Reaction-Diffusion and G-Equation Approaches Reconciled for Turbulent Premixed Combustion Modelling

Gianni Pagnini[1,2], Rinie Akkermans[1,3], Andrea Mentrelli[1,4], Nadine Buchmann[1,3]

[1] BCAM – Basque Center for Applied Mathematics, Bilbao, Basque Country, Spain
[2] Ikerbasque, Bilbao, Basque Country, Spain
[3] TU Braunschweig, Institute of Fluid Mechanics, Germany
[4] University of Bologna, Department of Mathematics & AM[2], Italy

Two classical approaches in turbulent premixed combustion modelling are the one based on the reaction-diffusion equation, e.g. [1], and that based on the G-equation, e.g. [2]. These two approaches can be considered alternatives to each other because the solution of the reactiondiffusion equation is generally a continuous smooth function that has an exponential decay, and it is non-zero in an infinite domain, while the G-equation, which is strongly connected to the front tracking technique named level-set method [3], generates a sharp function that is non-zero on a compact domain. However, it is here shown that these two approaches are indeed complementary and they can be reconciled. Starting from the G-equation and the random motion of the flame front, following the average procedure adopted by Oberlack et al. [4], an integral representation for the average progress variable is derived [5]. This averaged formulation takes into account the probability density function of the Lagrangian particle displacement within the turbulent flow [5]. The emerging evolution equation for such averaged progress variable is an equation of reaction-diffusion type. As the turbulent diffusion vanishes, the particle density tends to a Dirac delta function and the G-equation is recovered, while when a Gaussian density is assumed the Zimont equation is obtained [6]. This approach can be considered the natural three-dimensional generalization including the mean front curvature for the Zimont equation. The potentiality of the proposed approach are investigated analytically and numerically focusing on the effects on the front evolution of parabolic- [5], hyperbolic- [7] and fractional-diffusion [8,9] processes for the Lagrangian turbulent motion, as well as on the effects of the mean front curvature.

References

[1] Lipatnikov A. N., and Chomiak J. (2005). Self-similarly developing, premixed, turbulent flames: A theoretical study. Phys. Fluids 17: 065105

[2] Peters N. (1999). The turbulent burning velocity for large-scale and small-scale turbulence. J. Fluid Mech. 384: 107–132

[3] Sethian J. A., and Smereka P. (2003). Level set methods for fluid interfaces. Ann. Rev. Fluid Mech. 35: 341–372

[4] Oberlack M., Wenzel H., and Peters N. (2001). On symmetries and averaging of the G-equation for premixed combustion. Combust. Theory Modelling 5: 363–383

[5] Pagnini G., and Bonomi E. (2001). Lagrangian formulation of turbulent premixed combustion. Phys. Rev. Lett. 107: 044503

[6] Zimont V. L. (2000). Gas premixed combustion at high turbulence. Turbulent flame closure combustion model. Exp. Therm. Fluid Sci. 21: 179–186

[7] Zimont V. L., and Pagnini G. (2011). Lagrangian properties of turbulent diffusion with passive chemical reaction in the framework of the premixed combustion theory. Phys. Fluids 23: 035101

[8] Mentrelli A., and Pagnini G. (2014). Random front propagation in fractional diffusive systems. Commun. Appl. Ind. Math. `http://dx.doi.org/10.1685/journal.caim.504`

[9] Mentrelli A., and Pagnini G. (2015). Front propagation in anomalous diffusive media governed by time-fractional diffusion. J. Comput. Phys. 293: 427–441