The Origin and Evolution of Mechanical and Thermodynamic Disturbances Caused by Localized Energy Deposition in Gaseous Volumes

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1. Research Overview

Transient, spatially resolved thermal energy deposition into inert and reactive gas volumes is the source of thermodynamic and mechanical disturbances. Thermo-mechanical concepts and modeling, Kassoy [1-4], have been used to develop a quantified cause-effect relationship. When the energy deposition is "sufficiently" small the disturbances are described by classical acoustic wave equations (examples include among many, Chu [5], Rott [6], Liewen [7] and their cited references). In contrast the mathematical models used to describe thermo-mechanical physics incorporate a much wider range of energy deposition. Examples of these phenomena include shocks generated by lightning and explosions, detonation initiation, internal combustion engine knock, blast waves associated with supernovae, transient pressure variations in liquid propellant rocket engines and perhaps, coronal mass ejections from the Sun.

Formal mathematical methods are employed to quantify the thermo-mechanical response of gases in;

A. an unconstrained gaseous **micro-volume** embedded in a larger unheated environment, and
B. a fixed **macro-volume** of gas heated throughout.

The analyses are used to characterize the physical phenomena occurring within both volumes, as well as the consequences of processes internal to the micro-volume on the unheated environment. The non-dimensional compressible Euler equations, including a heat source term in the energy equation, are used to identify the relevant non-dimensional parameters. They include;

A. the energy parameter $\boldsymbol{\alpha}$, and
B. the time-scale parameter $\boldsymbol{\varepsilon}$.

The parameter α measures the amount of energy added during a defined energy deposition time-scale relative to the initial internal energy in the volume of interest. The time scale parameter is the ratio of the energy deposition time-scale to the acoustic time-scale of the volume. Results are given for the following combinations of the parameters;

A. $\alpha << O(1)$, $\varepsilon = O(1)$,
B. $\alpha = O(1)$, $\varepsilon = O(1)$,
C. $\alpha >>O(1)$, $\varepsilon = O(1)$,
D. $\alpha >>O(1$, $\varepsilon <<O(1)$,

Physically, cases A and B describe «weak» and «modest» energy addition, respectively when the two time-scales are comparable, while Cases C and D correspond to «strong» energy addition for comparable time-scales and for «very fast heating», respectively.

Cases A-D are interpreted asymptotically and limit process analysis is applied to the full non-dimensional Euler equations to derive «reduced» equations. **A physical interpretation of the mathematical results follows without explcit reference to describing equations**.

2. Overview of results for an inert gas

«Weak» heating, Case A, demonstrates that the non-dimensonal thermodynamic variables experience only only small O(α) disturbances from their initial values and the non-dimensional induced velocity is small, O(α), compared to the initial speed of sound in both the micro- and macrovolumes. The reduced equations devolve to linear non-homogenous acoustic wave equations as in Chu [5], for example. The non-homogeneous terms represent the effects of both instantaneous and historical heat deposition. The nondimensional efflux mass from the mircovolume expansion and the efflux speed are «weak», O(α), implying that only equally «weak» acoustic disturbances driven by the «piston effect» , (Liepmann and Rosko [8]) appear in the unheated gas surroundings. The acoustic field for the macrovolume depends on the shape of the container and the appropriate boundary conditions.

Case B leads to larger, O(1), thermodynamic variable changes and an induced velocity comparable to the initial speed of sound. The full compressible Euler equations describe the larger thermomechanical response in both micro- and macrovolumes, characterized by more signifcant microvolume expansion with the possibility of «weak» shocks being generated in the unheated gaseous environment by the «piston effect». As long as the macrovolume energy deposition is distributed throughout the space, O(1) pressure and velocity distubances will occur.

«Strong» energy addtion on a time-scale comparable with the initial acoustic time-scale in Case C causes an O(α)>>O(1) increase in temperature, an inversely large decrease in density and a very small change, O(1/α)<<O(1), in pressure. This *non-intuitive* nearly isobaric thermo-mechanical response occurs because the acoustic time-scale within either volume becomes very small as the gas heats up. As a result the energy addition time-scale is much longer than the hot acoustic time, and pressure disturbances can be nearly spatially homogenized. The reduced equations are nonlinear and non-hyperbolic. The transient temperature increase in a fluid particle, and the localized gas expansion (positive divergence of the induced velocity field), are proportional to the local power deposition. The mass efflux of high temperature gas relative to the initial mass in the microvolume is very small, O(1/α)<<O(1) because the density of the heated gas is O(1/α),.The dimensional induced speed is comparable with the initial speed of sound implying that microvolume expansion will drive weak shocks into the surrounding unheated gas. In contrast the internal induced Mach number is very small, O($1/\alpha^{1/2}$) the

result of a very large hot speed of sound. The macrovolume will experience large temperature variations but only very minor pressure distubances.

The asymptotic analysis for Case D, corresponding to «strong» energy addition on a time-scale short compared to the initial acoustic time-scale, demonstrates that the parameter combination $\varepsilon^2\alpha$ plays a crucial role in determining the physical response of the gas. The absolute temperature and pressure variations are both large $O(\alpha)$ while the induced speed is $O(\varepsilon\alpha)$.

Case D1

When $\varepsilon^2\alpha << O(1)$ the small density perturbation is $O(\varepsilon^2\alpha)$. The non-hyperbolic,reduced equations describe constant volume heating to a first approximation, with a pressure gradient driven induced velocity field of order $O(\varepsilon\alpha) << O(1/\varepsilon)$. This unusual, *non-intuitive* result implies that the expanding micro-volume can generate a wide range of mechanical disturbances in the external unheated gas, ranging from weak acoustic to strong shock waves depending on the individual sizes of α and ε. In a related way the induced speed in the fixed macro-volume can be quite large while the «large» pressure is characterized by $O(\alpha)$. The internal speed of sound will be very large, due to the large temperature, so that the relevant Mach number is is $O(\varepsilon^2\alpha)^{1/2} << O(1)$.

Case D2

A distinguished limt is defined for $\varepsilon^2\alpha = O(1)$. The describing equations are the full compressible non-linear Euler equations in terms of scaled variables for temperature, pressure and induced speed. The non-dimensional value of the latter is either $O(1/\varepsilon)>>O(1)$, for a given α, or $O(\alpha^{1/2})>>O(1)$, for a given ε. Unlike the previous example the density variation is $O(1)$. The relative mass efflux is $O(\varepsilon)$ because the heating time-scale is relatively short. In the context of a fixed macro-volume, «fast» and «large» energy release is the source of a «large» pressure increase from the initial value, a result that may be of interest in the study of reactive gas thermo-mechanics, described in Section 3 below.

3. Overview of results for a reactive gas

A one step, exothermic high activation energy Arrhenius reaction is used to drive the thermomechanical evolution of a reactive gas from an imposed spatially variable initial state. The compressible reactive Euler equations amended by a species equation with diffusion suppressed provide a viable mathematical model [1]. Thermal explosion theory, Kassoy [9] is used to describe the induction period for the reaction on a time-scale denoted by t_i. Three non-dimensional parameters are identified;

A. (t_i/t_A), where t_A is the characteristic acoustic time-scale in the initial state of the volume,

B. $\beta << O(1)$ is the non-dimensional high activation energy parameter familiar in thermal explosion theory,
C. H_R is the non-dimensional heat of reaction.

Asymptotic methods are used to find reduced forms of the full Euler equations in the context of thermal explosion theory when the initial state is described by small, $O(\beta)$, deviations from an undisturbed base state like that used in the inert gas study. The induction time-scale is defined in terms of the parameter β in the usual way [9]. In the limit $\beta \rightarrow 0$ the nonlinear convection terms are suppressed.

When $t_i/t_A = O(1)$ the reduced equations devolve to non-homogeneous linear wave equations for each of the $O(\beta)$ thermodynamic and species perturbation varibles, and a vector wave equation for the $O(\beta)$ induced velocity Non-linear reaction terms depending on the time-history of the rising temperature perturbation, the sources of transient spatially distributed exothermic heat release, drive the variations in all perturbation variables. Given experience with thermal explosion theory [9] it is possible that one kind of temperature perturbation evolution will be characterized by a gradual increase during the induction time period, followed by a classical logarithmic singularity at the finite «explosion time» leading to a very brief period of large temperature increase during which most of the heat of reaction is released. If the heat of reaction parameter $H_R >> O(1)$ the results described for Case D, above may be germane. Alternatively, the temperature increase may be limited by the cooling effect of gas expansion. Numerical solutions for the perturbation wave equations are needed to quantify the time-history for finite values of t_i/t_A.

If the induction time-scale is assumed short compared to the acoustic time-scale, the limit $t_i/t_A \rightarrow 0$ is used to derive the reduced equation set, intiially for $H_R = O(1)$. Compressibility effects are suppressed! The density and velocity perturbations are $O(t_i/t_A)^{1/2} << O(1)$ and $O(t_i/t_A) << O(1)$, respectively. The non-hyperbolic reduced equations feature a «classical», constant density thermal explosion [9] expression for the temperature perturbation and a pressure pertubation equal to the temperature perturbation. The induced velocity is driven by the gradient of the pressure perturbation and is compatable with the minor density variation. The analytical solution for the temperature perturbation includes a logarithmic singularity at a finite explosion time value. That singularity appears in all of the variable solutions. The singularity characteristics combined with the asymptotic expansions for the thermodynamic and velocity variable are used to show that the thermal explosion is followed by an exponentially short period relative to t_i, during which O(1) changes in all the variables occur if $H_R = (1)$ [9]. If $H_R >> O(1)$ the description above for Case D provides a physical understanding of the consequences of «fast», «large» energy deposition into a gas volume. In particular, the microvolume will experience a «large» rapid increase in temperature and pressure and a significant induced velocity leading eventually to the generation of strong mechanical disturbances in the unheated nearby gas. Presumably these waves can pressurize the entire larger environment. Similarly the entire macrovolume will experience strong mechanical disturbances if the assumed combustion process is distrubted throughout the volume. The complete analysis will be presented at the conference.

4. Preliminary Conclusions

The thermo-mechanical response of a gas to thermal energy deposition depends intimately on both the deposition time-scale (ε) and quantity of energy added (α) during that interval. Some combinations of the parameters are compatible with hyperbolic equations while others are not. Surprisingly, some responses are characterized by nearly constant volume physics, while others occur in a nearly isobaric manner. The results demonstrate that significant pressure variations can occur in fixed macro-volumes when the two parameters have appropriate values. The mechanism is not likely to be described by solutions to linear or nonlinear wave equations used in traditional combustion chamber stability studies. The novel message here is that pressure disturbances in combustion chambers can arise either from a distributed set of active discrete micro-volumes or from heating distributed throughout a macro-volume

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