Bigg|_s. \tag{13}$$

where $i$ is the solution element being chosen to evolve. In this case the energy component $(\rho E)$ is chosen. For example, when $A = 0$ in (4),

$$\rho_s E_s = \frac{4\rho_0 D_{ns}^2}{(\gamma^2 - 1)}, \quad \frac{d}{dD_{ns}}(\rho_s E_s) = \frac{8\rho_0 D_{ns}}{(\gamma^2 - 1)}. \tag{14}$$

The initial and boundary conditions are as follows. Initially, we have a planar oblique detonation traveling at an angle $\phi_e$ to the $r$ axis (fig. 1). Undisturbed by the wall, its path would be

$$z_s = \left(\frac{D_{CJ}}{\cos\phi_e}\right)\tau + (R - \xi)\tan\phi_e, \tag{15}$$

so that at $\tau = 0$,

$$\frac{\partial z_s}{\partial \xi} = -\tan\phi_e, \quad D_{ns} = D_{CJ}. \tag{16}$$

For the distributed reaction zone problem, the flow state at $(\xi, \eta)$ at $\tau = 0$ is populated with the ZND planar wave state one would find at a distance $\eta\cos\phi_e$ behind the shock. Note the planar solution for lab frame velocity at $(\xi, \eta)$, $U_{ZND}$, needs to be split into two components, namely,

$$u_z = U_{ZND}\cos\phi_e, \quad u_r = U_{ZND}\sin\phi_e. \tag{17}$$

Boundary conditions imposed in the formulation correspond to the oblique ZND solution along the top boundary ($\xi = 0$), a zero gradient condition at the outflow ($\eta = \eta_{min}$), imposition of the jump conditions through the algebraic dependence of the shock state on $D_{ns}$ at the front ($\eta = 0$) and lastly a reflection boundary condition on the wall $\xi = R$, where the normal speed $u_r = u_\xi = 0$.

## 3 Numerical method

We use a finite volume approach, second-order in time and space, with spatial discretization by a Lax-Friedrichs flux-splitting (LFFS) method. A two-stage, second-order Heun's method is used to update the shock slope, shock speed and interior solution vector. In each stage, first the update to the shock slope (12) and to the interior solution (8) is computed, and from those, the update to the shock speed (13) is computed. This is described in [5].

## 4 Results and Planned Work

For a CJ detonation obliquely impacting a wall at shallow angle, Bdzil and Short [2] have described a perturbation analysis that reveals the Mach reflection pattern that results from overdriving the CJ wave due to the wall interaction. The flow evolution can be described by the 2D (Burgers-like) system,

$$\left(\frac{\partial \mathcal{U}}{\partial \tau}\right)_{x^*,y^*} + \mathcal{U}\left(\frac{\partial \mathcal{U}}{\partial x^*}\right)_{y^*,\tau} + \left(\frac{\partial \mathcal{V}}{\partial y^*}\right)_{x^*,\tau} = 0\,, \tag{18}$$

$$\left(\frac{\partial \mathcal{V}}{\partial x^*}\right)_{y^*,\tau} - \left(\frac{\partial \mathcal{U}}{\partial y^*}\right)_{x^*,\tau} = 0. \tag{19}$$

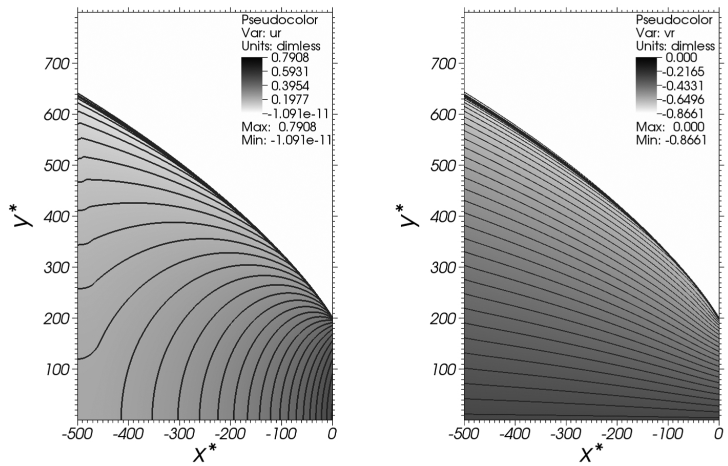


Figure 3: The $\mathcal{U}$ (left) and $\mathcal{V}$ (right) contours at $\tau = 200$, with $\alpha = 4/3$ and $\tan B = 1.0$.

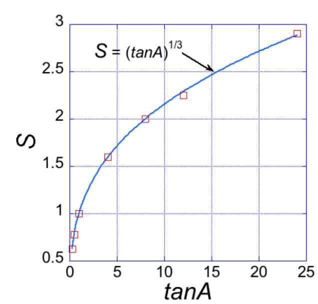


Figure 4: The speed of the triple point in the $y^*$-direction plotted as a function of the scaled incident shock-angle variable, $\tan B$.

Here $\mathcal{U}$ and $\mathcal{V}$ represent scaled velocity field perturbations off the 1D CJ state, while $x^*$ and $y^*$ are scaled coordinates related to the shock attached system described by $\xi$ and $\eta$. Along the rigid wall and shock, we have, respectively,

$$\mathcal{V} = 0, \quad -\frac{\partial \mathcal{V}}{\partial \tau} = \frac{\partial \mathcal{U}^2}{\partial y^*}, \tag{20}$$

while the initial conditions are

$$\mathcal{U}(\tau = 0, \cdots) = 0, \quad \mathcal{V}(\tau = 0, \cdots) = -\frac{\tan B}{\sqrt{\alpha}}, \tag{21}$$

where $B$ is related to the angle $\phi_e$ (fig. 1) and $\alpha$ is a function of $\gamma$. Figure 3 shows a typical Mach reflection pattern that develops in the flow following the CJ wave. The path and speed of the triple point can also be calculated (fig. 4).

For large angles of incidence, solutions of the reflection patterns behind a CJ wave will be obtained through the procedure described in §3. Figure 5 shows a verification example of the shock-attached formulation. Replacing the wall condition with the appropriate state for a 1D obliquely traveling detonation wave with

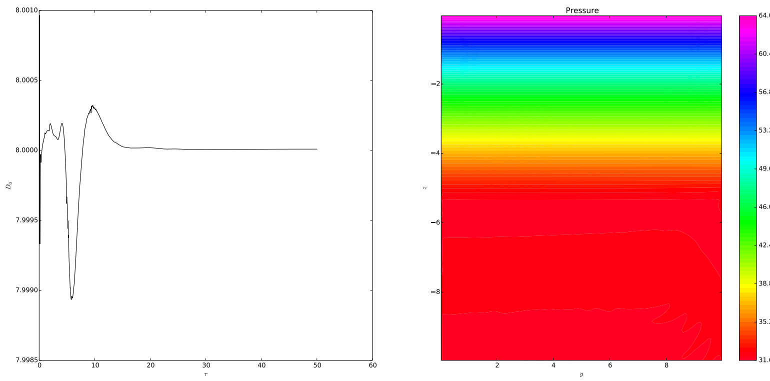

Figure 5: The evolution of a 1D obliquely traveling detonation wave with $D_{CJ} = 8$ mm/$\mu$s, $A = 0$ GPa and $\gamma = 3$. Left-hand image is the propagation speed in the shock attached $(\xi, \eta)$ frame and the right-hand side is the corresponding pressure distribution.

a distributed reaction zone, we can verify that the 1D structure is maintained with the correct propagation speed $(D_{CJ})$ (fig. 5).

For sufficiently small angles of incidence, we will compare the asymptotic results in Bdzil and Short [2] for a CJ wave with those of the full numerical solution. In addition, the procedure described in §3 will be used to examine the reflection patterns that develop when a detonation with a fully spatially distributed reaction zone impacts the wall as a function of wall incidence angle. Here we will also examine the effect of exothermic energy on the streamline turning properties of the flow.

## References


[1] Bdzil JB, Aslam TD, Henninger R, Quirk JJ. (2003) High-explosives performance: understanding the effects of a finite-length reaction zone. Los Alamos Science 28, 96-110.

[2] J.B. Bdzil & M. Short (2017). Theory of Mach Reflection of detonation at glancing incidence J. Fluid. Mech. 811, 269–314.

[3] Henrick AK. (2008). Shock-fitted numerical solutions of one-and two-dimensional detonation. PhD thesis, University of Notre Dame.

[4] Romick CM, Aslam TD. (2014). Two-dimensional detonation propagation using shock fitting. In Proc. 15th Intl. Symp. on Detonation, pages 380–389. Office of Naval Research.

[5] Chiquete C, Meyer CD, Short M & Quirk JJ. (2017). Calibration methodology for advanced programmed burn detonation wave propagation. Combust. Flame. submitted.