Edge Flame Assisting the Stabilization of Diffusion Flames in Mixing Layers

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Introduction

Many practical combustion devices are concerned with the stabilization of diffusion flames that are formed by injecting gaseous fuels into a co-flowing stream containing an oxidizer. A primary concern of these configurations is the attachment and lift-off characteristics of the diffusion flame relative to the rim of the injector. Depending on the stoichiometric conditions and the diffusive properties of the fuel and oxidizer, the diffusion flame may either be attached to the rim of the injector, lifted and stabilized at a downstream equilibrium position, or blown off by the flow. Under certain conditions the diffusion flame may undergo spontaneous oscillations, whereby the edge of the flame exhibits a back and forth motion along a direction that coincides with the diffusion flame surface. In all circumstances, the edge of the flame, which possesses a distinct structure that combines characteristics of both premixed an diffusion flames, is found to play a crucial role in determining the stabilization of the diffusion flame.

The dynamics of edge flames in mixing layers was studied by Kurdyumov and Matalon [1]. Their results show that the edge flame may display two different stabilization modes. The first is characterized by a stationary edge flame that is stabilized at a well-defined distance from the splitter plate, while the other is characterized by sustained oscillations with well-defined frequencies and amplitudes. Studies by Fernández et al. [2] and Higuera and Liñán [3] have investigated the effects of thin- and thick-walled splitter plate on the attachment of the diffusion flame. A factor that has not been thoroughly explored and discussed in the literature is the effect of unequal streams on the structural and dynamical properties of the edge flame. This has been the primary focus of a recent publication by Lu and Matalon [4], which is further explored in the present work.

Fundamental understanding of the attachment and lift-off characteristics of laminar jet diffusion flames could be achieved by considering the configuration shown in the schematic diagram of Figure 1. When two initially separated streams, one containing fuel and the other oxidizer, are led to pass a flat plate of zero thickness, Blasius boundary layers are developed on both sides of the plate. At the trailing edge of the splitter plate, the two boundary layers begin to merge and the resulting interdiffusion of fuel and oxidizer leads to the development of a mixing layer in the wake region where, upon successful ignition a diffusion

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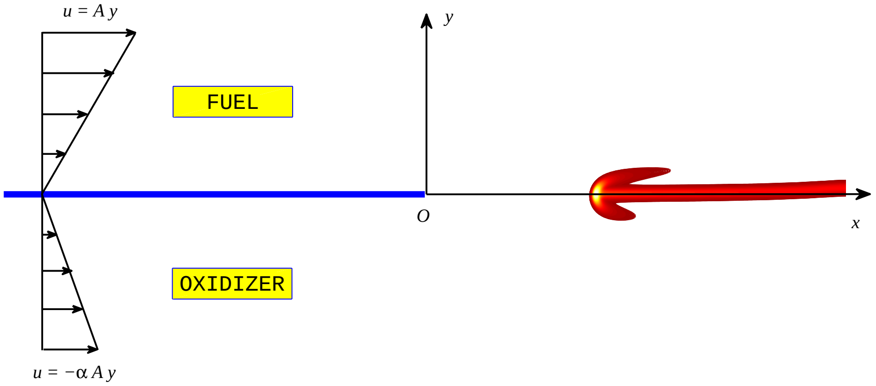


Figure 1: Schematic of an edge flame (illustrated by reaction rate contours) stabilized in the wake of two merging linear shear flows. The edge flame corresponds to stoichiometric conditions and equal Lewis numbers, with $\alpha < 1$, such that the asymmetry in flame position is solely due to the unequal strain rates in the two streams.

flame is established. The flow within a distance $l_{\rm NS} = L\,{\rm Re}^{-3/4}$ of the plate's trailing edge is described by the full Navier-Stokes equations, where L is the length of the plate and the Reynolds number ${\rm Re} = UL/\nu$ depends on the incoming uniform flow U and the kinematic viscosity ν. At larger distances the boundary layer assumptions hold and the flow is described by Goldstein's well-known similarity solution [5]. In either case, the flow near the trailing edge is well represented by uniform strain rates obtained from linearization of the Blassius solutions, as shown in the figure. Specifically, $u = Ay$ in the fuel stream and $u = -\alpha Ay$ in the oxidizer stream, with $0 \leq \alpha \leq 1$ the oxidizer-to-fuel shear strain rate ratio. Fuel and oxidizer mixing begins in the Navier-Stokes region. When the flame is attached to the splitter plate, or stabilized in its vicinity, upstream heat conduction towards the plate plays a significant role such that the thermal diffusion length $l_{\rm th}$ is comparable to the length scale $l_{\rm NS}$ associated with viscous diffusion. The focus on the dynamics must start with edge flames that develop in the Navier-Stokes region but may re-establish in the inner-wake region.

Formulation

For simplicity, the chemical reaction between the fuel and oxidizer is modeled by the one-step irreversible reaction and, avoiding the complexity associated with density variations, the conventional diffusive-thermal model is employed. Using the characteristic length $l_{\rm NS} = \sqrt{\nu/A}$ and velocity $U_{\rm NS} = \sqrt{\nu A}$ scales, the flow field in the Navier-Stokes region is described by the dimensionless equations

$$\nabla \cdot \mathbf{v} = 0, \qquad \mathbf{v} \cdot \nabla \mathbf{v} = -\nabla p + \nabla^2 \mathbf{v}, \tag{1}$$

where the pressure p has been scaled by $\rho U_{\rm NS}^2$ with ρ the density of the mixture. The combustion process is described by conservation equations for the fuel and oxidizer mass fractions $Y_{\rm F}$ and $Y_{\rm O}$ normalized by their values in the incoming streams $\tilde{Y}_{{\rm F}_0}$ and $\tilde{Y}_{{\rm O}_0}$, and by an energy equation for the entire mixture expressed in terms of the scaled dimensionless temperature $\theta = (T - T_0)/(T_{\rm a} - T_0)$, where T_0 is the temperature of the fuel and oxidizer streams (assumed to be the same) and T_a is the adiabatic flame temperature (for complete combustion, under stoichiometric conditions). In dimensionless form,

$$\begin{aligned} \mathrm{Pr}\frac{DY_\mathrm{F}}{Dt} &= \mathrm{Le}_\mathrm{F}^{-1}\,\nabla^2 Y_\mathrm{F} - \omega, \qquad \mathrm{Pr}\frac{DY_\mathrm{O}}{Dt} = \mathrm{Le}_\mathrm{O}^{-1}\,\nabla^2 Y_\mathrm{O} - \phi\omega, \\ \mathrm{Pr}\frac{D\theta}{Dt} &= \nabla^2\theta + (1+\phi)\omega, \end{aligned} \tag{2}$$

with the reaction rate given by

$$\omega = D\beta^3\, Y_\mathrm{F} Y_\mathrm{O} \exp\left\{\frac{\beta(\theta-1)}{(1+q\theta)/(1+q)}\right\}. \tag{3}$$

The parameters in these equations are the Prandtl number $\mathrm{Pr} = \nu/D_\mathrm{th}$ where D_th is the thermal diffusivity of the mixture, the fuel and oxidizer Lewis numbers $\mathrm{Le}_\mathrm{F} = D_\mathrm{th}/D_\mathrm{F}$ and $\mathrm{Le}_\mathrm{O} = D_\mathrm{th}/D_\mathrm{O}$ where D_F and D_O are the fuel-inert and oxidizer-inert binary mass diffusivities, the initial mixture strength $\phi = s\tilde{Y}_{\mathrm{F}_0}/\tilde{Y}_{\mathrm{O}_0}$ where s is the mass-weighted oxidizer-to-fuel stoichiometric coefficient, the adiabatic flame temperature $T_\mathrm{a} = T_0 + Q\tilde{Y}_{\mathrm{F}_0}/c_p(1+\phi)$ where c_p is the mixture specific heat (at constant pressure), the activation energy parameter $\beta = E(T_\mathrm{a} - T_0)/\mathcal{R}T_\mathrm{a}^2$ where E the activation energy and $\mathcal{R}$ the universal gas constant, the heat release parameter $q = (T_\mathrm{a} - T_0)/T_0$, and the Damköhler number $D = \big(\mathrm{Pr}\,\mathcal{B}\rho\tilde{Y}_{\mathrm{O}_0}/A\beta^3\big)e^{-E/\mathcal{R}T_\mathrm{a}}$ where $\mathcal{B}$ is a pre-exponential factor assumed constant. Note that the Damköhler number representing the ratio of the flow-to-chemical reaction times is controlled by the overall flow rate.

As a result of the constant density approximation, the flow field is unaffected by the combustion process and is solved a priori for various values of α. The combustion field is then obtained by solving (2) with α specified, for a given set of the remaining parameters. In this study $\beta = 10$, $Pr = 0.72$ and $q = 5$ and the Lewis numbers are assumed equal with the common value denoted by Le. The parameter that is varied is the Damköhler number, inversely proportional to the overall flow rate.

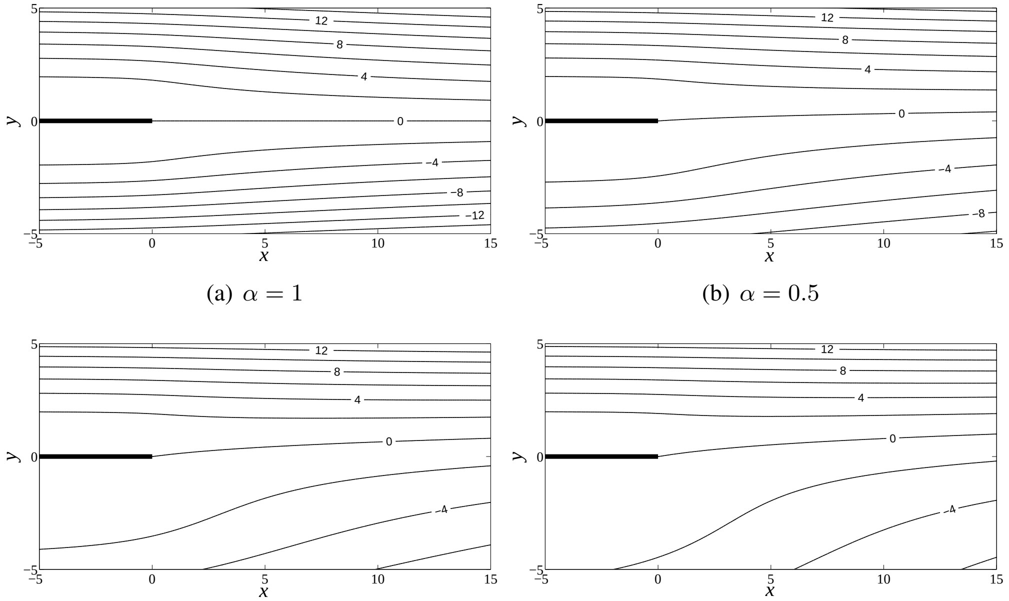
Flow Field

The flow field that satisfies no-slip boundary conditions along the splitter plate and matches Goldstein's inner-wake similarity solution far downstream is shown in Figure 2 for several values of α. The flow is illustrated by equally-spaced streamlines. When $\alpha = 1$, the flow field is symmetric with respect to the x axis, and the dividing streamline that emanates from the trailing edge coincides with the x-axis. When $\alpha < 1$, the imbalance in vorticity in the incoming streams causes the dividing streamline to deviate from the x-axis towards the fuel region, indicative of fluid entrainment from the lower oxidizer stream. The deviation of the dividing streamline increases as α decreases, implying enhancement of the entrainment effect.

Combustion field

When the fuel and oxidizer are supplied in stoichiometric conditions ($\phi = 1$) and there is no preferential diffusion ($\mathrm{Le}_\mathrm{F} = \mathrm{Le}_\mathrm{O}$), the edge flame in the mixing region of co-flowing streams of equal speeds ($\alpha = 1$) is necessarily symmetrical with respect to the x-axis. Asymmetrical solutions are obtained when $\alpha \neq 1$.

We first consider the case of a splitter plate held at a constant temperature, equal to the gas temperature of the two streams. In Fig. 3 we show the flame standoff distance as a function of the Damköhler number for



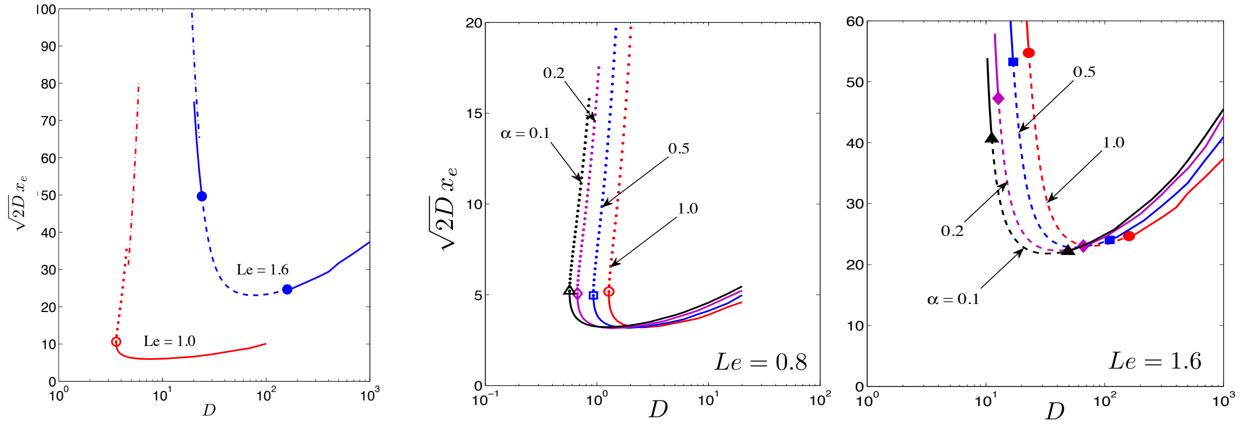
(a) $\alpha = 1$ (b) $\alpha = 0.5$

(c) $\alpha = 0.2$ (d) $\alpha = 0.1$

Figure 2: Flow field illustrated by selected streamlines, for several values of α.

two values of the Lewis number Le, with $\alpha = 1$ in (a) and $0 < \alpha < 1$ in (b). Rescaling x_e was done to remove the dependence of the distance x on the flow rate, exhibited in the original scale. With the new scale, distances are measured in units of the thickness l_f of a laminar premixed flame propagating in a mixture consisting of fuel and oxidizer in the same proportions as those available at the edge of the flame. We note that the response curves are markedly different for low ($\mathrm{Le} = 1$) and high ($\mathrm{Le} = 1.6$) Lewis numbers.

Figure 3(a) shows that for mixtures with $\mathrm{Le} = 1$, the response curve is multi-valued with the upper branch (the dotted section beyond the turning point) corresponding to unstable states that are physically unattainable. The solid curve corresponds to stable states with the edge flame remaining near the tip of the splitter plate for a wide range of flow rates. The flame under such conditions is practically attached to the plate. Upon increasing the flow rate (or decreasing D) the edge flame remain stabilized near the splitter plate and is blown off when the gas velocity exceeds a critical value (identified by the symbol ○). Thus, for mixture of low Lewis numbers the diffusion flame remains practically attached to the splitter plate until it is blown off by the flow. The response curve for mixtures with $\mathrm{Le} = 1.6$ is single valued and the entire response curve corresponds to *globally stable* states. Upon increasing the flow rate (or decreasing D) the edge flame approaches the splitter plate, reaches a minimum distance and then lifts off rapidly and stabilizes at exceedingly large distances from the splitter plate. The dashed segment of the curve (between the symbols ●) shows that stabilization often occurs with the edge flame undergoing a back and forth motion near the splitter plate with a well-defined frequency. These results qualitatively agree with the observation of Chung and Lee [6] that for lighter fuels, such as methane and ethane, jet diffusion flames remains attached to the rim of the nozzle and are blown off when the flow exceeds a critical value, while jet flames of heavy fuels, such as propane and butane, are lifted off the rim and stabilized at significantly large distances when increasing the flow rate; see also [7, 8].



(a) (b)

Figure 3: Standoff distance of an edge flame as a function of the Damköhler number (inversely proportional to the overall flow rate) (a) for $\phi = \alpha = 1$ and two distinct Lewis numbers and (b) for decreasing values of α and for two distinct Lewis numbers. The curves in (a) also illustrate the transition of an edge flame from the NS to the inner-wake region; the latter represented by a dot-dashed lines.

Figure 3(a) also shows the evolution of the edge flame from the Navier-Stokes to the inner-wake region. The curve marked by a dashed-dotted line, which corresponds to flames located at distances that are sufficiently remote from the splitter plate, were obtained by assuming that the flow satisfies Goldstein's similarity solution; all other parts of the curve were obtained with a flow obtained by solving the NS equations. The transition from one regime to the next occurs smoothly, as it should.

In Fig. 3(b) the dependence of the standoff distance on the Damköhler number is shown for increasing values of α and for low ($Le = 0.8$) and high ($Le = 1.6$) Lewis numbers. The oxidizer is introduced in a co-flowing stream of equal velocity when $\alpha = 1$, and is fully entrained from the ambience when $\alpha \to 0$; the latter simulates a fuel jet in a quiescent atmosphere. For low Lewis numbers ($Le = 0.8$), the flame standoff distance remains within $5l_f$ for all values of α, with the critical Damköhler number corresponding to extinction decreasing slightly when reducing the co-flowing oxidizer stream (i.e., when decreasing α). Similarly, for high Lewis numbers ($Le = 1.6$), the minimum distance reached before liftoff, which is nearly constant, is reached at a smaller Damköhler number when reducing the co-flowing oxidizer stream.

We consider next, a thermally active splitter plate and examine the dependence of the flame standoff distance on the overall flow rate, or Damköhler number. The key parameter in this case is r_λ the ratio of wall-to-gas conductivities. The upper bound $r_\lambda \to \infty$ corresponds to a plate that has infinite thermal conductivity, or vanishing thermal resistance. This case is the "cold plate limit" discussed above. The other extreme, $r_\lambda = 0$, corresponds to to a plate that has vanishing conductivity, or infinite thermal resistance, in which case there is no heat transfer between the plate and the gaseous mixture and the plate serves as an adiabatic wall. In this case the edge flame, upon increasing the flow rate, gets much closer to the plate and is either blownoff for mixtures of low Lewis numbers, or lifts off and stabilizes downstream for mixtures of high Lewis numbers. Of greater interest are the cases of finite r_λ in which case the plate is significantly heated by the heat transferred from the edge flame at its tip, heating the fuel and oxidizer streams and leading to super-adiabatic temperature flames. Due to computation limitations we have only considered the case of a virtual air plate, corresponding to $r_\lambda = 1$, which illustrates the distinct flame behavior not observed in the limiting cases of an adiabatic or a cold plate. Results of this case will be presented at the meeting.

References

[1] V. N. Kurdyumov and M. Matalon. Stabilization and onset of oscillation of an edge-flame in the near-wake of a fuel injector. *Proc. Combust. Inst.*, 31:909–917, 2007.

[2] E. Fernández, V. Kurdyumov, and A. Liñán. Diffusion flame attachment and lift-off in the near wake of a fuel injector. *Proc. Combust. Inst.*, 28:2125–2131, 2000.

[3] F. J. Higuera and A. Liñán. Flow field of a diffusion flame attached to a thick-walled injector between two coflowing reactant streams. *J. Fluid Mech.*, 329:389–411, 1996.

[4] Z. Lu and M. Matalon. Structure and dynamics of edge flames in the near wake of unequal merging shear flows. *Combustion Theory and Modelling*, 20(2):258–295, 2016.

[5] S. Goldstein. Concerning some solutions of the boundary layer equations in hydrodynamics. *Proc. Cambridge Phil. Soc.*, 26:1–30, 1930.

[6] S. H. Chung and B. J. Lee. On the characteristics of laminar lifted flames in a nonpremixed jet. *Combust. Flame*, 86:62–72, 1991.

[7] B. J. Lee and S. H. Chung. Stabilization of lifted tribrachial flames in a laminar nonpremixed jet. *Combust. Flame*, 109:163–172, 1997.

[8] S. H. Chung. Stabilization, propagation and instability of tribrachial triple flames. *Proc. Combust. Inst.*, 31:877–892, 2007.