Incomplete Detonation in Type Ia Supernovae

A. Khokhlov
The University of Chicago, USA
I. Dominguez
The University of Granada, Spain

We review the dynamic properties of low-density incomplete carbon-oxygen (CO) detonations in Type Ia supernovae (SNe Ia). An incomplete detonation produces chemical elements from O^{16} to Si-peak elements, whereas the Fe-peak elements are synthesized when the nuclear reactions proceed to chemical equilibrium. Properties of incomplete detonations are fundamental for current theoretical understanding of the explosion mechanism of SNe Ia.

Incomplete detonation is sensitive to ambient density, overdrive, and geometry, and is especially sensitive to the initial composition of CO matter. An incomplete detonation may be viewed as a quasi-steady detonation wave in CO mixtures with carbon mass fraction $X(C^{12}) \simeq 0.5$ or greater, which is expected in SNe Ia exploding in binary systems with non-degenerate stellar companions. For CO mixtures with $X_C \leq 0.3$, expected in SNe Ia in binary white dwarf stellar systems, the detonation must be treated as an unsteady explosive burning taking place in rapidly varying background conditions.

2 Physical model of a thermonuclear detonation in supernovae

Explosive burning of carbon-oxygen in SNe Ia generally proceeds through three distinct kinetic stages: C-burning - burning of C^{12} and synthesis of O^{16}, Ne^{20}, Mg^{24}, and Si^{28}, the subsequent O-burning - burning of O^{16} and the conversion of all products of C-burning into Si-group elements and, finally, Si-burning - burning of Si-group elements into the elements of the Fe-peak. The characteristic time-scales of these stages differ by orders of magnitude with C-burning being the shortest and Si-burning being the longest. Incomplete burning with nuclear reactions terminating after C-burning or O-burning explains the presence of intermediate mass chemical elements observed in SNe Ia at maximum light. See [1] for further references on SNe Ia modeling and on the modeling of incomplete detonations.

The detonation can be described by the compressible reactive flow Euler equations of fluid dynamics,

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0, \tag{1}$$

$$\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot \rho \mathbf{u}\mathbf{u} + \nabla P = 0, \tag{2}$$

$$\frac{\partial E}{\partial t} + \nabla \cdot ((E + P)\,\mathbf{u}) = \rho \sum_{k=1}^{N} Q_k R_k, \tag{3}$$

Correspondence to: ajk@oddjob.uchicago.edu & inma@ugr.es

$$\frac{\partial \rho Y_k}{\partial t} + \nabla \cdot (\rho \mathbf{u} Y_k) = \rho R_k, \quad k = 1, ..., N, \tag{4}$$

where ρ is mass density, $\mathbf{u}$ is fluid velocity, $E = \rho e + \rho u^2/2$ is total energy density, e is the internal energy per unit mass, P is the pressure, Y_k and R_k are the chemical variables (mole numbers of reactants per unit mass) and the reaction terms, respectively, and Q_k are the binding energies of individual nuclei. In the simulations reported in this paper we used the α-network which consisted of $N = 13$ nuclei He^4, C^{12}, O^{16}, Ne^{20}, Mg^{24}, Si^{28}, S^{32}, Ar^{36}, Ca^{40}, Ti^{44}, Cr^{48}, Fe^{52}, and Ni^{56}. The network took into account reactions between α-particles and heavier nuclei $C^{12} + He^4 \rightarrow O^{16}$, $O^{16} + He^4 \rightarrow Ne^{20}$, ..., $Fe^{52} + He^4 \rightarrow Ni^{56}$; binary reactions $C^{12} + C^{12}$, $C^{12} + O^{16}$, $O^{16} + O^{16}$, the triple-alpha reaction $3He^4 \rightarrow C^{12}$ and their inverse reactions. The equation of state included the contributions from ideal Fermi-Dirac electrons and positrons with arbitrary degeneracy and relativism, equilibrium Planck radiation, and ideal Boltzmann nuclei.

The simulations were performed in a square "tube" with length L and cross-section W^2 in a reference frame moving with the detonation velocity D. A constant supersonic inflow of unburned matter with the velocity $-D$ was imposed on the upstream, right X-boundary and a zero gradient outflow on the downstream, left X-boundary. Symmetry was assumed in Y- and Z-directions. The initial steady-state ZND detonation solution was mapped onto the computational domain with the leading shock placed at $x = 0.9\,L$ facing the upstream. A small perturbation was introduced behind the leading shock in order to trigger multi-dimensional perturbations of the flow. The equations were integrated using a second-order accurate, conservative, Godunov-type, adaptive mesh refinement code. Euler fluxes were calculated using a Riemann solver and a monotone Van Leer reconstruction. The reaction terms were integrated together with the energy equation using an unconditionally stable stiff integration method with sub-sycling. During calculations the mesh was dynamically refined around the shocks, contact discontinuities, and in regions of large gradients of density, pressure, and composition of of H^4, C^{12}, O^{16}, Ne^{20}, Mg^{24} and Si^{28} [1–3].

3 Detonation propagation and stability

Numerical simulations show that the incomplete C-detonation is highly unstable because of the reduced energy release and the low post-shock temperature. As a result, the propagation is highly dependent on dimensionality and is qualitatively different in one, two, and three dimensions, as illustrated in Figures 1 and 2 below for a detonation with $X(C^{12}) = 0.5$. The one dimensional detonation is characterized by strong pulsations of post-shock pressure with the period of pulsations increasing with decreasing the overdrive f. The pulsation "period" of the CJ C-detonation is so large that the meaningful propagation becomes impossible. The two-dimensional cellular detonation exhibits marginal propagation which maintained by "reflections" of the transverse waves off the symmetry "walls" of the numerical tube. The period of the detonation is dependent on the tube width W. The three-dimensional detonation calculated at densities $10^6 > \rho_0 > 3 \times 10^5$ g/cm^3 shows an irregular but very robust propagation with the characteristic size of the largest structures (cells) of the order $\simeq 5x_C$ where x_C is the half-reaction length of the ZND detonation. The cell size obtained in the simulations with varying numerical resolution of $10 - 40$ computational cells per x_C was is practically independent of the resolution and of the W. The average distribution of physical parameters in a three-dimensional detonation is very close to the distribution of parameters in the corresponding ZND detonation.

The situation is different in carbon-poor mixtures, as illustrated for $X(C^{12}) = 0.3$ and $\rho_0 = 3 \times 10^6$ g/cm^3 in Figure 3. The average post-shock temperature in detonations with $X(C^{12}) = 0.3$ is lower compared to a detonation with $X(C^{12}) = 0.5$ due to a lower energy release in the leading $C^{12} + C^{12}$ reaction. As a result the detonation zone is more unstable and is characterized by a higher amplitude

of temperature fluctuations. The peak temperature near the triple-points becomes so high that the shock collisions trigger O-burning which converts matter to Si-group elements and almost doubles the energy release. The O-burning quickly spreads through the reaction zone of the incomplete C-detonation so that the C-detonation without the subsequent O-burning becomes impossible. The same behavior was observed in simulations at lower $\rho_0 = 10^6$ g/cm^3. However, the size of the reaction zone at this density is already $\simeq 0.1$ of the entire star. It is thus impossible to treat a detonation at $\rho_0 \leq 10^6$ g/cm^3 as a quasi-steady detonation wave.

3 Summary

The two necessary conditions for an incomplete C-detonation to propagate in a form of a quasi-steady detonation wave are (1) Large separation of the ZND reaction time-scales of C-burning, O-burning, and Si-burning stages. (2) A sufficiently small temperature perturbations inside the reaction zone of a cellular C-detonation incapable of triggering O-burning inside the reaction zone. Both conditions are met for C-detonations in a carbon-rich mixture $X(C^{12}) = 0.5$ typical of the outer layers of SNe Ia exploding in binary systems with non-degenerate secondary stars. The first condition is fulfilled but the second condition is violated in carbon-poor mixtures $X(C^{12}) \leq 0.3$ typical of the outer layers of SNe Ia exploding in the double-degenerate binary systems. As a result, the detonation in double-degenerate systems must be treated in a fully non-stationary manner.

That the detonation wave behavior is highly dependent on the dimensionality of the problem suggests that the outcome of an incomplete detonation in SNe Ia should critically depend on geometry of the exploding star which may be deformed by rotation, and on the direction of propagation up or down the density gradient. The dynamics of the initiation of the detonation and of the DDT (in case of precursor deflagration) should be affected by geometry and exact location of hot spots. Chemical composition of the outer layers should play a crucial role in the outcome of the explosion. The quantitative modeling of low-density incomplete detonation in SNe Ia, especially in doubly-degenerate binary stellar systems, requires time-dependent, three-dimensional reactive flow simulations with numerically resolved C- and O-burning zones.

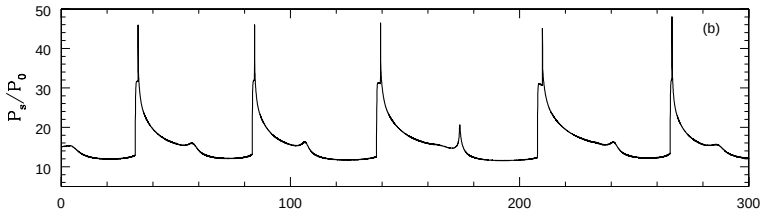
Acknowledgements. The work was supported in parts by the NSF grants AST-0709181 and TG-AST090074, BES and ASCR programs of the DoE Office of Sciences under contract DE-SC0002954 and the INCITE time allocations at the ALCF at Argonne National Laboratory, which is supported by the DoE Office of Science under contract DE-AC02-06CH11357 (A.K.), and the Spanish Ministry for Education Mobility Programme within the framework of the National Plan I+D+I 2008-3011, and Spanish Ministry for Science an Innovation project AYA2008-04211-C02-02 (I.D.).

References

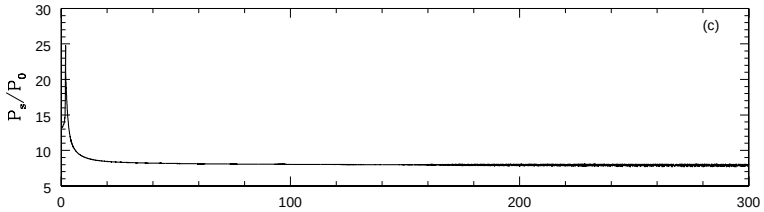
[1] Dominguez I and Khokhlov A (2011). Incomplete Carbon-Oxygen Detonation in Type Ia Supernovae. ApJ 730:87.

[2] Khokhlov A (1998). Fully Threaded Tree Algorithms for Adaptive Refinement Fluid Dynamics Simulations. JCP 143:519.

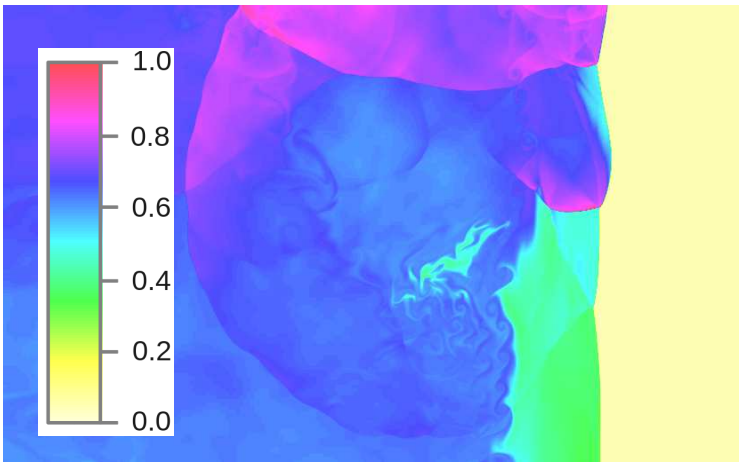
[3] Khokhlov A, Dominguez I, Bacon C, etal. (2012). Three-dimensional Simulations of Thermonuclear Detonation with alpha-Network: Numerical Method and Preliminary Results. Advances in computational astrophysics: methods, tools, and outcome. Proceedings of a conference held at Cefalu, Italy 13 - 17 June 2011. Astronomical Society of the Pacific Conference Proceedings, Vol. 453:107. Edited by R. Capuzzo-Dolcetta, M. Limongi, and A. Tornambe.



(a)

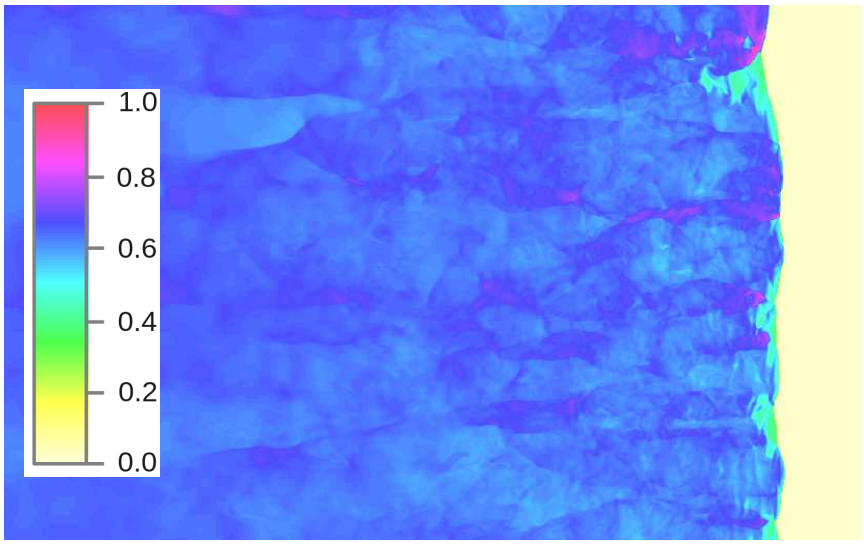


(b)

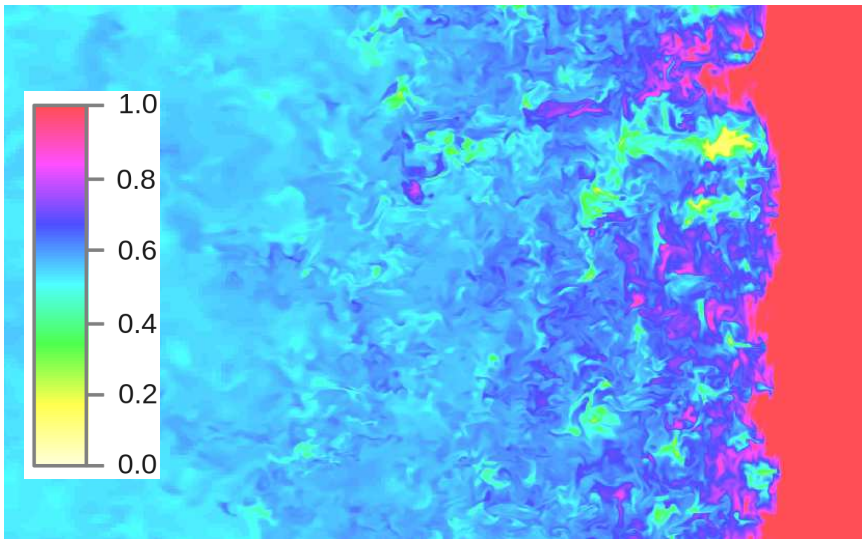


(c)

Figure 1: (a) and (b) - Post-shock pressure $P_s(t)$ for a one-dimensional galloping detonation in $\frac{1}{2}C^{12} + \frac{1}{2}O^{16}$ at $\rho_0 = 3 \times 10^6$ g/cm^3 and $f = 1.13$ and $f = 1$, respectively; t_C is the half-reaction time-scale of a ZND detonation. (c) - Marginal two-dimensional detonation in $\frac{1}{2}C^{12} + \frac{1}{2}O^{16}$ at $\rho_0 = 10^6$ g/cm^3. The plot shows $(T/4 \times 10^9 K)$ at $t = 142.3 t_C$.



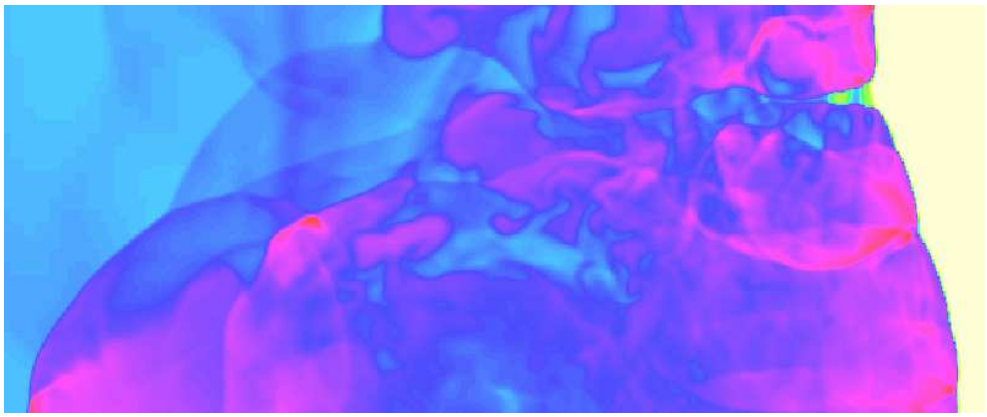
(a)



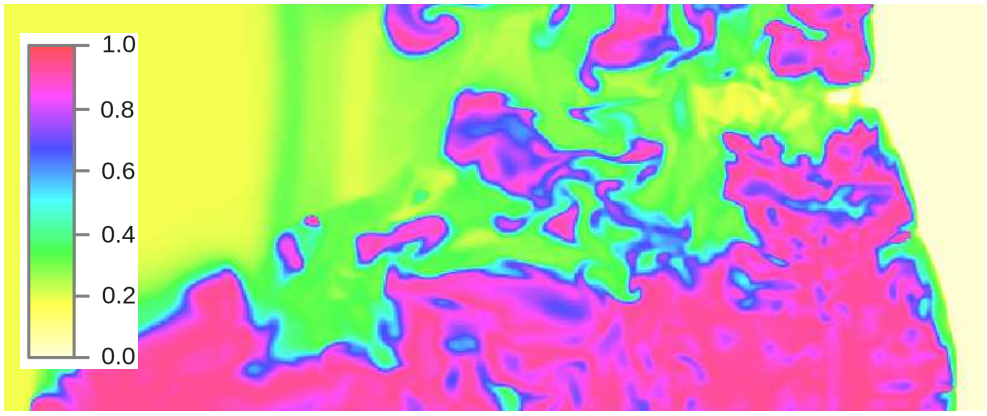
(b)

Figure 2: Three-dimensional cellular detonation in $\frac{1}{2}C^{12} + \frac{1}{2}O^{16}$ at $\rho_0 = 10^6$ g/cm^3 at $t = 50t_c$. (a) - $T/4 \times 10^9 K$ and (b) - $X(C^{12})/0.5$ in the XZ-plane passing through the centerline of the computationa domain.

(a)



(b)



(c)

Figure 3: Three-dimensional detonation in $0.3C^{12} + 0.7O^{16}$ mixture at $\rho_0 = 3 \times 10^6$ g/cm^3. (a) and (b) - $T/4 \times 10^9 K$ at $t = 0.07$ s and $t = 0.085$ s, respectively. (c) mass fraction of silicon $X(Si^{28}/0.9$ at $t = 0.085$ s, shown in the XZ-plane passing through the centerline of the computationa domain.