

The critical conditions for the onset of pool-fire puffing

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1 Introduction

Pool fires are known to exhibit a self-sustained oscillatory behavior, shedding large toroidal coherent structures at a well established frequency, a phenomenon referred to in the literature as “puffing”. This behavior influences the rate of air entrainment, the radiated heat output, the flame height and also the spreading of the flame. Pool-fire puffing has been studied extensively in the literature (e.g. [1–9]), and it is well known that under normal conditions of temperature and pressure typical hydrocarbon fuel pools of a few centimeters in diameter puff with a frequency on the order of 10 Hz. Nevertheless, no detailed account has been given of the critical conditions for the *onset* of puffing. In that regard, for the physically similar problem of flickering jet diffusion flames, the authors have recently shown that this oscillatory behavior corresponds to a hydrodynamic global instability of the flow, the onset of which can only be predicted with a stability analysis that is also global in nature, taking into account the 2D nonparallel character of the flow [10].

The objective of the present work is to apply the same technique to a small laminar pool fire to determine the critical conditions for puffing. Contrary to a jet diffusion flame, where momentum is injected with the fuel, and consequently the jet Reynolds number plays a role along with the Froude number, in a pool fire the momentum that is transferred to the flow by the evaporation of fuel is negligible. Therefore, a single parameter, expressed as a Rayleigh number, will be shown to characterize the flow—apart from the inherent thermochemical parameters. In particular, puffing occurs when this Rayleigh number exceeds a critical value. For given burning conditions, the latter is directly equivalent to a critical pool diameter.

In the next section, a detailed formulation will be given to describe the dynamics of small laminar pool fires, accounting for the nonunity Lewis number and vaporization characteristics of typical liquid hydrocarbon fuels. The global linear stability analysis is presented in section 3, including the prediction for the critical Rayleigh number. Finally, concluding remarks are given in section 4.

2 Governing equations for the description of laminar pool-fire dynamics

Consider the burning of a round pool of liquid fuel with radius a in a quiescent air atmosphere with temperature T_A and density ρ_A . In the presence of buoyancy, the density differences of order unity associated with the temperature increase caused by the chemical heat release induce velocities of order $v_g = ga^2/\nu_A$,

where g is the gravitational acceleration and ν_A is the ambient kinematic viscosity. The Reynolds number based on this velocity is the Grashof number $Gr = ga^3/\nu_A^2$, measuring the relative effects of convective acceleration and viscous forces. The accompanying Rayleigh number $Ra = Pr Gr = ga^3/(\nu_A \kappa_A)$, based on the thermal diffusivity of the ambient air κ_A is the corresponding Peclet number that measures the ratio of convective and diffusive energy transport rates. These two parameters, related through the Prandtl number $Pr = \nu_A/\kappa_A \simeq 0.7$, characterize the structure and the dynamics of pool fires. In particular, the onset of puffing occurs at a critical value $Gr_c = Ra_c/Pr$, to be determined below.

The pool radius a and the characteristic velocity v_g will be used as scales for the dimensionless axisymmetric cylindrical coordinates $\mathbf{x} = (x, r)$ and velocity $\mathbf{v} = (v_x, v_r)$. The time and the pressure differences from the quiescent ambient hydrodynamic distribution are scaled with $t_g = a/v_g$ and $\rho_A g a$, giving the variables t and p , respectively. The continuity and momentum equations then become

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \quad (1)$$

$$\frac{Ra}{Pr} \rho \frac{D\mathbf{v}}{Dt} = -\nabla p + (1 - \rho) \mathbf{e}_x + \nabla \cdot [\mu(\nabla \mathbf{v} + \nabla \mathbf{v}^T)], \quad (2)$$

where $D/Dt = \partial/\partial t + \mathbf{v} \cdot \nabla$ denotes the substantial derivative and ρ and μ are the density and viscosity scaled with their ambient values.

The reaction between the fuel and the oxygen of the air occurs according to the global irreversible step $F + sO_2 \rightarrow (1+s)P + q'$, where s and q' are the mass of oxygen consumed and the amount of heat released per unit mass of fuel burnt. Our analysis employs the fast-reaction limit in which the chemical-reaction terms in the conservation equations for reactants and energy appear as Dirac delta distributions along a flame surface separating an outer ambient region with no fuel vapor from an inner region surrounding the pool surface with no oxygen. To facilitate the analysis of the fast-reaction limit it is convenient to construct two chemistry-free conservation equations by eliminating the reaction terms through appropriate linear combinations [11, 12]. In this manner, the conservation equations become

$$Ra \rho \frac{D\xi}{Dt} = \nabla \cdot (\rho \kappa \nabla \xi), \quad (3)$$

$$Ra \rho \frac{DZ}{Dt} = \frac{1}{Le} \nabla \cdot (\rho \kappa \nabla \tilde{Z}), \quad (4)$$

involving a normalized excess thermal and chemical enthalpy ξ , which is zero in the air atmosphere and unity on the fuel surface, a diffusion-weighted mixture fraction $\tilde{Z}$, and the classical mixture fraction Z , defined respectively as

$$\xi = \frac{T - 1 + (q/S)(Y_O - 1)}{T_B - 1 - q/S}, \quad \tilde{Z} = \frac{SY_F/Le_F - Y_O + 1}{S/Le_F + 1}, \quad Z = \frac{SY_F - Y_O + 1}{S + 1}. \quad (5)$$

In the former, T is the temperature scaled with T_A , and Y_F and $Y_O = Y_{O_2}/Y_{O_{2A}}$ are the mass fraction of fuel vapor and oxygen, the latter normalized with its value $Y_{O_{2A}} \simeq 0.232$ in air. The thermochemical parameters s and q' defined above appear through the reaction-rate factors $S = s/Y_{O_{2A}}$ and $q = q'/(c_p T_A)$, where c_p denotes the specific heat at constant pressure, assumed to be constant in the following analysis. The parameter S , on the order of $S = 15$ in hydrocarbon combustion, represents the amount of air needed to burn the unit mass of fuel. The other parameter, q , is the heat release per unit mass of fuel scaled with the ambient enthalpy, which takes fairly large values for typical fuels, such that $q/S \sim 7$.

Just like the other transport properties, the thermal diffusivity κ has been scaled with its ambient air value. The presumed power laws

$$\mu = \rho\kappa = T^\sigma \quad (6)$$

will be used below for the temperature variation of the transport properties, with $\sigma = 0.7$. A Fickian model has been used for the diffusion velocities of the reactants, a sufficiently accurate approximation in fuel-air combustion, where the dominant presence of nitrogen molecules simplifies the description of molecular transport. Whereas a unity Lewis number has been employed for oxygen, an excellent approximation under most conditions, a non-unity Lewis number $Le_F > 1$ is used for the fuel, as is needed for accuracy in computations of most hydrocarbon and alcohol fuels. The modified Lewis number Le in (4) is defined as $Le = (S + 1)/(S/Le_F + 1)$.

The motion induced is very slow, and involves small spatial pressure variations that can be neglected in the first approximation when writing the equation of state

$$\rho T = [Y_F (W_A/W_F - 1) + 1]^{-1}. \quad (7)$$

The molecular mass of most liquid fuels W_F is significantly larger than that of the air W_A , so that the density differences induced by the presence of the fuel vapor have been considered for increased accuracy in (7).

To solve (3) and (4) coupled with (1) and (2) and supplemented with (6) and (7) we need to relate T , Y_F , and Z with ξ and $\tilde{Z}$. These relations can be obtained using the fast-reaction condition $Y_F Y_O = 0$ of non-coexistence of Y_O and Y_F , which are simultaneously zero at the flame, given by the iso-surface $Z = Z_S = 1/(1 + S)$, or $\tilde{Z} = \tilde{Z}_S = 1/(1 + S/Le_F)$, as follows from the definitions (5) of $\tilde{Z}$ and Z . Thus,

$$\text{for } \tilde{Z} \geq \tilde{Z}_S : \quad Y_O = 0, \quad Y_F = \frac{Z - Z_S}{1 - Z_S} = \frac{\tilde{Z} - \tilde{Z}_S}{1 - \tilde{Z}_S}, \quad T - 1 = (T_B - 1)\xi + \frac{q}{S}(1 - \xi); \quad (8)$$

$$\text{for } \tilde{Z} \leq \tilde{Z}_S : \quad Y_F = 0, \quad 1 - Y_O = \frac{Z}{Z_S} = \frac{\tilde{Z}}{\tilde{Z}_S}, \quad T - 1 = (T_B - 1)\xi + \frac{q}{S}\left(\frac{\tilde{Z}}{\tilde{Z}_S} - \xi\right). \quad (9)$$

Equations (8) and (9) provide piece-wise linear relations for the evaluation of Y_F , Y_O , and Z in terms of $\tilde{Z}$. In addition, the last relation in the equations gives T in terms of $\tilde{Z}$ and ξ .

The boundary conditions that complete the definition of the problem are those corresponding to a quiescent surrounding air atmosphere, an impermeable and chemically inert surface surrounding the fuel pool, and fuel vaporization at the pool surface occurring with a prescribed boiling temperature T_B and dimensionless latent heat of vaporization $l_v = L_v/(c_p T_A)$:

$$v_x = v_r = p = \xi = \tilde{Z} = 0 \quad \text{as } |\mathbf{x}| \rightarrow \infty \quad \text{for } x \neq 0 \quad (10)$$

$$v_x = v_r = \partial\xi/\partial x = \partial\tilde{Z}/\partial x = 0, \quad \text{at } x = 0 \quad \text{for } r > 1, \quad (11)$$

$$\left. \begin{array}{l} v_r = \xi - 1 = 0 \\ -T_B^\sigma \partial\xi/\partial x = \alpha Ra \rho v_x \\ -T_B^\sigma \partial\tilde{Z}/\partial x = Le_F Ra \rho v_x (1 - \tilde{Z}) \end{array} \right\} \quad \text{at } x = 0 \quad \text{for } r < 1, \quad (12)$$

where $\alpha = (l_v + T_B)/(q/S + 1 - T_B)$.

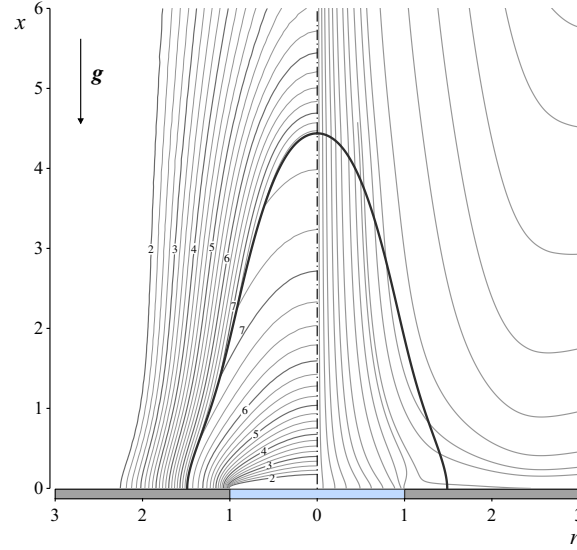


Figure 1: Steady solution of a pool fire for $Ra = 1000$, $S = 7.44$, $Le_F = 2.60$, $T_B = 1.12$ and $q/S = 7.7$. Left: isotherms with equispaced temperature levels $T = (2, \dots, 7)$, indicated on the corresponding contours, with equally-spaced intermediate values in between. Right: streamlines. The stoichiometric line $\tilde{Z} = \tilde{Z}_S$ is represented by the thick black line.

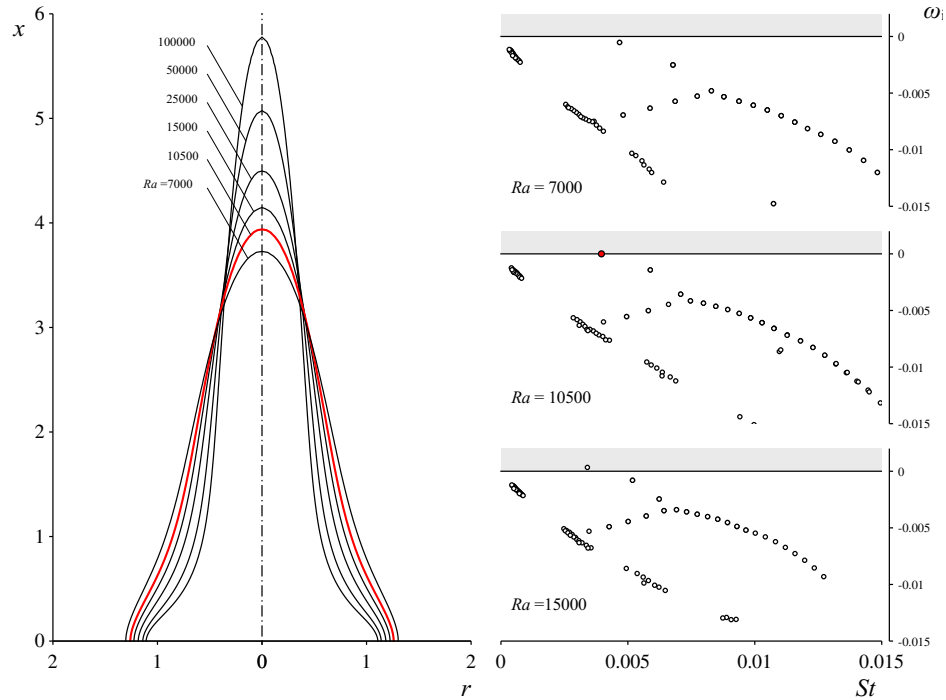


Figure 2: (a) Steady methanol flame shapes $\tilde{Z} = \tilde{Z}_S$ for different Rayleigh numbers (indicated in figure), and (b) accompanying eigenvalue spectra for stable ($Ra = 7000$), marginally stable ($Ra = Ra_c = 10500$), and unstable ($Ra = 15000$) configurations. The leading eigenvalue is marked in red in the critical case, with its associated flame shape also depicted in red.

3 Puffing as a global hydrodynamic instability and the determination of its onset

Following the recent analysis of flickering diffusion flames [10], we consider the puffing of pool fires as the manifestation of an axisymmetric hydrodynamic global instability, the onset of which can be determined by means of a linear global stability analysis. It is useful to recast the equations of motion, together with the corresponding boundary conditions, equation of state, and constitutive relations, in the compact form

$$\frac{\partial \mathbf{q}}{\partial t} = \mathcal{F}(\mathbf{q}), \quad (13)$$

where $\mathbf{q} = (\mathbf{v}, p, \tilde{Z}, \xi)$ is the state vector and $\mathcal{F}$ is a nonlinear differential operator, representing the steady counterpart of the continuity (1), momentum conservation (2), and passive scalars conservation equations (3) and (4). First, a basic state $\bar{\mathbf{q}} = (\bar{\mathbf{v}}, \bar{p}, \bar{\tilde{Z}}, \bar{\xi})$ is defined as the steady solution of the governing equations, i.e. $\mathcal{F}(\bar{\mathbf{q}}) = 0$. Figure 1 shows an example of such a basic state. Then, small perturbations are added in the form of global 2D temporal Fourier modes as $\mathbf{q} = \bar{\mathbf{q}} + \varepsilon \hat{\mathbf{q}} e^{-i\omega t}$ with $\omega = \omega_r + i\omega_i$. Substitution of this modal decomposition in (13) and subsequent linearization yields at $\mathcal{O}(\varepsilon)$ a generalized eigenvalue problem that determines $(\omega, \hat{\mathbf{q}})$. If the growth rate of the most unstable eigenvalue is positive, perturbations will grow in time and the flow is asymptotically unstable. On the contrary, if all eigenvalues have negative growth rates, the flow is asymptotically stable. For a description of the numerical methods that are used to discretize the governing equations, find the basic state, and solve the eigenvalue problem, we refer to [10].

Figure 2 shows how the steady flame shape and corresponding eigenvalue spectrum changes with the Rayleigh number Ra , which is, once a certain fuel—methanol in this case—is chosen, the only governing parameter of the problem. The growth rate increases with the Rayleigh number, becoming positive at a critical value $Ra_c = 10500$. Recalling the definition of the Rayleigh number, $Ra = ga^3/(\nu_A \kappa_A)$, it can be seen that for a methanol pool fire burning in an air atmosphere under normal temperature and pressure, this critical Rayleigh number translates into a critical diameter $2a = 1.4$ cm.

Figure 2(b) also shows that the frequency ω_r of the instability mode, represented in dimensionless form as $St = \omega_r/\pi$, decreases with the Rayleigh number. This is in agreement with experimental observations [8] that find that the characteristic frequency of the self-sustained puffing oscillations decreases with the size of the pool fire. Furthermore, at criticality, our computations for methanol show that $St = 0.004$, which corresponds to a puffing frequency under normal conditions of 9 Hz.

4 Concluding remarks

We have given a realistic description of the dynamics of small laminar pool fires, accounting for the nonunity Lewis number and vaporization characteristics of typical liquid hydrocarbon fuels. In the description, once a certain fuel has been chosen, the only remaining governing parameter is the Rayleigh number Ra . By means of a linear global stability analysis, we have shown that above a certain critical value Ra_c , a global instability of the flow sets in, leading to periodic oscillations of the flow field with a frequency that decreases with Ra .

Our computations show that for methanol, $Ra_c = 10500$, which, on Earth in an air atmosphere under normal temperature and pressure, is equivalent to a critical pool diameter $2a = 1.4$ cm. Furthermore, the puffing frequency under these conditions is found to be 9 Hz. Although no specific experimental determination of the onset of pool fire puffing has been found in the literature to compare our result with, it is encouraging that

the smallest pool fires characterized in puffing experiments are typically of the order of a few centimeters in diameter, with puffing frequencies of the order of 10 – 15 Hz [8, 9]. It can be anticipated that the boundary conditions on the wall surrounding the fuel pool have a considerable influence on the critical values. In that regard, for comparison with experiments it may be interesting to change the adiabatic condition (11) to one of constant temperature, as this is easier to achieve in an experiment. The authors are currently setting up an experiment to verify the critical conditions, and are extending the analysis to other fuels.

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