# Application of Dynamic Mode Decomposition for Stabilization of Reactive Flow in a Subscale Combustor with an Injector

Young Jun Kim, Guillaume Jourdain, Chae Hoon Sohn<sup>\*</sup> Department of Mechanical Engineering, Sejong University Seoul, 05006, Republic of Korea

## Abstracts

The acoustic optimization of a swirl coaxial jet injector is investigated to tackle combustion instabilities. The least damped modes are extracted by applying dynamic mode decomposition (DMD) and the injector length is optimized to damp the second longitudinal mode. The sensitivity of heat release perturbation to velocity perturbation at the frequency of the second longitudinal mode is investigated by combining two concepts of the Crocco's model and the inhomogeneous wave equation, leading to computation of flame transfer function (FTF). The gain of FTF shows that the sensitivity of heat release fluctuation to inlet velocity fluctuation is minimal in the chamber with the optimized injector length. Dynamic mode decomposition combined with the inhomogeneous wave equation and the Crocco's model appears to be a valuable tool to compute efficiently flame transfer function and evaluate the stability of a combustion chamber.

## 1. Introduction

Combustor design is achieved through several stages which take into account, for example, the mechanical constraints and the required power output. Apart from those main design requirements, an acoustic optimization is needed to avoid thermo-acoustic instabilities [1]. The acoustic optimization of the combustor cab be achieved by re-designing injectors as well as installation of dampers. The injector is used as a device enabling acoustic damping of potentially dangerous thermo-acoustic modes. It has been shown in the previous works that injectors can act as a half-wave resonator and damp pressure wave significantly if they are tuned properly [2, 3] and then, combustion is stabilized.

In the present study, the wide range of injector length is investigated by performing dynamic mode decomposition [4,5] and the injector is optimized in the aspect of combustion stability. As a result, the optimal injector length is found with respect to maximum acoustic damping. In addition, flame transfer

### Kim, Y. J.

function [6,7] is computed by evaluating the sensitivity of heat release rate to the velocity fluctuation at the inlet of the combustor, which can be a quantitative parameter of thermo-acoustic instability induced by perturbed flames.

## 2. Chamber and injector geometry

The subscale chamber [2, 8] has the cylindrical shape with a single injector at the chamber inlet. The chamber has a diameter,  $D_{ch}$ , of 142 mm and a length,  $L_{ch}$ , of 537 mm. An injector, a gas-centered swirl coaxial injector, is mounted at the inlet of the chamber. The injector is shown in Fig 1. Fuel in prevaporized kerosene, which is injected through peripheral holes.



Figure 1. Geometry of the gas centered swirl coaxial injector.

In the present study, the adjustable parameter for injector tuning is the injector length,  $L_{inj}$ , which varies from 120 mm to 160 mm, and the other geometric parameters of  $d_{orif}$ ,  $d_{inl}$ ,  $d_{inj}$ , and the recess length,  $R_l$ , are fixed to be 6.6 mm, 10 mm, and 8 mm, respectively. The chamber geometry and the computational grids are shown in Fig 2 and the number of grids in the entire domain is selected after grid-dependency check, which is over 0.4 million.



Figure 2. Geometry and computation grids of the model chamber and the injector.

#### Kim, Y. J.

Dynamic mode decomposition is a method to extract linear fluctuations out of instantaneous fields of data set collected from CFD analysis or measured data. It was validated in our previous work [5]. The result of pressure fluctuation p' is substituted into the inhomogeneous wave equation (IWE) [9] to get heat release fluctuation q' as described in Eq. (1).

$$\bar{\rho}\bar{c}^2 \frac{\partial}{\partial x_i} \left(\frac{1}{\bar{\rho}} \frac{\partial p'}{\partial x_i}\right) - \frac{\partial^2 p'}{\partial t^2} = q'(1-\bar{\gamma}), \qquad (1)$$

where  $t, x_i, \overline{\gamma}, \overline{\rho}$ , and  $\overline{c}$  denote time, spatial coordinate, mean heat capacity ratio, mean density and mean speed of sound, respectively. The Eq. (1) is combined with the n- $\tau$  model (or Crocco's model) [10] to evaluate flame sensitivity, where heat release fluctuation is correlated with the inlet velocity fluctuation in the form,

$$q' = n u'_{inlet}(t - \tau), \qquad (2)$$

where he parameters, n and  $\tau$ , are evaluated depending on the frequency, f, at which each pressure wave is oscillating. They are called sensitivity index or gain and time lag, respectively. The obtained heat release rate fluctuation q'(f) and u'(f) at a specific frequency are employed to calculate flame transfer function [6,7] by using the following equation,

$$F(\omega) = \frac{q'(f)}{u'_{inlet}(f)}.$$
(3)

## 4. Results and Discussion

First, by applying DMD, the fluctuation of pressure p' and velocity u' are obtained to substitute into Eqs. (1) and (2) and the parameters of n and  $\tau$  are obtained. The local values of gain, n, and time lag,  $\tau$ , are calculated at the frequency of the 2L mode in the combustion chamber mounted by a 130 mm long injector. And, The hydrodynamic wavelength can be estimated by the velocity fluctuation as demonstrated in Fig. 3.



Figure 3. Fields of the parameters, *n* and  $\tau$ , for flame transfer function (FTF) at the 2L-mode frequency ( $f_{2L}=1,611$ Hz) with the injector length of 130 mm: (a) gain, (b) time lag, and (c) velocity fluctuation for the 2L mode and iso-temperature contours (dashed line).

The gain averaged in a whole domain is then computed for every injector design case and it is normalized

by the gain calculated with the injector length of 120 mm, which is a start length. The normalized gains are calculated as a function of the injector length. The presents results are compared with those from a simplified 1-dimensional thory [11]. The minimum value of the averaged gain is found for the injector length of 130 mm. It implies that the reactive flow shows the least sensitivity to acoustic fluctuations at the length and the interaction is minimal between velocity fluctuation and heat release fluctuation at the 2L mode frequency. The 1-dimensional theory shows much discrepancy although it has the similar qualitative trend compared with the present results. The present tool would be a viable method in analyzing relative stability rather than the 1-dimensional theory.

# 5. Conclusion

For combustion stabilization, acoustic optimization of an injector in a sub-scale combustion chamber has been numerically conducted by applying dynamic mode decomposition to get flame transfer function. The frequency-fixed heat release fluctuation in flame transfer function (FTF) was calculated by combining Crocco's model with the inhomogeneous acoustic wave equation. The data used in the combined equation were obtained by application of DMD. As a result, the local gain and the local time lag were computed to evaluate FTF and from the averaged gain, the optimal injector length of 130 mm has been found for the second longitudinal mode. With the length, combustion is the most stabilized. Dynamic mode decomposition can be a useful tool in evaluating sensitivity of flame response to perturbation.

## Acknowledgments

This work was supported by Advanced Research Center Program (NRF-2013R1A5A1073861) through the National Research Foundation of Korea (NRF) grant funded by the Korean government (MSIP) contracted through Advanced Space Propulsion Research Center at Seoul National University. CHS was partially supported by the National Research Foundation of Korea (NRF) grant funded by the Korean government (MSIP) (Grant No. NRF-2015M1A3A3A02009957).

# References

[1] Feldman KTJ.1968. Review of the literature on Rijke thermoacoustic phenomena. J Sound Vib. 7(1):83–89.

[2] Sohn CH, Park IS, Kim SK, Kim HJ.2007. Acoustic tuning of gas-liquid scheme injectors for acoustic damping in a combustion chamber of a liquid rocket engine. J Sound Vib. 304:793-810.

[3] Park IS, Sohn CH. 2010. Nonlinear acoustic damping induced by a half-wave resonator in an acoustic chamber. Aerosp. Sci. Technol. 14:442-450.

[4] Schmid PJ. 2010. Dynamic mode decomposition of numerical and experimental data. J Fluid Mech. 656:5–28.

[5] Jourdain G, Eriksson LE, Kim SH, Sohn CH. 2013. Application of Dynamic Mode Decomposition to Acoustic-Modes Identification and Damping in a 3-Dimensional Chamber with Baffled Injectors. J Sound Vib. 332(18):4308-4323.

[6] Truffin K, Poinsot T. 2005. Comparison and extension of methods for acoustic identification of burners. Combust. Flame 142:388–400.

[7] Duchaine F, Poinsot T. 2010. Sensitivity of flame transfer functions of laminar flames. Proc. The Summer Program, Center for Turbulence Research. 251-258.

[8] Sohn CH, Seol WS, Shibanov AA, Pikalov VP. 2004. On the method for hot-fire modeling of high frequency combus-tion instability in liquid rocket engines. KSME Int'l J. 18:1010-1018.

[9] Poinsot T, Veynante D. 2001. Theoretical and numerical combustion. Philadelphia: RT Edwards.[10] Crocco L, Cheng S. 1956. Theory of Combustion Instability in Liquid Propellant Rocket Motors. Butterworths Scien-tific Publications.

[11] Truffin K, Poinsot T. 2005. Comparison and extension of methods for acoustic identification of burners. Combust. Flame142:388-400.