Traveling Vortex in a Natural Convection Field

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1 Introduction

Natural convection governs the heat, mass, and momentum transfers of fire in an enclosure. Real scale room fire has large ceiling height, the Grashof number is order of 10^{12} that is large enough to be turbulent [1].
In turbulent flow, the velocity vector which has its scalar and orientation fluctuates with time.
In converging flow, there is no limitation on the converge angle of stream lines. In diverging flow, there are some limitations on the diverge angle for viscous flow [1]. These limitations may define power spectrums of scalar length and crossing angle of velocity vectors.
A simple numerical simulation of a square space heated from the floor produces a typical natural convection field. If the flow field is laminar, a steady flow field will be seen. If the flow field is turbulent, vortices will be seen.
If a vortex blob [2–4] locates between adjoining points, the velocity vectors at these pints cross with an angle close to π. For most part of flow field, the velocity vectors does not cross and the angle is zero.
Once flow field is given, by taking the inner product of these velocity vectors, the crossing angle is determined.

2 Numerical Simulation

A simple numerical simulation of viscid flow field [5] was carried out to investigate the flow and thermal field near the combustible surface. Due to limitation of simulation capability, a control volume with ceiling, walls, and floor boundaries is used. In this numerical simulation, two-dimensional model of a confined volume was employed. Based on these reference quantities, the following dimensionless variables are constructed.

$$X = x/H,\ Y = y/H,\ T = t/(H^2/\nu),\ U = u/(\nu/H),\ V = v/(\nu/H) \tag{1}$$

Square meshes of size 300 x 300 are chosen for this simulation. The heat release in the flame was replaced with a high temperature surface on the floor boundary at Y = 1. Each simulation carried out from 0 to 8.0 x 10^{-5} in nondimensional time T. The governing equations for the fluid motion are the continuity, the Navier-Stokes adding buoyant force, the energy equations and the equation of state. Simplifications are made by using the Boussinesq approximation.

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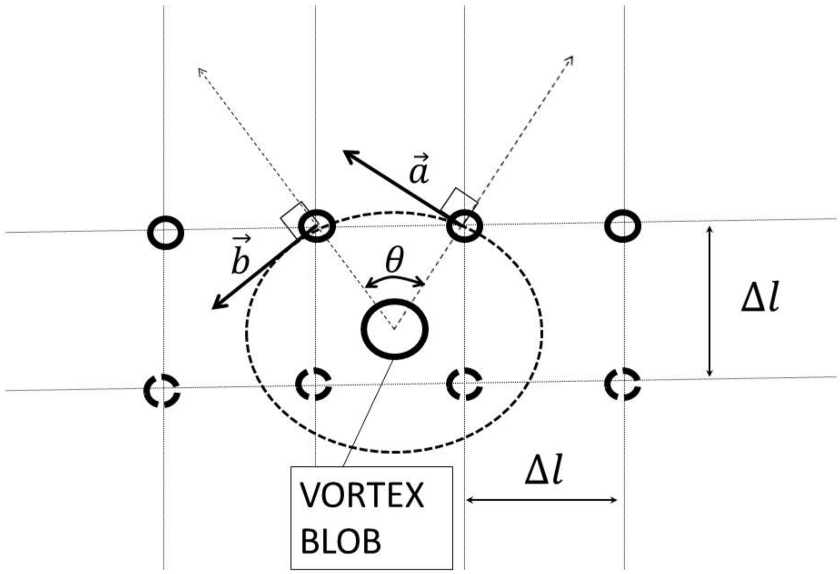

Figure 1: Vortex Blob and Two Dimensional Grid Space

3 Inner Product of Velocity Vectors

In a two dimensional grid space of Δl, velocity vectors $\vec{a}$ and $\vec{b}$ at adjoining points near a vortex blob are crossing with an angle θ as shown in figure 1. The angle θ is less than $2\arctan(\frac{1}{2})$ = 0.927 where the vortex blob is on the adjoining grid line. The inner product of these vectors p is

$$p = \vec{a}\,\vec{b} = |\vec{a}||\vec{b}|\cos(\theta) \tag{2}$$

The crossing angle θ is

$$\theta = \arccos(\frac{\vec{a}\,\vec{b}}{|\vec{a}||\vec{b}|}),\ 0 \le \theta \le \pi \tag{3}$$

If vortex blob is close to these two points, the angle θ becomes larger than 0.927.
Figure 2 shows number density n of adjoining points which velocity vectors cross with an angle θ for G_r = 1.0 x 10^7 to 3.0 x 10^{11} in a logarithmic scale. For $G_r > 1.0$ x 10^{12}, the simulation does not converge. Number density is normalized by the total cell count 90000.
Two groups of plots are seen in this figure. One group G1 is plots from G_r = 1.0 x 10^7 to 1.0 x 10^9, which the number density n becomes below the limiting value of 1.0 x 10^{-5} at θ = 0.927. Other group G2 is plots from G_r = 1.0 x 10^{10} to 3.0 x 10^{11}, which the number density n is above the limiting value at θ = 0.927. The plot of G_r = 3.0 x 10^9 is between these two groups.
The presence of inner products which angle θ is larger than 0.927 indicates that vortices are entering the flow field for $G_r > 3.0$ x 10^9.
The number density n is proportional with $\theta^{-2.8}$ for group G1, $\theta^{-2.25}$ for group G2. The number density n is constant for $\theta < 0.02$ among group G1.
Vortex blob is locates near the grid where inner product of velocity vectors gives an angle larger than 2 rad.

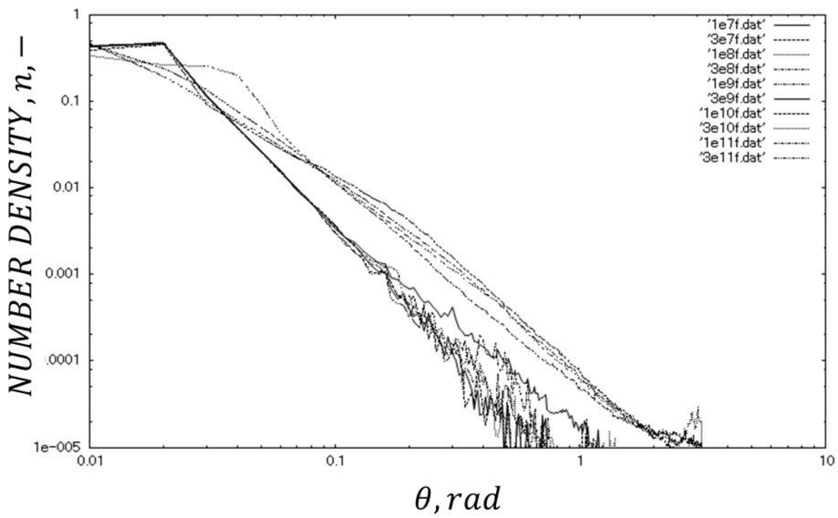

Figure 2: Number Density with θ

4 Vortex Blob

Figures 3 and 4 show horizontal and vertical vortex blob locations with time for G_r = 3.0 x 10^8. After T = 1.0 x 10^{-5}, a pair of vortex blobs appears corners on the floor. At T = 5.0 x 10^{-5}, vortex blobs appear near ceiling and move to floor, other vortex blobs move from floor to ceiling.
Figures 5 and 6 show horizontal and vertical vortex blob locations with time for G_r = 3.0 x 10^{10}. At T = 3.0 x 10^{-6}, vortex blobs appear near ceiling and move to floor, other vortex blobs move from floor to ceiling. Vortex blobs at corner on floor move to the center in horizontal plain and rise from T = 1.0 x 10^{-5} to 2.3 x 10^{-5}. Many vortex blobs appear on the floor at T = 2.3 x 10^{-5}. and rise from T = 2.3 x 10^{-5} to 4.0 x 10^{-5} in vertical cross section. Vortex blobs seem to corride from T = 4.0 x 10^{-5} to 7.0 x 10^{-5}.
Figure 7 shows number of vortex blobs M with time T for G_r = 1.0 x 10^8 to 3.0 x 10^{11} in a logarithmic scale. Numbers of vortex blobs M are 2, 4, 20-30, 50-200 for periods of a pair, two pairs, rising, and interacting vortex blobs. The numbers of 20-30 and 50-200 depend on the used cell number of numerical simulation.

5 Discussion

Inner product of two adjoin velocity vectors gives a crossing angle θ <0.927 for $G_r < 1.0$ x 10^{10}. Vortex blobs are on the boundaries to define the boundary conditions. As Grashof number G_r increases, vortex blobs move into the control volume and determined crossing angle θ increases beyond 0.927. The number density n distribution with crossing angle θ changes with Grashof number G_r.
A uniform number density n distribution appears at $\theta < 0.2$ for $G_r < 1.0$ x 10^{10},this uniform distribution may be one of the limitations on diverge angle for viscous flow.

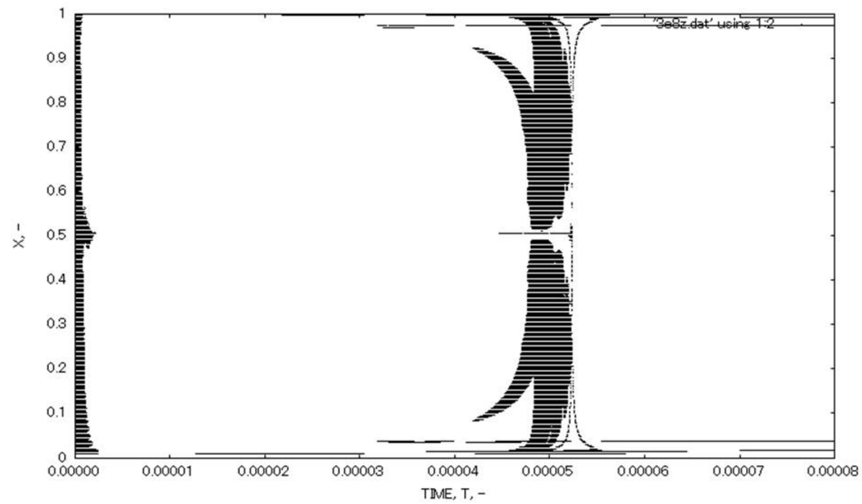

Figure 3: Horizontal Vortex Blob Location with Time (G_r = 3.0 x 10^8)

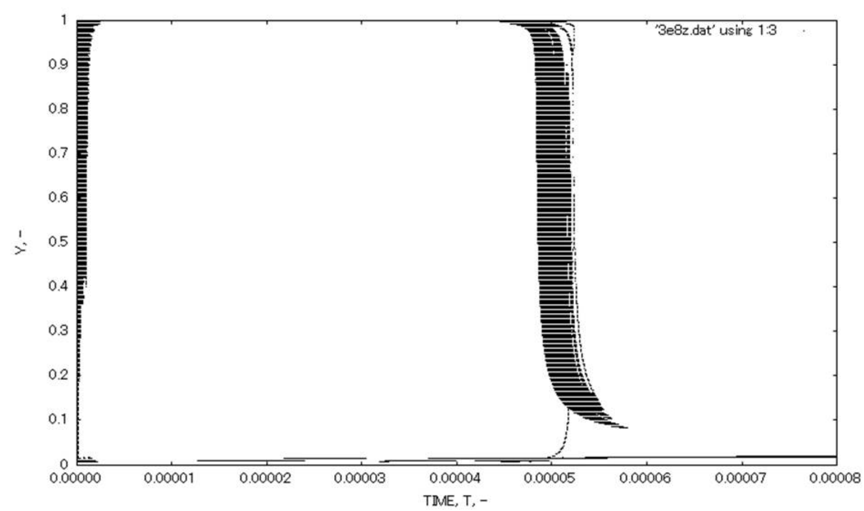

Figure 4: Vertical Vortex Blob Location with Time (G_r = 3.0 x 10^8)

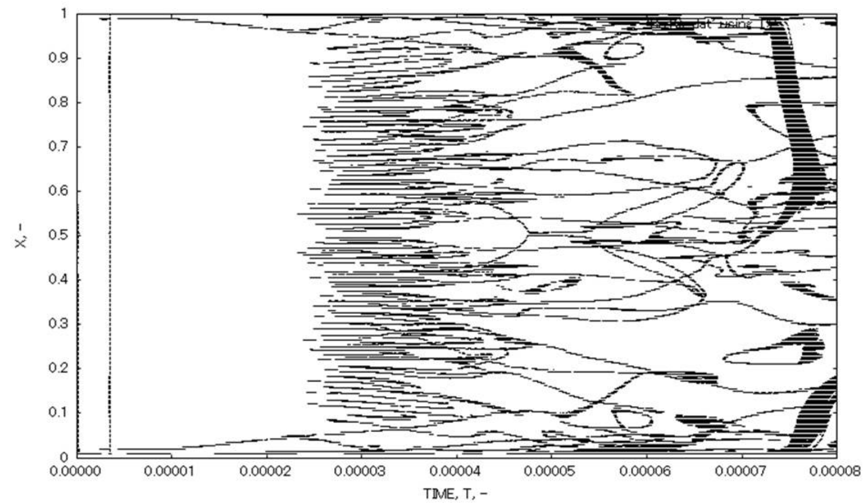

Figure 5: Horizontal Vortex Blob Location with Time (G_r = 3.0 x 10^{10})

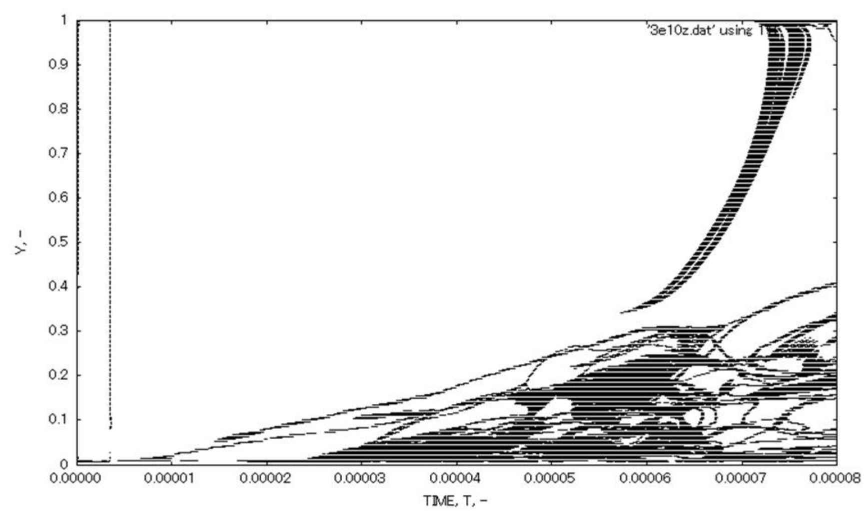

Figure 6: Vertical Vortex Blob Location with Time (G_r = 3.0 x 10^{10})

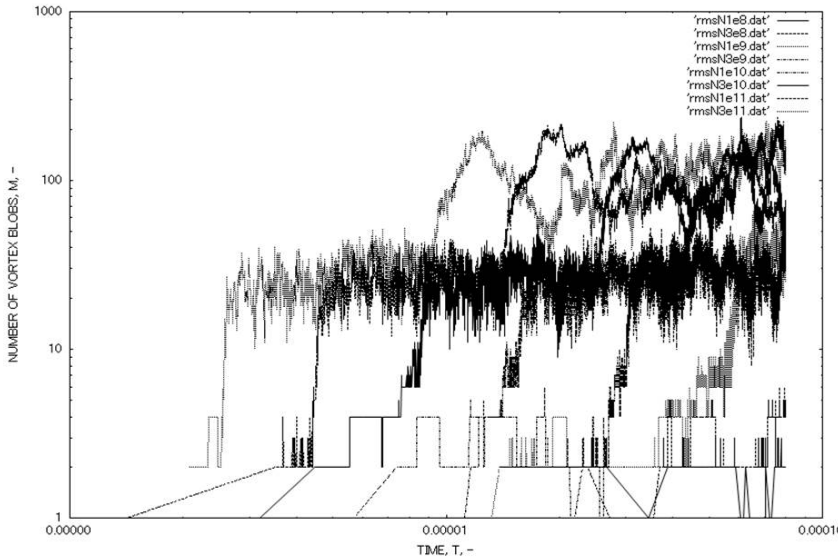

Figure 7: Number of Vortex Blobs with Time

6 Conclusion

Inner product of velocity vectors gives the crossing angle θ which is an index to determine the presence of vortex blob. Using the distribution of crossing angle, an uniform region is seen for $\theta < 0.2$ from G_r = 1.0 x 10^7 to 1.0 x 10^9. The trace of vortex blobs shows a series of behavior, a pair, two pairs, 10-30, and 50-300 of vortex blobs in the control volume from G_r = 1.0 x 10^{10} to 3.0 x 10^{11}.

References

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[3] A. F. Ghoniew, A. J. Chorin, A.K. Oppenheim, Numerical Modeling of Turbulent Flow in a Combustion Tunnel, Philosophical Transactions of the Royal Society of London, Series A, Vol. 304 (1982).

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