# Smoldering Spread Velocity along a Thin Solid in a Narrow Channel

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## **1** Introduction

Smoldering combustion is the slow, low-temperature, flameless burning of solid fuels, and it is common to charring fuels [1]. Pyrolysis of such a solid fuel results in gaseous and solid (char and ash) products, and smoldering combustion is characterized by char oxidation on its surface without significant oxidation of the gaseous products.

Smoldering combustion tends to occur when oxygen supply is limited. In a narrow space or in a microgravity environment, for example, oxygen supply due to natural convection is suppressed, leading to smoldering combustion [2–4]. Since smoldering spread often occurs at early stages of fires, understanding its mechanism helps us predict their initial growth rates. Furthermore, the smoldering of a curing sheet in a narrow space is a potential fire scenario in plants [5]. Smoldering combustion is also common in wildland fires in peatlands such as those causing haze problems in Southeast Asia and Northeast Europe [6].

This study focuses on the smoldering spread along a thin solid in a narrow channel because wellcontrolled experiments can be conducted relatively easily (see Refs. [3–5, 7–9]), enabling detailed comparison between experimental data and theoretical predictions. Although a number of theoretical studies on the phenomenon have been conducted [8–13], most of them focused on the occurrence of fingering instability, whereas the smoldering spread velocity, which is the most important information for predicting fire growth rates, has been rarely addressed. In particular, detailed comparison between experimentally measured spread velocities and theoretical predictions has never been reported to the best of our knowledge. The spread velocity is influenced by fingering instability [8, 12], and a theoretical model that takes into account the instability mechanism must be considered. This study reports the comparison between experimentally measured spread velocities and theoretical predictions of such a model.

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## 2 Experimental setup

The same experimental setup as our previous study [9] was used. Either a filter paper sheet  $(0.54 \text{ g/cm}^3)$  or a low-density paper sheet  $(0.15 \text{ g/cm}^3)$  of 80 mm wide  $\times$  300 mm long was burned in a narrow channel between two parallel plates. The vertical location of the paper sheet was at the center of the channel. An oxidizer flow of a specified velocity was supplied in the opposite direction to the smoldering spread.

The paper edge on the downstream side of the oxidizer flow was uniformly ignited using a slot burner. In this study, the following three experimental parameters were varied: the channel height,  $d_g$ , from 7 to 12 mm; the oxidizer velocity, u, from 15 to 450 mm/s; and the oxygen mass fraction in the oxidizer stream,  $Y_{0,u}$ , from 0.23 to 0.36. Ten runs were conducted under each condition, and the smoldering spread velocities reported below are the average values.

#### **3** Results and Discussion

Figure 1 shows smoldering spread velocities, v, at two different channel heights,  $d_g$ , as functions of oxidizer velocity, u. The error bars show the standard deviations. Two trends can be noted from the figure. Firstly, the spread velocity first increases and then decreases with an increase in oxidizer velocity. Secondly, the spread velocity increases with an increase in channel height. These trends can be explained by the unperturbed, one-dimensional theory developed in Ref. [9] that yields

$$v^{(0)} = \frac{2u}{\beta m} \ln\left(\frac{u_{\rm r}}{u}\right) - \frac{NuD_{\rm th}\lambda_{\rm g}}{m\rho_{\rm g}c_{\rm g}d_{\rm g}^2 u} \tag{1}$$

where  $\beta$  is the Zel'dovich number,  $m \equiv \rho_s c_s d_s / \rho_g c_g d_g$  is the weighted density ratio ( $\rho$  is the density, c is the specific heat,  $d_s$  is the solid thickness, and subscripts g and s denote the gas and the solid phases, respectively),  $u_r$  is the reference velocity at which v = 0 when there is no heat loss, Nu is the Nusselt number used in the heat loss term,  $D_{\rm th} \equiv (\lambda_{\rm g} d_{\rm g} + \lambda_{\rm s} d_{\rm s}) / \rho_{\rm g} c_{\rm g} d_{\rm g}$  is the thermal diffusivity, and  $\lambda$  is the thermal conductivity. Figure 1 shows that the predictions of the one-dimensional theory are smaller than the measured velocities. This is because of the following two reasons: (1) fingering instability occurred in the experiments especially when u was relatively small (see Fig. 1), and reactions at smoldering fronts were enhanced by the instability mechanism, whereas the one-dimensional theory cannot consider any instability effects by its nature; (2) the theory assumes constant values for thermophysical properties, and those at the adiabatic smoldering temperature (assumed to be 700 K [4]) were used as in Ref. [9]. The predicted spread velocity increased close to the measured data when the thermophysical properties were evaluated at 300 K. However, we do not further discuss the temperature at which thermophysical properties should be evaluated because one-dimensional experiment for comparison is not possible. Hence, the values at the adiabatic smoldering temperature are used throughout this study. The choice of thermophysical properties quantitatively changes model predictions but does not alter the conclusions of this study. Furthermore, the discussion below is limited to the regime where spread velocity increases with oxidizer velocity because of the apparent occurrence of fingering instability in that regime; the major objective of this study is to discuss the influences of fingering instability on spread velocity.

Figure 2 shows the relationship between the rate of solid consumption, calculated as  $vd_sw\rho_s f_b$  (w is the solid width, and  $f_b$  is the fraction burned), and the rate of oxygen supply,  $ud_gw\rho_g Y_{0,u}$ . A linear relationship exists between them, indicating that the overall mass balance is roughly satisfied under the conditions tested. The information shown in Figure 2, however, cannot be used for predicting spread

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velocity because the fraction burned depends on fingering instability (see Figure 1) and is not known a priori, necessitating a stability analysis.





Figure 1. Left: smoldering spread velocity and oxidizer velocity; right: fingering patterns obtained at  $d_g = 10 \text{ mm}$ , u = 19 mm/s (top) and 50 mm/s (bottom). Experiments were conducted at  $Y_{0,u} = 0.23$  (air).



The present analysis is based on the same conservation equations as Ref. [9],

$$\left(\rho_{\rm g}c_{\rm g}d_{\rm g} + \rho_{\rm s}c_{\rm s}d_{\rm s}\right)\frac{\partial T}{\partial t} + \rho_{\rm g}c_{\rm g}d_{\rm g}u\frac{\partial T}{\partial x} = \left(\lambda_{\rm g}d_{\rm g} + \lambda_{\rm s}d_{\rm s}\right)\nabla^2 T + QW - \frac{Nu\lambda_{\rm g}}{d_{\rm g}}(T - T_{\rm u}) \tag{2}$$

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$$\rho_{\rm g} d_{\rm g} \frac{\partial Y_{\rm O}}{\partial t} + \rho_{\rm g} d_{\rm g} u \frac{\partial Y_{\rm O}}{\partial x} = \rho_{\rm g} d_{\rm g} D \nabla^2 Y_{\rm O} - W \tag{3}$$

which were obtained by reducing a three-dimensional system to the two-dimensional one using a similar technique to Kagan and Sivashinsky [10]. Here, x is the longitudinal direction, whereas y is the transverse direction. The last term on the right hand side of Eq. (2) is a heat loss term. A first-order reaction of oxygen is assumed for W, whereas pyrolysis reaction is not considered; see Ref. [14] for the effects of pyrolysis.

Introducing the dimensionless system of  $(\xi, \eta) = u_r(x, y)/D_{th}$ ,  $U = u/u_r$ , and  $\tau = 2u_r^2 t/\beta m D_{th}$  (hence  $V = \beta m v/2u_r$ ), a lengthy, but straightforwad linear stability analysis with the use of activation-energy asymptotics leads to the following dispersion relation:

$$\omega = U^2 \left[ \left( \alpha + \ln \frac{1}{U} + \frac{\kappa}{U^2} - 1 \right) \Delta - 4\Delta^2 - 4\Delta^3 \right]$$
(4)

where  $\Delta = (\sqrt{1 + 4U^{-2}k^2} - 1)/2$ . The scaled Lewis number,  $\alpha$ , and the scaled (dimensionless) heat loss coefficient,  $\kappa$ , are defined by the following equations:

$$\alpha = \frac{\beta(1 - Le)}{2} \text{ with } Le = \frac{D_{\text{th}}}{D}, \qquad \kappa = \frac{\beta N u D_{\text{th}} \lambda_g}{2\rho_g c_g d_g^2 u_r^2}$$
(5)

The transformation of  $k = U\hat{k}$  and  $\omega = U^2\hat{\omega}$  leads to the following one-parameter dispersion relation:

$$\widehat{\omega} = (F-1)\widehat{\Delta} - 4\widehat{\Delta}^2 - 4\widehat{\Delta}^3 \tag{6}$$

where  $\hat{\Delta} = (\sqrt{1+4\hat{k}^2}-1)/2$  and  $F = \alpha + \ln(1/U) + \kappa/U^2$ . Noting that the dispersion relation expresses the relative growth rate of wavenumber k to the unperturbed front propagating at  $V^{(0)}$ , and assuming that the spread velocity is proportional to the characteristic velocity  $\omega/k$  of a certain critical wavelength, the transformation leading to Eq. (6) indicates that the scaled relative spread velocity defined as

$$\hat{V} = \frac{V - V^{(0)}}{U}$$
(7)

depends only on parameter F. Figure 3 plots  $\hat{V}$  computed from spread velocities of different series of experiments against F. Each error bar shows the standard deviaion of 10 runs under the same condition; the error associated with evaluating  $V^{(0)}$  was not estimated because of the difficulty discussed above in connection with Eq. (1). It appears that all the experimental data can be expressed by a single function of the sole parameter F, supporting the present analysis. The scaled relative velocity increases with an increase in F because smoldering front becomes more unstable, and the oxidation reaction is enhanced by the instability. At F = 1 (stability threshold), on the other hand,  $\hat{V}$  should be equal to zero; in other words, the spread velocity should be equal to the unperturbed value without instability. However, the experimental data shown in Figure 3 appear to approach a higher value than zero as  $F \rightarrow 1$ . This is because of the difficulty in evaluating  $V^{(0)}$ .

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Figure 3 also plots the smoldering velocities obtained by the numerical simulations described in Ref. [8]. Because of numerical difficulties, the range of F tested in the present numerical simulations does not cover experimental conditions. Nevertheless, the numerical results do not contradict the experimental data.

Near the stability threshold (F = 1), Eq. (6) is simplified to

$$\widehat{\omega} = (F-1)\widehat{k}^2 - 4\widehat{k}^4 \tag{8}$$

Eq. (8) is reduced to the following parameter-free form via the further transformation of  $\hat{\omega} = (F-1)^2 \tilde{\omega}$ and  $\hat{k} = (F-1)^{1/2} \tilde{k}$ :

$$\widetilde{\omega} = \widetilde{k}^2 - 4\widetilde{k}^4 \tag{9}$$

This indicates that

$$\hat{V} \sim (F-1)^{3/2} \tag{10}$$

Although Eq. (10) is accurate only when F is close to unity, it can well correlate the numerical and experimental data when choosing an appropriate proportionality coefficient. Unlike Figure 2, Figure 3 provides predictive information because the value of F depends only on material properties and experimental parameters.



Figure 3. Scaled relative spread velocity  $\hat{V}$  and parameter *F*. The same symbols as Figure 2 are used for experimental data, while  $\blacklozenge$  shows numerical predictions.

#### 4 Conclusions

Smoldering spread along a thin solid in a narrow channel was studied with the major focus on predicting the spread velocity. A number of different series of experiments were conducted, and the spread velocities were measured. A linear stability analysis was then conducted to identify a governing parameter, F. It was shown that all the experimental data can be expressed by a single function of F. Finally, Eq. (10) was derived for predicting the dependence of spread velocity on F. It was found that Eq. (10) can correlate all the numerical and experimental data reasonably well.

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