Experimental Assessment Of The Displacement And Consumption Speeds In Flame/Vortex Interactions

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1 Introduction

The notion of laminar flamelets implies that turbulent flames are characterized only by their geometrical and kinematic properties. Flame geometry is well described in terms of surface density (surface per unit of volume), curvature, stretch, etc. As far as kinematic features are concerned one has to distinguish between the flame displacement speed noted S_d (the speed at which a particular iso-scalar propagates relative to the flow velocity) and the consumption speed S_c (the speed at which the reactants are consumed). S_c is a global quantity as it relates to the integrated fuel reaction rate over the flame normal direction. It appears notably in flame surface density models of the sub-grid scale reaction rate. S_d depends on the iso-scalar used to track the flame and emanates when characterizing the spatio-temporal evolution of the interface (as in the G-equation for instance).

However, it is well known that S_c and S_d can be locally affected by the flame geometrical properties (for instance stretch and curvature) and *vice-versa*. Consequently, the flamelet hypothesis requires some closure equations providing an explicit relationship between flame speeds and the interface geometrical properties. In this goal, one needs appropriate tools of investigation among which experiments in well controlled situations are particularly insightful.

While the displacement speed and the flame geometrical properties are measurable, a proper experimental assessment of the consumption speed is challenging as the fuel reaction rate is not accessible. Some studies [1] reveal that surrogates of the heat release such as CO_2^*, CH^*, OH^*, HCO can be employed. However, these surrogates are unequally appropriate [1], and the relationship between fuel reaction rate and measured intensity of these surrogates may not hold in general. Here, we propose a novel approach inspired by previous work on spherical flames [2, 3], tailored for simultaneous Mie scattering tomography - PIV measurements.

This paper aims at presenting the method and its validation. Application of the technique is discussed in [4]

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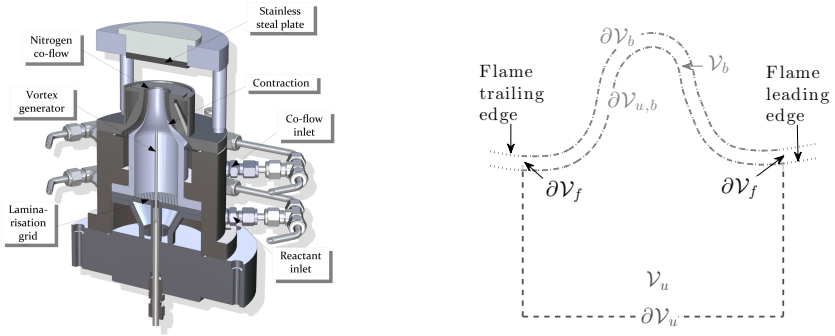

Figure 1: (left) Schematic of the Flame-Vortex Interaction Burner (FVIB). (right) Schematic of the material volume over which the transport equation of Y_f^* is integrated

2 Analytical considerations

A proper method to quantify S_c is to start by integrating the fuel mass fraction equation over a given control volume [2,3]. This method is generally referred to as the integral approach [2]. Let $Y_f^* = (Y_f - Y_{f,b})/(Y_{f,u} - Y_{f,b})$ be the reduced fuel mass fraction where $Y_{f,u}$ and $Y_{f,b}$ are the fuel mass fractions in the fresh and burned gases, respectively. Using the Leibniz-Reynolds transport theorem together with the Green-Ostogradsky theorem, the integration of the transport equation of Y_f^* over a given material volume $\mathcal{V}(t)$ of frontier $\partial\mathcal{V}(t)$ leads to

$$\frac{\mathrm{d}}{\mathrm{d}t}\int\limits_{\mathcal{V}(t)} \rho Y_f^* \mathrm{d}^3V + \oint\limits_{\partial\mathcal{V}(t)} \left[\rho Y_f^* \left(\boldsymbol{u} + \boldsymbol{v}_f - \boldsymbol{w}\right)\cdot\boldsymbol{n}\right] \mathrm{d}^2A = \int\limits_{\mathcal{V}(t)} \dot{\omega}_f^* \mathrm{d}^3V \tag{1}$$

where ρ is the density, $\boldsymbol{u}$ is the fluid velocity, $\boldsymbol{v}_f$ the diffusion velocity of Y_f^* and $\boldsymbol{w}$ the absolute displacement speed of the volume boundary $\partial\mathcal{V}(t)$. $\boldsymbol{n}$ is the outward-pointing unit-normal to $\partial\mathcal{V}(t)$. $\dot{\omega}_f^* = \dot{\omega}_f/(Y_{f,u} - Y_{f,b})$ is the reaction rate of the reduced fuel mass fraction. Because some (if not all) quantities appearing in Eq. (1) are not directly and simultaneously measurable, the control volume $\mathcal{V}$ has to be carefully defined. A schematic of the control volume $\mathcal{V}$ is presented in Fig. 1. $\mathcal{V}$ is decomposed into two volumes $\mathcal{V}_u$ and $\mathcal{V}_b$ which are characterized below:

• The volume $\mathcal{V}_u$ is the volume enclosed between boundaries $\partial\mathcal{V}_u$ and $\partial\mathcal{V}_{u,b}$. It covers all the fresh gases ($\rho = \rho_u$, $Y_f^* = 1$) until the leading edge of the flame $\partial\mathcal{V}_{u,b}$ where $Y_f^* = 1 - \mathcal{O}(\epsilon)$. The boundary $\partial\mathcal{V}_u$ is static so that $\boldsymbol{w}\cdot\boldsymbol{n} = 0$. We also impose $\boldsymbol{v}_f\cdot\boldsymbol{n} = 0$ at $\partial\mathcal{V}_u$ and $\partial\mathcal{V}_{u,b}$, i.e. zero gradient of Y_f^* at the boundaries.

• The volume $\mathcal{V}_b$ is the volume enclosed between boundaries $\partial\mathcal{V}_b$, $\partial\mathcal{V}_{u,b}$ and $\partial\mathcal{V}_f$. It comprises the flame volume from the leading edge $\partial\mathcal{V}_{u,b}$ where $\rho = \rho_u$, $Y_f^* = 1$ to the trailing edge $\partial\mathcal{V}_b$ where $\rho \approx \rho_b$ and $Y_f^* = 0 + \mathcal{O}(\epsilon)$. Therefore, $\oint_{\partial\mathcal{V}_b} \rho Y_f^* (\boldsymbol{u} - \boldsymbol{w} + \boldsymbol{v}_f)\cdot\boldsymbol{n} = 0$ since Y_f^* is zero at the level of $\partial\mathcal{V}_b$. The flux at the boundary $\partial\mathcal{V}_f$ will be determined later.

With this definition of $\mathcal{V}$, Eq. (1) can be further simplified by integrating Eq. (1) over $\mathcal{V}_u$ and then proceed with the integration over the volume $\mathcal{V}_b$. In the unburned gases volume $\mathcal{V}_u$, we have $\rho = \rho_u$ and $Y_f^* = 1$

together with $\omega_f^* = 0$ everywhere. Therefore, Eq. (1) writes

$$\begin{aligned}\text{Eq.}(1)|_{\mathcal{V}_u} &= \rho_u \frac{\mathrm{d}}{\mathrm{d}t} \int\limits_{\mathcal{V}_u(t)} \mathrm{d}^3V + \rho_u \oint\limits_{\partial\mathcal{V}_u(t)} \boldsymbol{u}\cdot\boldsymbol{n}\mathrm{d}^2A + \rho_u \oint\limits_{\partial\mathcal{V}_{u,b}(t)} \underbrace{(\boldsymbol{u}-\boldsymbol{w})\cdot\boldsymbol{n}}_{S_d^u} \mathrm{d}^2A \\ &= \rho_u \frac{\mathrm{d}\mathcal{V}_u}{\mathrm{d}t} + \rho_u \oint\limits_{\partial\mathcal{V}_u(t)} \boldsymbol{u}\cdot\boldsymbol{n}\mathrm{d}^2A + \rho_u\langle S_d^u\rangle A_f = 0 \end{aligned} \tag{2}$$

where the brackets stand for the area weighted values, i.e. $\langle S_d^u\rangle = \int S_d^u \mathrm{d}^2A/A_f$. A_f is the surface of the leading edge $\partial\mathcal{V}_{u,b}$. Therefore, the budget in the unburned volume $\mathcal{V}_u$ yields an expression for the area weighted displacement speed $\langle S_d^u\rangle = \langle(\boldsymbol{u}-\boldsymbol{w})\cdot\boldsymbol{n}\rangle$

$$\langle S_d^u\rangle = -\frac{1}{A_f}\frac{\mathrm{d}\mathcal{V}_u}{\mathrm{d}t} - \frac{1}{A_f}\oint\limits_{\partial\mathcal{V}_u(t)} \boldsymbol{u}\cdot\boldsymbol{n}\mathrm{d}^2A \tag{3}$$

The first term on LHS of Eq. (1) over $\mathcal{V}_b$ cannot be measured as we have no information about Y_f^* and ρ in the flame. One solution consists in expressing the first term on LHS of Eq. (1) in the flame curvilinear basis

$$\frac{\mathrm{d}}{\mathrm{d}t}\int\limits_{\mathcal{V}_b(t)} \rho Y_f^* \mathrm{d}^3V = \frac{\mathrm{d}}{\mathrm{d}t}\int\limits_{A_f} \underbrace{\left[\int\limits_{\eta} \rho Y_f^* \mathrm{d}\eta\right]}_{\rho_u\mathcal{F}} \mathrm{d}^2A_f = \rho_u \frac{\mathrm{d}}{\mathrm{d}t}\langle\mathcal{F}\rangle A_f \tag{4}$$

where η is the flame normal coordinates. Then, following [3], $\rho_u\mathcal{F}$ can be estimated by taking the average value of ρY_f^* between the unburned and burned gases, viz.

$$\mathcal{F} = \frac{1}{\rho_u}\left(\frac{\rho_u Y_{f,u}^* + \rho_b Y_{f,b}^*}{2}\right)\delta_L = \frac{\delta_L}{2} \tag{5}$$

where δ_L is the flame thickness whose appropriate definition may be here $\eta(Y_f^* = 0.99) - \eta(Y_f^* = 0.01)$. The second term on LHS of Eq. (1) integrated over $\mathcal{V}_b$ leads to

$$\oint\limits_{\partial\mathcal{V}_{u,b}(t)+\partial\mathcal{V}_f(t)} \left[\rho Y_f^*\left(\boldsymbol{u}+\boldsymbol{v}_f-\boldsymbol{w}\right)\cdot\boldsymbol{n}\right]\mathrm{d}^2A = -\rho_u\langle S_d^u\rangle A_f + \oint\limits_{\partial\mathcal{V}_f}\left[\rho Y_f^*\left(\boldsymbol{u}+\boldsymbol{v}_f-\boldsymbol{w}\right)\cdot\boldsymbol{n}\right]\mathrm{d}^2A \tag{6}$$

Here, we neglect the diffusion velocity $\boldsymbol{v}_f\cdot\boldsymbol{n}$ at the boundary $\partial\mathcal{V}_f$ assuming that the species gradient is oriented in the flame normal direction. Second, we have $\boldsymbol{w}\cdot\boldsymbol{n} = 0$ at $\partial\mathcal{V}_f$. Finally, we take the average value of $\rho Y_f^*\boldsymbol{u}\cdot\boldsymbol{n}$ between the burned and fresh regions to obtain:

$$\oint\limits_{\partial\mathcal{V}_f}\left[\rho Y_f^*\left(\boldsymbol{u}+\boldsymbol{v}_f-\boldsymbol{w}\right)\cdot\boldsymbol{n}\right]\mathrm{d}^2A = \rho_u\mathcal{F}\pi\Delta v_r \tag{7}$$

using the axisymmetry of our configuration. v_r is the tangential velocity close to the flame in the unburned gases and Δ is the diameter of the control volume. The term on RHS of Eq. (1) is written in the coordinate system attached to the flame and leads to

$$\int\limits_{\mathcal{V}_b(t)} \dot{\omega}_f^* \mathrm{d}^3V = -\rho_u\langle S_c\rangle A_f \tag{8}$$

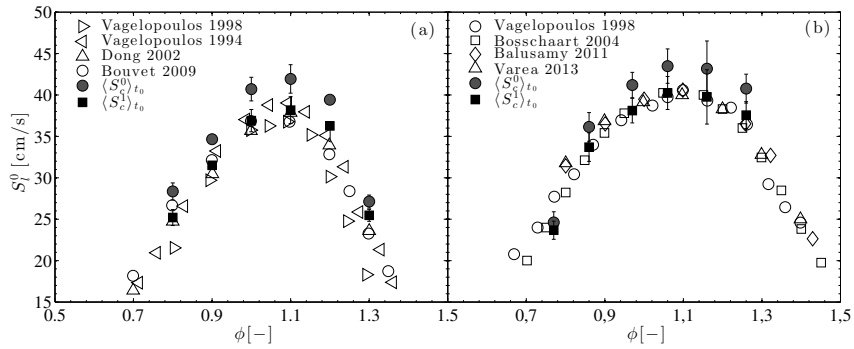

Figure 2: Comparison of the present assessement of the consumption speed for methane (a) and propane/air mixtures (b) at different equivalence ratios ϕ with some data from the litterature.

Finally, the integration of the mass budget Eq. (1) over $\mathcal{V}_b$ leads to

$$\langle S_c \rangle = \langle S_d^u \rangle - \mathcal{F} \left(\frac{1}{A_f} \frac{\mathrm{d}A_f}{\mathrm{d}t} + \frac{1}{A_f} \pi \Delta v_r \right) \tag{9}$$

To obtain Eq. (9), we assumed that $\langle \mathcal{F} \rangle = \mathcal{F} = \mathrm{const}$. This hypothesis will be tested later using DNS of a flame/vortex interaction. Eq. (9) provides an explicit relationship between the displacement and the consumption speeds. This expression can be used in experiments as all the terms appearing in Eq. (9) are measurable.

3 Experimental and numerical validation

The adequacy of Eq. (9) for measuring the flame consumption speed is first tested experimentally for steady stagnation point flames in the so-called Flame/Vortex Interaction Burner (FVIB) (Fig. 1). The FVIB and the measurement techniques have been fully described elsewhere and the reader is referred to [5] for detailed informations.

We will focus on the estimation of $\langle S_c^0 \rangle_{t_0}$, i.e. the consumption speed at steady state assuming $\mathcal{F} = 0$ and $\langle S_c^1 \rangle_{t_0}$, the consumption speed at steady state inferred with $\mathcal{F} = \delta_L/2$), for methane/air and propane/air mixtures. Even though the flames considered here are strained, we compare directly our estimates to S_l^0, i.e. the extrapolated value of S_c or S_d^u to zero stretch generally reported in papers. This seems reasonable for the low values of the strain rate measured in our experiments, K_r being about 80. Results (Fig. 2) indicate a substantial difference between $\langle S_c^1 \rangle_{t_0}$ and $\langle S_c^0 \rangle_{t_0}$, with $\langle S_c^1 \rangle_{t_0}$ being systematically closer to experimental data taken from the literature. The contribution of the mass flow rate through $\partial \mathcal{V}_f$ is between 7% to 10% depending on the type of fuel and equivalence ratio. Present estimates of $\langle S_c^1 \rangle_{t_0}$ are in excellent agreement with experimental data from the literature. The small differences that are observed are about 5% which may be attributed to experimental uncertainties and the effect of stretch.

These first tests show that the present method is reliable for measuring the flame consumption speed at least at stationary state. Second, the zero flame thickness assumption ($\mathcal{F} = 0$) appears to be inadequate and the improvement provided by assuming $\mathcal{F} = \delta_L/2$ is significant and provides trustworthy results.

Eq. (9) is further validated using 2D DNS of flame/vortex interactions. Simulations have been carried out using the AVPB solver. The third-order scheme TTGC [6] was used. The boundary conditions were

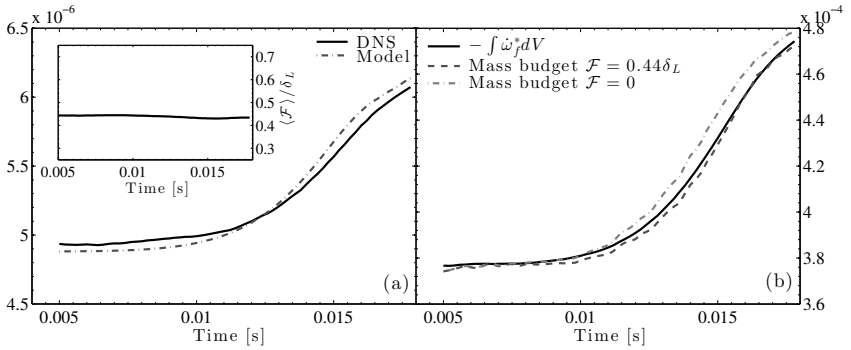

Figure 3: (a) Time evolution of the mass contained within the flame as inferred from the DNS (LHS on Eq. (10)) or the model with $\mathcal{F} = \text{const} = 0.44\delta_L$ (RHS on Eq. (10)). The figure in caption represents the ratio $\mathcal{F}/\delta_L$. (b) Volume integrated fuel reaction rate as inferred from the DNS, compared to the mass budget using either $\mathcal{F} = 0$ or $\mathcal{F} = \text{const} = 0.44\delta_L$.

implemented with the Navier-Stokes Characteristic Boundary Conditions (NSCBC) [7]. Only half of the domain was simulated taking advantage of the symmetry. Methane combustion kinetics was described using the 19 species scheme of [8]. The initial velocity field is prescribed using a Taylor type (gaussian) vortex with similar vortex/flame characteristics than in the experiments. The assumptions that have to be tested through the DNS are the following. First, we have to check if the fuel mass fraction within the flame is accurately captured by stating

$$\iiint_{\mathcal{V}_b} \rho Y_f^* \mathrm{d}^3V = \rho_u \langle \mathcal{F} \rangle A_f \tag{10}$$

Second, we have to check whether $\mathcal{F}$ is constant in time or if the flame is thickened or shrunk. Finally, the ratio $\mathcal{F}/\delta_L$ itself needs to be determined and compared to the model of [3], i.e. $\mathcal{F} = \delta_L/2$.

Results are presented in Fig. 3(a) where we first test the validity of Eq. (10). The mass within the flame is inferred from DNS and compared to the model of [3] using $\mathcal{F} = \text{const}$. One observes very marginal differences between the model and the DNS suggesting that $\mathcal{F} = \text{const}$ is appropriate. This is further confirmed by plotting the ratio $\mathcal{F}/\delta_L$ as a function of time (inset in 3(a)) which appears to be nearly constant. The flame zone is thus only very weakly affected by thickening or shrinking effects which are moderate in our case. The average value for $\mathcal{F}$ is about $0.44\delta_L$ which is in fairly good agreement with the expected theoretical value of $0.5\delta_L$.

An overall test of Eq. (9) is also portrayed in Fig. 3(b). The volume integrated fuel reaction rate is compared to the one calculated by $\rho_u \langle S_c \rangle A_f$, where $\langle S_c \rangle$ is calculated from Eq. (9). Fairly good agreement is observed thus validating the proposed assumptions. The reaction rate measured assuming $\mathcal{F} = 0$ is also displayed indicating that a good estimation of the mass ($\mathcal{F} = 0.44\delta_L$) within the flame thickness is mandatory for accurately estimating the fuel consumption speed.

3 Summary and outcomes

The main result of the present study is a method for experimentally evaluating the displacement and consumption speeds during flame/vortex interactions. It is based on the integration of the fuel mass fraction over

a well chosen volume. Validation is carried out using both experiments and 2D DNS. This technique is not dedicated to flame/vortex configurations only but can be employed in other situations as long as the flame thickness remains on average constant while being stretched and curved. The method is thus not suitable when there is significant flame extinction or large variations of flame thickness. The possible extension to other flows deserves to be investigated further and is likely to help significantly for assessing the sensitivity of flame speed to e.g. stretch and curvature.

The present paper is devoted to the presentation and validation of the method. Eqs. (3) and (9) were employed to experimentally investigate the sensitivity of the displacement and consumption speeds to stretch and curvature in the FVIB. These results provide a clear confirmation of the existence of the two Markstein numbers [9, 10], one characterizing the dependence of flame speed to curvature, the other to stretch. This outcome is the object of another paper [4]

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