Viscous Solutions of the Triple Shock Reflection Problem

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1 Introduction

Irregular detonations have shown the ability to propagate with losses beyond the predicted limits. This has been ascribed to the presence of reflected shock waves [1] in the detonation cellular structure periodically colliding, creating hot spots and increasing reaction rates. The reflected shocks propagate transverse to the front and form a triple shock structure with the detonation front ahead (incident shock) and behind (Mach stem) the reflected wave. A contact surface (slip line) separates the flow shocked by the Mach stem from the flow shocked by both the incident and reflected waves. The three shocks and contact surface join at the triple point. The triple shock structure can be formed following the reflection of a shock wave on a surface or plane of symmetry. Shock strength, the isentropic exponent of the gas, and angle of incidence are important factors to the resulting reflection configuration, as well as the initial boundary conditions [2]. Thorough reviews of shock reflection have been written by Ben-Dor [3] and Hornung [4].

The triple shock reflection problem occurs naturally with the collision of detonation cells, as seen in figure 1. Here a detonation wave travels from left to right. Two triple shock structures travel towards each other, one from the bottom and the other from the top, as seen in the first two frames. The structures then reflect off each other and move apart, as seen in the next three frames. The reaction front, seen as a bright textured structure trailing the shocks, becomes more closely coupled with the shock front following a reflection, causing it to become locally over-driven, sustaining the detonation and cellular structure. Cellular dynamics have been well studied in the past [5–9].

Numerous studies [10–16] have examined reactive shock reflections, seeking insight on the phenomena responsible for the creation of locally over-driven detonations, and have come upon a handful of candidates. Adiabatic compression from the incident shock reflection may sufficiently heat the gas to reduce induction times, coupling the reaction and shock fronts; jet formation may entrain combustion radicals from reacted zones to the zone behind the Mach stem where subsequent mixing with unburnt gases may increase reaction rates; Richtmyer-Meshkov instabilities may arise from the interaction between pressure waves and density gradients, accelerating mixing; and Kelvin-Helmholtz instabilities along shear layers may also accelerate mixing. These mechanisms have been suggested as causes of increased reaction rates leading to detonation re-initiation. While these studies have been able to capture and study reactive shock reflections and have clarified the importance certain phenomena over others, the cause of re-initiation still remains unclear.

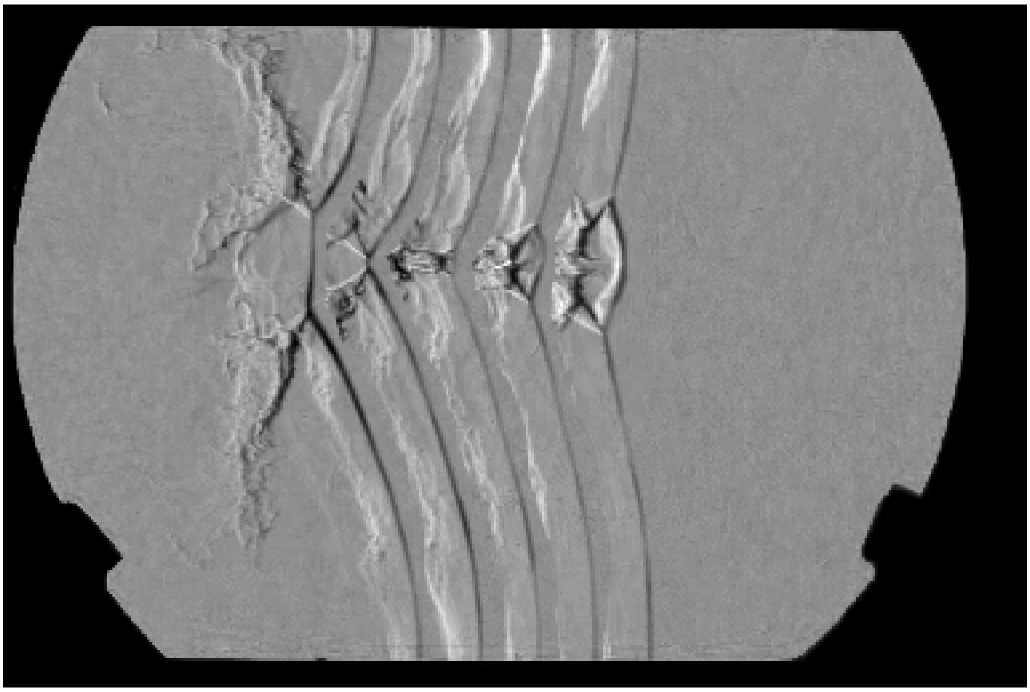


Figure 1: Superimposed schlieren photographs (cropped, with grain extraction and stretched contrast) of two triple points colliding in a detonation, from Bhattacharjee [13] (CH_4+$2O_2$, $\hat{p}_0 = 3.5$ kPa, $\hat{T}_0 = 300$ K, $\Delta\hat{t} = 11.53$ μs, channel height = 203.2 mm)

Inviscid numerical simulations of shock reflections have predicted bifurcation of the Mach stem under certain conditions [17, 18], caused by a strong jet and vortex and leading to an acceleration of the Mach stem and creation of new cells. This aspect of shock reflection has, however, been absent in experiments. The validity of the inviscid flow assumption has also come into question recently [19] as mechanisms for turbulent mixing may significantly increase reaction rates behind the detonation front.

This study looks at the effect of viscosity on the resolved triple shock reflection for conditions relevant to detonations. The triple-shock reflection reproduces the geometry of cell collision more accurately than the reflection of a single shock front studied previously, and removes certain numerical difficulties associated with creation of reflection boundaries and grid alignment.

2 Numerical method

The numerical simulations solve the compressible Navier-Stokes equations in two dimensions.

A resolution study was performed on a one-dimensional shock with Mach number $M = 4$ through a quiescent gas with isentropic exponent $\gamma = 1.2$. The shock thickness appeared to converge at a resolution of approximately 100×2^3 grids per unit length for a kinematic viscosity $\nu = 0.01$.

Figure 1 shows experimental results of a detonation passing through stoichiometric methane-oxygen, performed by Bhattacharjee [13]. Conditions calculated from the second frame of figure 1 were imposed as initial conditions onto the domain. They consist of an ideal triple-shock solution with an incident Mach stem of strength $M = 5.362$ and an angle $\alpha_M = 51.95°$ to the horizontal, and a shock normal to the reflecting boundary. The triple-point's frame of reference in the x direction was used, with the unshocked gas traveling at a velocity $u_0 = -4.36136$. An isentropic exponent $\gamma = 1.2$ was used, corresponding approximately to the post-shock state of the pre-reflection Mach stem (zone 3). The triple

Table 1: Initial conditions, zones refer to figure 2

	Unshocked (zone 0)	Incident shock (zone 1)	Reflected shock (zone 2)	Mach shock (zone 3)
ρ	1	6.74488	11.0494	8.16127
u	-4.36136	-0.646618	-1.28933	-0.302763
v	0	0	-0.63216	-3.176625
p	1	17.2013	31.2724	31.2724

point was positioned 0.5 unit lengths above the reflecting surface to allow the viscous shock structure to fully develop prior to reflection. The initial conditions are tabulated in table 1 and shown in figure 2.

Simulations were run using the `mg_g` computational package developed by Mantis Numerics Ltd. A second order accurate exact Godunov scheme was used to solve convective terms, and diffusive terms were solved explicitly [20]. A maximum domain size of 80 by 64 unit lengths was modeled, covered by a Cartesian mesh with 125 by 100 grid points and using nine levels of adaptive mesh-refinement with a relative tolerance of 0.01 between levels [21]. The bottom wall was parallel to the mesh and used a symmetry boundary condition while the remainder were free. The domain is sized in the y direction to allow enough room for the reflection to grow to the desired size before boundary error reaches the triple point.

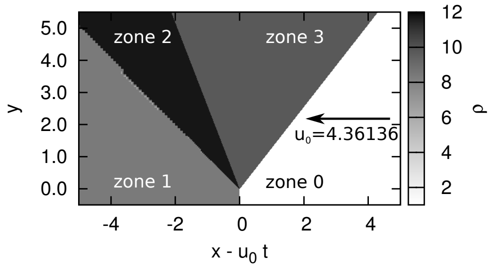


Figure 2: Initial conditions

3 Results

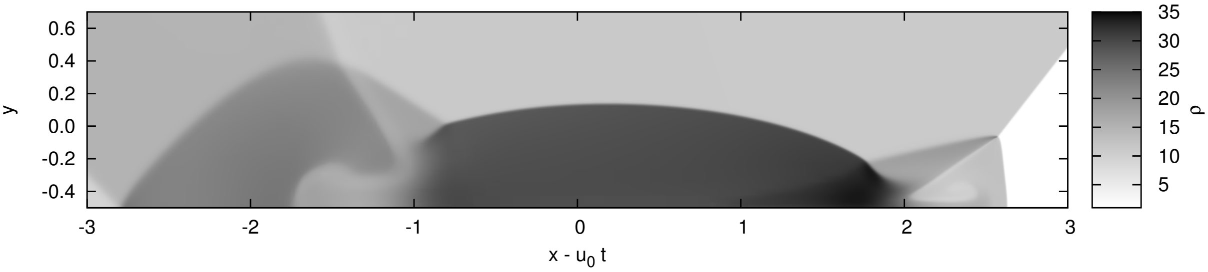
Cropped simulation results are shown in figure 3. A double Mach reflection is formed, seen on the right side of figure 3a, as the pre-reflection Mach stem (shock separating zones 0 and 3 of figure 2) reflects. The resulting Mach reflection has an triple point trajectory of $5.4°$ from the horizontal. The Mach stem travels at an average velocity $D = 8.41$ relative to the unshocked gas, yielding a Reynolds number of Re $= 12749.4$ which is comparable to the Reynolds number of the Mach stem at the induction length, approximated from the experiments to be Re $= 16400$. There is a wall jet and vortex behind the Mach stem, however it does not bifurcate [17]. Kelvin-Helmholtz instabilities have not yet developed along the contact surface.

A second reflection occurs, seen on the left of figure 3a, caused by the reflection of the transverse wave on the plane of symmetry. It takes the configuration of a regular reflection and cannot be captured in the previously mentioned studies, by design, until the second reflection event. The two reflections are joined at the pre-reflection contact surface where the reflected wave of the regular reflection curves across the contact surface and joins the Mach reflection's reflected wave. Kelvin-Helmholtz instabilities are also absent on this contact surface. The surface is deflected after being shocked by the curved reflected wave, and curls near the axis of symmetry.

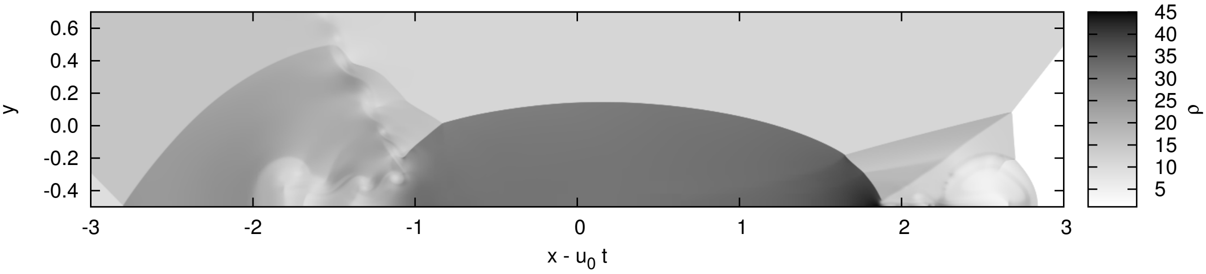
The inviscid result is shown in figure 3b. A large vortex is present at the Mach stem, bifurcating it. The triple point trajectory is superior to the viscous case. Kelvin-Helmholtz instabilities develop quickly along the pre-reflection contact surface, but are still absent, at these scales, along the contact surface behind the Mach stem.

The temperature profile of the viscous simulation is plotted in figure 3c and reveals the high temperatures along the contact surface, jet and vortex behind the Mach stem. This feature is absent in the inviscid case, seen in figure 3d, however both vortices layer cool and warm gas.

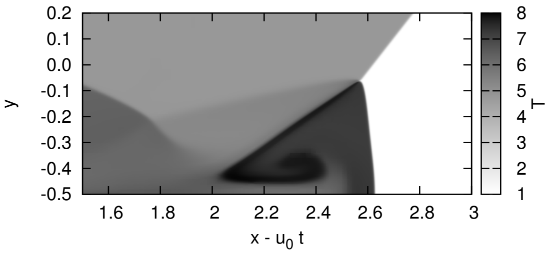
The time evolution of the viscous Mach reflection is shown in figures 3e and 3f. The wall jet and vortex behind the Mach stem grow with Reynolds number, impinging on the Mach stem and causing it to begin bulging. The Mach stem eventually bifurcates when Re $\gtrsim$ 160000, and the bulging becomes more severe, as shown in figure 3f. The Mach stem in the region of the triple point shows strong curvature.



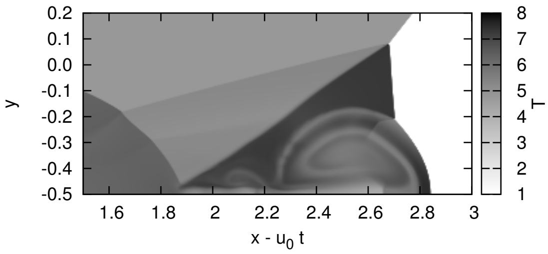
(a) $t = 0.84375$, Re=12749.4



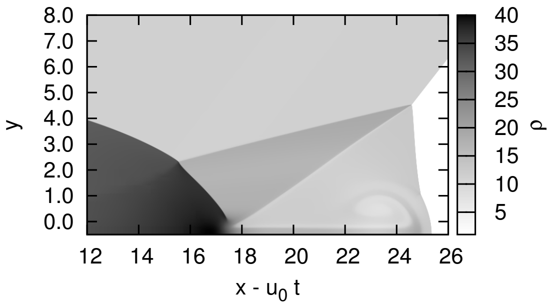
(b) $t = 0.84375$, inviscid



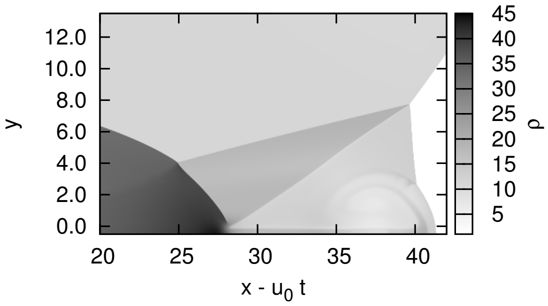
(c) $t = 0.84375$, Re=12749.4



(d) $t = 0.84375$, inviscid



(e) $t = 6.7887$, Re=133121



(f) $t = 10.827$, Re=223712

Figure 3: Simulations results: density ρ and temperature T

4 Conclusion

The reflection of the triple-shock configuration was studied numerically under conditions similar to those present in the creation of detonation cells. A double Mach reflection occurred from the reflection of the incident Mach stem. A regular reflection occurred when the transverse wave reflected.

Inviscid simulations showed bifurcation of the Mach stem and a large vortex. The length of the Mach stem was greater than the viscous case, however, Kelvin-Helmholtz instabilities are not present along the slip line at the time observed and resolution used. The formation of Kelvin-Helmholtz instabilities along the pre-reflection contact surface occurred rapidly.

When viscosity was considered, no Kelvin-Helmholtz instabilities were seen and there was no bifurcation of the Mach stem for Reynolds numbers corresponding to the induction length. However, a forward jet was present and the highest temperatures were seen along the contact surface and jet. Increasing Reynolds number eventually lead to bifurcation of the Mach stem, but Kelvin-Helmholtz instabilities remained absent.

5 Acknowledgements

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References

[1] J. H. Lee, *The detonation phenomenon*. Cambridge University Press Cambridge, 2008, vol. 2.

[2] S.-M. Lau-Chapdelaine and M. I. Radulescu, "Non-uniqueness of solutions in asymptotically self-similar shock reflections," *Shock Waves*, vol. 23, no. 6, pp. 595–602, 2013.

[3] G. Ben-Dor, *Shock wave reflection phenomena*. Springer, 2007.

[4] H. Hornung, "Regular and mach reflection of shock waves," *Annual Review of Fluid Mechanics*, vol. 18, pp. 33–58, 1986.

[5] V. Subbotin, "Collision of transverse detonation waves in gases," *Combustion, Explosion, and Shock Waves*, vol. 11, no. 3, pp. 411–414, 1975.

[6] V. N. Gamezo, D. Desbordes, and E. S. Oran, "Formation and evolution of two-dimensional cellular detonations," *Combustion and Flame*, vol. 116, no. 1, pp. 154–165, 1999.

[7] J. M. Austin, "The role of instability in gaseous detonation," Ph.D. dissertation, 2003.

[8] R. A. Strehlow, "Detonation structure and gross properties," *Combustion Science and Technology*, vol. 4, no. 1, pp. 65–71, 1971.

[9] M. Radulescu, G. Sharpe, J. Lee, C. Kiyanda, A. Higgins, and R. Hanson, "The ignition mechanism in irregular structure gaseous detonations," *Proceedings of the Combustion Institute*, vol. 30, no. 2, pp. 1859–1867, 2005.

[10] A. Teodorczyk, "Fast deflagrations and detonations in obstacle-filled channels," *Journal of Power Technologies*, vol. 79, 1995.

[11] S. Ohyagi, T. Obara, S. Hoshi, P. Cai, and T. Yoshihashi, "Diffraction and re-initiation of detonations behind a backward-facing step," *Shock Waves*, vol. 12, no. 3, pp. 221–226, 2002.

[12] T. Obara, J. Sentanuhady, Y. Tsukada, and S. Ohyagi, "Reinitiation process of detonation wave behind a slit-plate," *Shock Waves*, vol. 18, no. 2, pp. 117–127, 2008.

[13] R. R. Bhattacharjee, "Experimental investigation of detonation re-initiation mechanisms following a mach reflection of a quenched detonation," Ph.D. dissertation, University of Ottawa, 2013.

[14] S. Lau-Chapdelaine, "Numerical simulations of detonation re-initiation behind an obstacle," Ph.D. dissertation, University of Ottawa, 2013.

[15] L. Maley, R. Bhattacharjee, S. Lau-Chapdelaine, and M. I. Radulescu, "Influence of hydrodynamic instabilities on the propagation mechanism of fast flames," *Proceedings of the International Combusion Symposium*, 2013.

[16] Y. Lv and M. Ihme, "Computational analysis of re-ignition and re-initiation mechanisms of quenched detonation waves behind a backward facing step," *Proceedings of the Combustion Institute*, 2014.

[17] P. Mach and M. Radulescu, "Mach reflection bifurcations as a mechanism of cell multiplication in gaseous detonations," *Proceedings of the Combustion Institute*, vol. 33, no. 2, pp. 2279–2285, 2011.

[18] H. Glaz, P. Colella, I. Glass, and R. Deschambault, "A detailed numerical, graphical, and experimental study of oblique shock wave reflections," *Lawrence Berkeley National Laboratory*, 1985.

[19] M. I. Radulescu, G. J. Sharpe, C. K. Law, and J. H. Lee, "The hydrodynamic structure of unstable cellular detonations," *Journal of Fluid Mechanics*, vol. 580, pp. 31–81, 2007.

[20] S. Falle, "Self-similar jets," *Monthly Notices of the Royal Astronomical Society*, vol. 250, no. 3, pp. 581–596, 1991.

[21] S. Falle and S. Komissarov, "An upwind numerical scheme for relativistic hydrodynamics with a general equation of state," *Monthly Notices of the Royal Astronomical Society*, vol. 278, no. 2, pp. 586–602, 1996.