Disturbance Energy Analysis of Turbulent Swirling Premixed Flame in a Cuboid Combustor

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1 Introduction

Combustion driven oscillation caused in industrial combustion systems, such as gas turbine combustors, has been eagerly investigated for several decades. The sustained attention to combustion instability has grown due to a demand for development of low NO_x emission combustors with lean premixed combustion technologies, since one of the most critical problems in lean premixed combustion is its propensity to induce combustion oscillation driven by thermoacoustic instability. However, it still remains challenging to accurately predict the onset of combustion oscillation and to control a combustion field due to complexity of interactions between flame, acoustic, velocity and other scalar fluctuations. In addition, presence of turbulence in practical combustion systems also makes it difficult to establish theoretical, numerical and experimental approaches to elucidation of critical factors in the combustion instability. The most fundamental and classical understanding of combustion driven oscillation was first shown by Rayleigh [1]. Rayleigh criterion shown below is a commonly used necessary condition for thermoacoustic instability to occur, stating that a combustion field is unstable when the pressure and heat release rate fluctuations are in phase,

$$\int_{\Omega} p' \omega'_T d\boldsymbol{x} > 0 , \quad (1)$$

where p' and ω'_T denote fluctuations of pressure and heat release rate, respectively, and Ω is a combustor domain. Above criterion is often interpreted as a condition for acoustic energy growth as the product $p'\omega'_T$ appears in a source term of a classical acoustic energy equation. Even though the Rayleigh criterion is one of the tools widely used for predicting the onset of combustion instability, its validity in turbulent combustion fields is not clear in terms of acoustic energy growth. This is because the classical acoustic energy equation is derived by imposing a lot of assumptions on exact governing equations. In order to construct an energy corollary and stability criterion, which can be applied to turbulent combustion fields, there have been some attempts to extend the classical acoustic energy to more generalized forms by

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defining disturbance energies [2-9]. Moreover, recent developments in high performance computing have enabled us to conduct three-dimensional direct numerical simulations (DNS) of turbulent flames with detailed chemistry. It implies that detailed analyses of disturbance energy evolution become possible, since DNS provide us access to all scalar/vector variables required to examine a disturbance energy equation which is applicable to combustion fields [8, 9]. In the previous study [10], budget of an exact disturbance energy [8] in turbulent swirling flames was investigated and complete budget closure was demonstrated. The objective of this study is to gain insight into substantial factors in combustion instability and to clarify the effect of swirling intensity on disturbance generation by investigation into components of dominant disturbance source terms.

2 DNS of Turbulent Swirling Premixed Flame

DNS of turbulent swirling premixed flame in a cuboid combustor is conducted employing detailed kinetic mechanism for hydrogen-air combustion. The computational methods and conditions of three-dimensional DNS employed in the present study are the same as those in the previous works [10, 11], and only explained briefly here. The size of the cuboid combustor ($L_x \times L_y \times L_z$) is 15 × 10 × 10 mm^3. The computational domain is discretized by 769 × 513 × 513 grid points. In the present study, DNS is carried out for two swirl number cases (S = 0.6 and 1.2) under two equivalence ratio conditions (ϕ = 0.6 and 1.0). Mean axial velocity of inflow mixture is 200 m/s and inflow Reynolds numbers (Re_{in}) are 6100 for ϕ = 0.6 and 5486 for ϕ = 1.0. Intensity of a velocity perturbation imposed on the inflow velocity is 13.2 m/s. Temperature of the inflow mixture is set to be 700 K. Wall surface temperature is assumed to be 450 K for ϕ = 0.6 and 700 K for ϕ = 1.0. In this paper, only the results for ϕ = 1.0 are shown. Figure 1 shows instantaneous flow fields obtained by DNS, where white iso-surfaces represent vortical structures visualized by the second invariant of a velocity gradient tensor, orange surfaces are volume rendered heat release rate (ω_T), and color map on the walls shows distribution of pressure (p). For S = 0.6, large-scale helical vortices can be observed in upstream region, whereas fine-scale eddies emerge in downstream region due to turbulence transition. As for S = 1.2, a significant convolution of the flame surface by a large-scale vortex is observed near the inlet, while fine-scale eddies also contribute to small-scale distortion of flame surfaces. Regular patterns of pressure distribution suggest that specific thermoacoustic modes are excited in the combustor [10, 11].

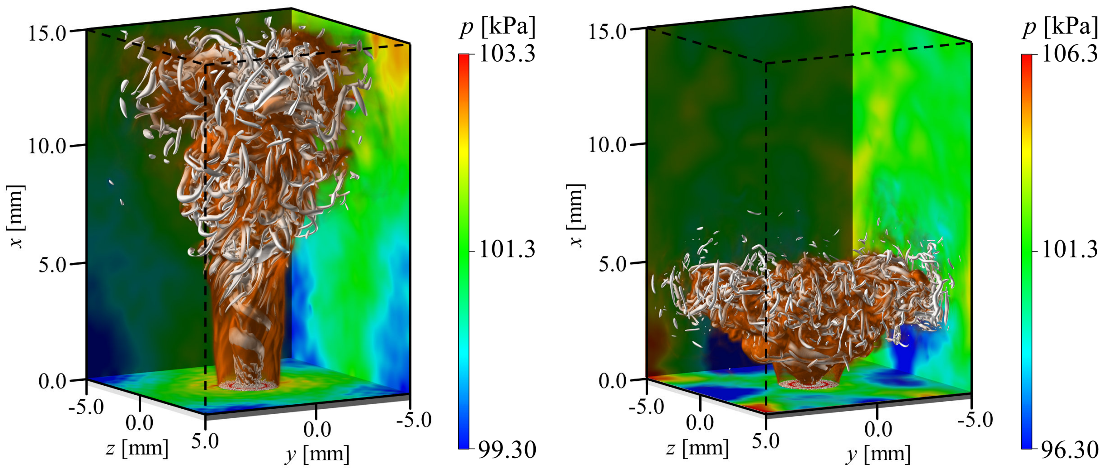

Figure 1. Instantaneous iso-surfaces of the second invariant of a velocity gradient tensor (iso-surfaces at 1% of a maximum value are visualized), volume rendered ω_T, and distribution of p on the walls (left: S = 0.6, right: S = 1.2).

3 Disturbance Energy Source Terms in Turbulent Swirling Flame

The conservation equation of the exact disturbance energy suggested by Giauque et al. [8] is described by the following equation:

$$\frac{\partial E_d}{\partial t} + \nabla \cdot \boldsymbol{W} = D, \tag{2}$$

where E_d, $\boldsymbol{W}$ and D respectively denote disturbance energy density, flux and source terms. The source term D is decomposed into seven sources as shown below based on their physical meanings.

$$D = D_\xi + D_s + D_Q + D_{Q*} + D_\psi + D_{\psi*} + D_{Y_k} + \varepsilon, \tag{3}$$

$$D_\xi = -\boldsymbol{m}' \cdot (\boldsymbol{\xi} \times \boldsymbol{u})' - \overline{\boldsymbol{m}' \cdot (\boldsymbol{\xi} \times \boldsymbol{u})'}, \tag{4}$$

$$D_s = -s'\boldsymbol{m}' \cdot \nabla \overline{T} - \overline{s'\boldsymbol{m}'} \cdot \nabla T + s'\overline{\boldsymbol{m}} \cdot \nabla T' - \boldsymbol{m} \cdot \overline{T'\nabla s'}, \tag{5}$$

$$D_Q = T'Q' + \overline{T'Q'}, \tag{6}$$

$$D_{Q*} = T'Q*' + \overline{T'Q*'}, \tag{7}$$

$$D_\psi = \boldsymbol{m}' \cdot \boldsymbol{\psi}' + \overline{\boldsymbol{m}' \cdot \boldsymbol{\psi}'}, \tag{8}$$

$$D_{\psi*} = \boldsymbol{m}' \cdot \boldsymbol{\psi}*' + \overline{\boldsymbol{m}' \cdot \boldsymbol{\psi}*'}, \tag{9}$$

$$D_{Y_k} = \sum\nolimits_{k=1}^{n} [g_k'\Omega_k' + \overline{g_k'\Omega_k'} + (g_k'Y_k + \overline{g}_k Y_k')\nabla \cdot \boldsymbol{m}' + \overline{(g_k Y_k)'\nabla \cdot \boldsymbol{m}'}]. \tag{10}$$

Here, $\boldsymbol{m}$, $\boldsymbol{u}$, $\boldsymbol{\xi}$, s, T, g, Y, and n denote momentum, velocity, vorticity, entropy, temperature, chemical potential, mass fraction and number of species, respectively. Subscript k indicates an index of species. Q, $Q*$, $\boldsymbol{\psi}$, $\boldsymbol{\psi}*$, and Ω_k are defined as follows: $Q = [\nabla \cdot \lambda \nabla T - \nabla \cdot \Sigma_k(\rho h_k Y_k \boldsymbol{V}_k) + \Phi + \omega_T]/T$, $Q* = -[\omega_k - \nabla \cdot (\rho \Sigma_k Y_k \boldsymbol{V}_k)]/T$, $\boldsymbol{\psi} = (\nabla \cdot \boldsymbol{\tau})/\rho$, $\boldsymbol{\psi}* = \Sigma_k(g_k \nabla Y_k)$, $\Omega_k = \omega_k - \nabla \cdot (\rho Y_k \boldsymbol{V}_k) - \nabla \cdot (\boldsymbol{m} Y_k)$, where, λ, ρ, h, $\boldsymbol{V}$, Φ, ω, and $\boldsymbol{\tau}$ represent thermal conductivity, density, sensible enthalpy, diffusion velocity, viscous dissipation, reaction rate and viscous stress tensor, respectively. Variables with a prime are fluctuations around their time-averaged value denoted with an overline. The final term in eq. (3) is usually ignored and is not explicitly shown in [8] as it becomes zero when the sampling duration is long enough. Figure 2 shows time series of the above source terms integrated over the computational domain. For both cases, the most dominant terms are D_s and D_Q, which contribute to disturbance evolution as production and sink terms, respectively. Further investigation into these source terms are conducted in the rest of this paper in order to clarify critical factors in disturbance generation and effects of swirling intensity on disturbance sources.

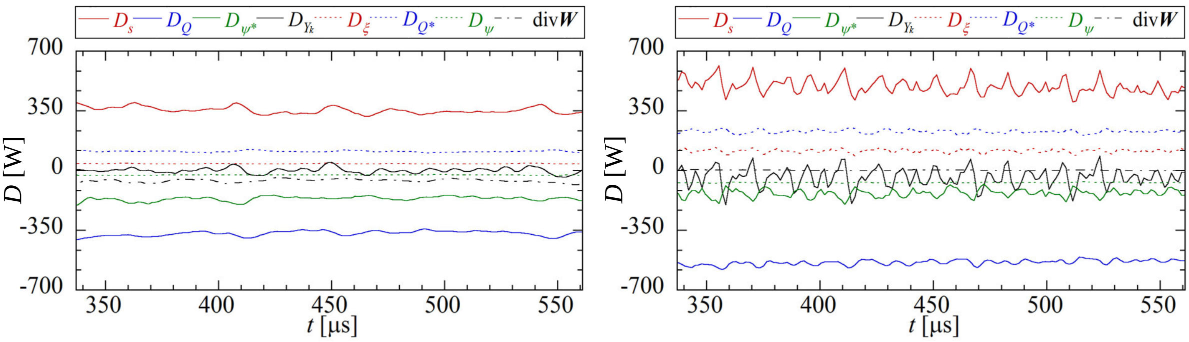

Figure 2. Time series of disturbance energy source terms (left: S =0.6, right: S =1.2)

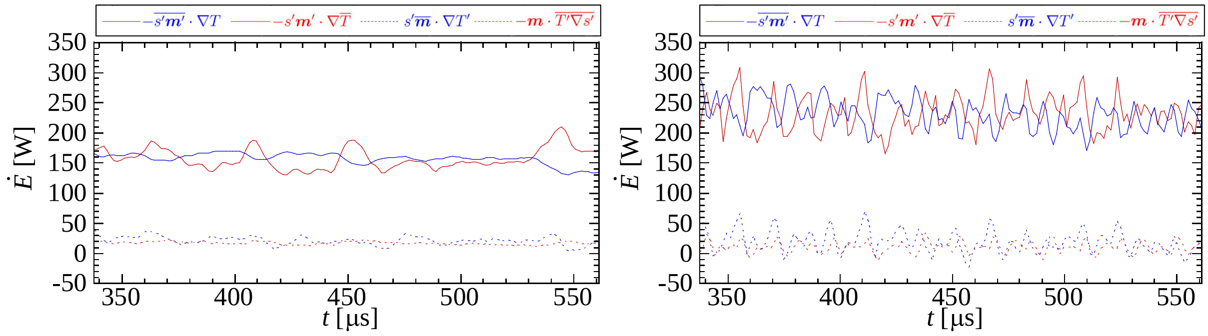

Figure 3. Components of source term D_s related to entropy fluctuation (left: S =0.6, right: S =1.2).

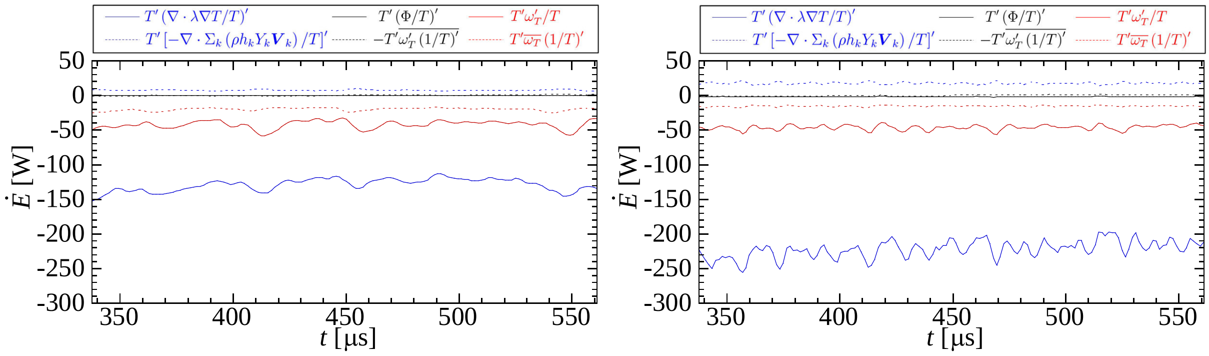

Figure 4. Components of source term D_Q related to heat transfer fluctuation (left: S =0.6, right: S =1.2).

4 Discussion on the Dominant Source Terms D_s and D_Q

Even though it is clear that the source terms D_s and D_Q are dominant for disturbance energy evolution from Fig. 2, the definitions of them (Eq. (5) and (6)) are much more complicated than the conventional disturbance energy source terms [1, 3, 7]. This leads to difficulty in constructing a simple stability criterion which works well even in turbulent combustion fields. Thus, an investigation into the components of the source terms might be useful for simplification of the exact terms. Figure 3 shows time series of components of D_s. Comparing the mean contributions of these components, it is found that the magnitude of the first and second terms in Eq. (5) is much greater than that of the other terms. Here, the time-average of D_s is written as follows:

$$\overline{D_s} = -2\overline{s'\boldsymbol{m}'}\cdot\nabla\overline{T} + \overline{\boldsymbol{m}}\cdot\overline{s'\nabla T'} - \overline{\boldsymbol{m}}\cdot\overline{T'\nabla s'}\,, \qquad (11)$$

indicating that the mean contributions of the first and second terms in Eq. (5) are exactly the same. It should also be noted that the mean contribution of the first term in Eq. (11) strongly depends on swirling intensity, while the contributions of the other terms are almost zero regardless of the swirl number. This result might come from $\boldsymbol{m}'$ included in the first term in Eq. (11) because flame oscillation is more intensive for the higher swirl number case due to the large-scale ring-shaped vortices near the inlet. Thus, higher swirl number could make the difference between the first and the other terms in Eq. (11) more significant. In fact, as shown by Giauque et al.[8], all the terms in Eq. (11) have almost the same magnitude for a laminar case. This result supports the above discussion. It is also worth noting that the first term in Eq. (5) corresponds to a classical production term $-\bar{p}s'\boldsymbol{u}'\cdot\nabla\bar{s}/RC_p$ introduced by Nicoud and Poinsot [7] in a zero Mach number limit, where R is a gas constant and C_p is a specific heat at constant pressure. This implies that the production term by entropy non-uniformity remains important in a turbulent combustion field with

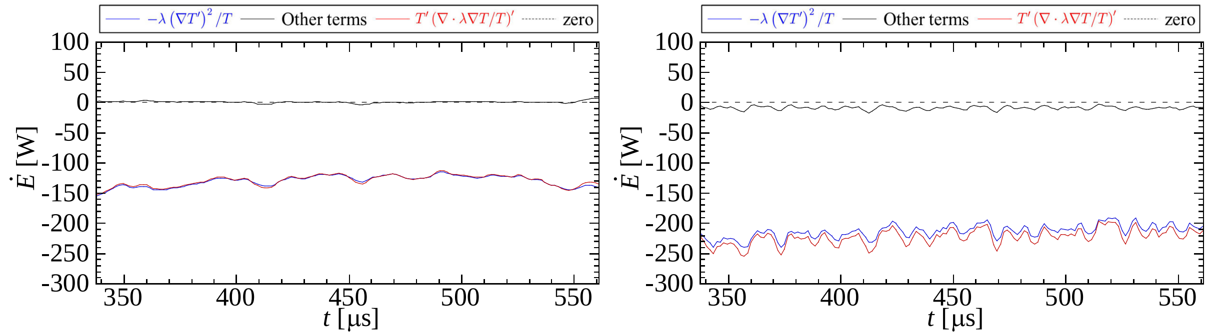

Figure 5. Comparison between $T'(\nabla\cdot\lambda\nabla T/T)'$, $-\lambda(\nabla T')^2/T$ and the other terms in Eq. (13) (left: S =0.6, right: S =1.2).

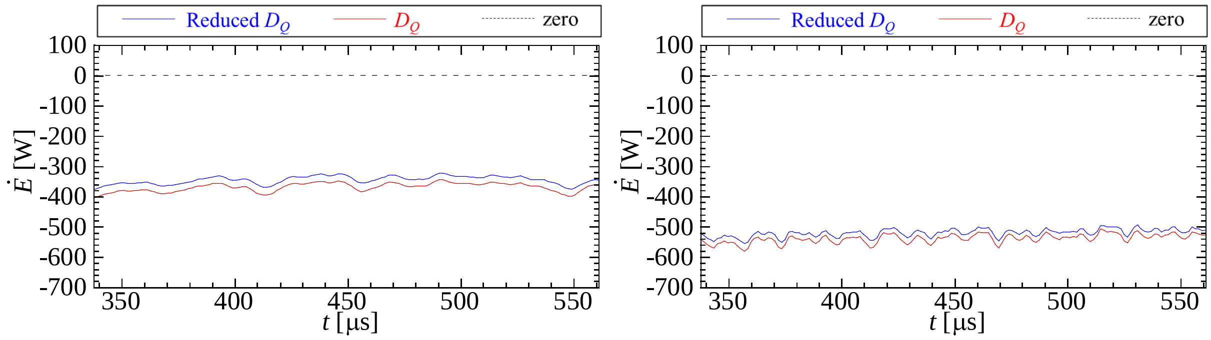

Figure 6. Comparison between D_Q and reduced D_Q (left: S =0.6, right: S =1.2).

non-zero mean flow, even though the disturbance energy equation suggested by Nicoud and Poinsot [7] is derived under a zero Mach number assumption.

For analysis of the components of D_Q, it is useful to expand the term $T'Q'$ into six terms as shown below,

$$T'Q' = T'(\nabla\cdot\lambda\nabla T/T)' - T'(\nabla\cdot\sum_k \rho Y_k h_k \mathbf{V}_k/T)' + T'(\Phi/T)' + T'\omega_T'/T + T'[\overline{\omega}_T(1/T)' - \overline{\omega_T'(1/T)'}]. \tag{12}$$

Here, the fourth term is analogous to a modified Rayleigh term shown by [3, 7] and it reduces to the classical Rayleigh term [1] under the condition stated by Brear et al. [9]. The contributions of above components are shown in Fig. 4. It is observed that the most dominant component of D_Q is the first term in Eq. (12), which comes from heat conduction. The second most dominant term is the Rayleigh like term, which is usually regarded as the most important source term for thermoacoustic instability. For the present cases, this term behaves as a sink of disturbance and its mean magnitude is not significantly affected by the swirling intensity, fluctuating around -50 W. In contrast with this term, the magnitude of the heat conduction term is strongly affected by the swirl number, implying that an intensive swirling flow tends not only to produce more disturbance though the first term of $\overline{D_s}$, but also to attenuate the disturbance by the heat conduction term in Eq. (12). In order to understand the role of the heat conduction term as a sink of disturbance, this term is expanded as shown below,

$$T'(\nabla\cdot\lambda\nabla T/T)' = -\lambda(\nabla T')^2/T + \lambda\nabla\cdot T'\nabla T'/T + T'(\nabla\lambda\cdot\nabla T' + \nabla\cdot\lambda'\nabla\overline{T} - \nabla\cdot\overline{\lambda'\nabla T'})/T + T'[(\overline{\nabla\cdot\lambda\nabla T})(1/T)' - \overline{(\nabla\cdot\lambda\nabla T)'(1/T)'}]. \tag{13}$$

Here, the first term in the above equation is identically negative. As shown in Fig. 5, this dissipative term has much greater magnitude than that of the other terms in Eq. (13). The fluctuation of temperature gradient mostly comes from motions of flame surfaces, implying that strong turbulence can increase this dissipative term through flame distortion by eddy motions. Thus, higher swirling intensity enlarges the contribution of heat conduction term. The above results suggest that D_Q could be reduced to the following form by keeping only the dominant dissipative terms and the heat release term,

$$D_Q \approx -\lambda(\nabla T')^2/T + T'\omega'_T/T + \overline{[-\lambda(\nabla T')^2/T + T'\omega'_T/T]}. \quad (14)$$

As shown in Fig. 6, above equation provides a reasonable approximation of D_Q for both swirl number conditions. The above results also clearly show that dissipative term by heat conduction cannot be neglected especially in flow fields with strong turbulence.

5 Conclusions

In this study, the exact disturbance energy equation is examined based on the DNS results of turbulent swirling premixed flame. Investigation of the source terms of the disturbance energy reveals that the terms related to entropy and heat transfer fluctuations (D_s and D_Q) are the dominant production and sink terms of disturbance energy. Further investigation of D_s shows that the terms including momentum fluctuation have the largest magnitude and their contributions increase under higher swirl number condition. Analysis of the components of D_Q reveals that the term caused by heat conduction is most dominant and the Rayleigh like term $T'\omega'_T$ only behaves as a sink in the present cases. Dissipation of the disturbance energy mainly comes from the fluctuation of temperature gradient, which is also significantly affected by swirling intensity. The results of the present analysis suggest that D_Q could be reduced to the form of Eq. (14).

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