Rayleigh-Taylor Unstable Flames: Speed and Structure

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1 Rayleigh-Taylor Unstable Flames in Type Ia Supernovae

Type Ia supernovae are extremely bright stellar explosions that are especially important to astronomers because they can be used as standard candles to measure cosmological distances [1, 2]. Because there is an empirically known relation between the absolute luminosity of a Type Ia supernova and the time evolution of that luminosity, the luminosity (and, therefore, the distance to the supernova) can be determined [3]. However, the accuracy of this measuring procedure is limited by the natural variability of Type Ia supernovae luminosities. So, in order for astronomers to measure distances as accurately as they would like, Type Ia explosions must be thoroughly understood. Cosmological distances are used to determine the amount of dark energy in the universe, so making these measurements as accurately as possible is very desirable [4].

Excitingly, Type Ia supernovae are fundamentally a combustion problem. In one possible explosion scenario, a white dwarf star accretes matter from a nearby companion star. As the white dwarf becomes more massive, it also becomes more compact, triggering convection in its core; eventually, carbon and oxygen fusion begins [5, 6]. This fusion takes place in very thin, premixed flames, possibly on the surface of rising buoyant bubbles. Initially, the flame speed is very subsonic. The flame surface is Rayleigh-Taylor unstable because the fuel is more dense than the ash and parts of the flame propagate outward radially, against the direction of gravity. The Rayleigh-Taylor instability deforms the flame, dramatically increasing the surface area and the flame speed [7–18]. The perturbed flame also baroclinically produces turbulence, which is initially deposited within the flame and is left behind as the flame continues propagating. Next, by an unknown process, a deflagration-to-detonation transition (DDT) occurs, unbinding the star and causing the explosion. All of this happens in a few seconds.

Ideally, this whole explosion process would be modeled using a fully-resolved simulation of the entire star, but the difference in length scales makes this impossible. A white dwarf is about the size of the Earth, but the flame width is from 10^{-4} to 10^{2} cm [19]. As a result, full-star simulations must use subgrid models for the flame behavior on small scales (practically, scales less than a few kilometers). In particular, a subgrid model is needed for the flame speed, and usually, a DDT triggering condition [20] also must be included.

Unfortunately, it is far from clear what flame speed subgrid model should be used. So far, two major types of models have been suggested; they differ in their intuition about the processes that control the flame. Recall that the flame surface is affected by both the Rayleigh-Taylor instability and the

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turbulence generated by the flame front itself. Turbulence-based flame speed subgrid models assume that the flame is controlled by its own self-generated turbulence [21–24]. These models are adapted directly from traditional turbulent combustion theory and assume that the RT unstable flame will behave like a traditional turbulent flame (i.e. a flame moving through turbulence). But does a RT unstable flame actually behave like a turbulent flame? Physically, the two types of flames are quite different. A turbulent flame is forced to interact with every turbulent eddy it encounters, while an RT unstable flame may not interact with its self-generated turbulence at all. Clearly, the assumption that RT flames are like turbulent flames needs to be thoroughly tested. How similar RT unstable flames are to traditional turbulent flames is also important when trying to determine the DDT mechanism in the white dwarf. It has been generally assumed that RT unstable flames follow the traditional turbulent combustion regimes; in particular, that the flame will enter the reaction zones regime when the Gibson scale is less than the flame width. Then, it is hypothesized that the DDT could be due to the Zel'dovich gradient mechanism. But can RT unstable flames enter the reaction zones regime? Another proposed DDT mechanism [25, 26] also relies on RT unstable flames behaving like turbulent flames. But are RT unstable flames actually similar enough to turbulent flames that turbulent flame speed models, combustion regimes and detonation mechanisms should be used to model them?

The second type of flame speed subgrid model assumes that the RT instability controls the flame behavior [8, 13]. In this model, the flame speed is set by competition between flame surface area growth (due to the instability) and flame surface area destruction by regions of high curvature (cusps). This process is known as "self-regulation". The flame surface is assumed to be self-similar. On average, the flame is expected to propagate at the same speed as an equivalently-sized buoyant bubble. This model has the advantage of accounting for the RT instability, but it does not include possible effects of the self-generated turbulence. It also doesn't account for local changes to the flame structure and burning rate. In particular, when the fire-polishing scale (the smallest scale that the flame can be perturbed on by the RT instability) is smaller than the flame width, the flame could be densely covered by cusps and the local burning rate could be much different than the laminar flame speed. In other words, different "regimes" are not postulated by this model so it can't be used to hypothesize about the DDT.

In reality, elements of both the turbulence-based and RT-based views are probably necessary to fully describe Rayleigh-Taylor unstable flames. The flame speed is likely to be determined by both Rayleigh-Taylor and turbulent stretching and wrinkling of the flame front. Both processes may increase the flame surface area and modify the local burning rate. In addition to the flame speed, it is important to understand whether the flame transitions between different combustion regimes. However, the situation here is necessarily more complex than in traditional turbulent combustion. This is because there are two processes that affect the flame, and therefore two length scales that should be compared to the flame width (or, more exactly, the appropriate time scale comparison). For turbulence, this is the Gibson scale (ℓ_G); for the RT instability, this is the fire-polishing scale (ℓ_{FP}). When either scale is on the order of the flame width, the global flame speed may be modified by the presence of densely packed areas of high curvature (cusps), which modify the local burning rate [8, 27] . In addition, small-scale turbulence may also be able to "puff-up" the flame, increasing the flame width, causing a transition from flamelets to reaction zones.

In order to understand these processes in the supernova, it makes sense to divide the star into three radial zones depending on the ordering of the important length scales in the problem: the flame width (δ), the Gibson scale, and the fire-polishing scale, which all vary with stellar radius. In the dense, central part of the star, Zone 1, we have $\ell_{fp} > \ell_G > \delta$, so the internal flame structure (and so the local burning rate) should be unaffected by the RT instability and self-generated turbulence. Moving outwards in radius to Zone 2, we have $\ell_{fp} > \delta > \ell_G$. In this zone, the RT instability shouldn't affect the local flame structure, but turbulence may do so. Finally in the outer parts of the star, Zone 3, we have $\delta > \ell_{fp} > \ell_G$. In this zone, the local structure of the flame may be affected by both the RT instability and self-generated turbulence. Because different physical effects come into play as the order of the length scales in the problem changes, a different subgrid model may be appropriate for each zone. In this talk, I will discuss simulations with parameters appropriate for each zone (focusing on Zones 2 and 3) and identify the physical mechanisms that change the flame speed and structure.

2 Simulations

The most direct way to study Rayleigh-Taylor unstable flames is using flame-in-box simulations with a simplified model flame. In these simulations, the flame surface is initialized between fuel and ashes in a simple, rectangular, 3D computational domain with gravity and allowed to evolve in time (see Figure 1). This model simplifies both the fluid equations and the treatment of the reaction. We used the Boussinesq approximation to reduce the fully compressible Navier-Stokes equations to an incompressible form which accounts for the density variation across the flame surface only in the buoyancy term. This is a good approximation for Type Ia flames because the density difference across the flame front is relatively small. We avoided the complexities of a full reaction chain by using a simple model reaction (a form of the bistable reaction). All simulations used Nek5000 [28], a freely-available, open-source, highly-scalable spectral element code currently developed by P. Fischer and collaborators at Argonne National Laboratory (ANL).

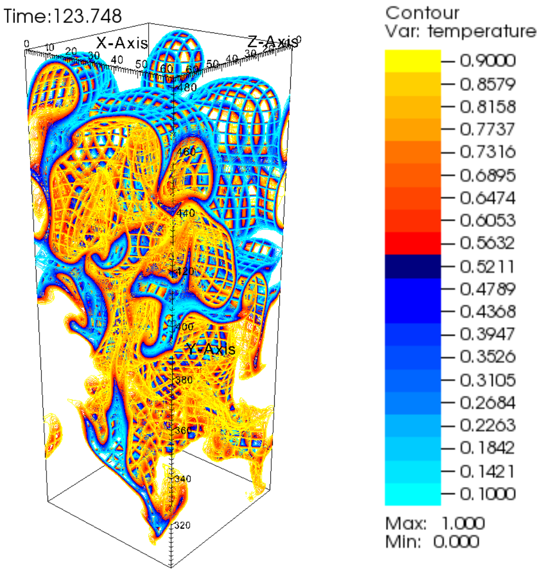

Figure 1: Rayleigh-Taylor Unstable Flame (Zone 3). Contours are of temperature, which acts as a progress variable ($T = 0$ is fuel and $T = 1$ is ash). The flame is propagating upwards in the box and gravity is pointing downwards. Many cusps are visible on the flame surface.

Generally, flame speed models predict the flame speed as a function of other parameters in the problem. Turbulence-based models predict the flame speed from the root-mean-square (rms) turbulent velocity (u'); RT-based models predict the flame speed from the Froude number, which is a measure of the strength of the RT instability. In order to test these models, we ran a parameter study of simulations with a variety of Froude numbers and, as a result, a range of rms velocities. Specifically, we varied the Froude number by varying the non-dimensional gravity (G) and the domain size (L). We also varied the Prandtl number (Pr). The Gibson scale and the fire-polishing scale naturally change as a result of varying these three fundamental parameters. The ordering of the scales in the simulations is the equivalent of white dwarf Zones 2 and 3.

3 Summary of Results

In this section, we give a short summary of some of our results [29]: measurements of the flame width and flame speed model tests.

First, we used measurements of the flame width to test whether or not Rayleigh-Taylor unstable flames follow the traditional turbulent combustion regimes. According to traditional regime theory, flames affected by turbulence such that $l_G < \delta$ (assuming equidiffusive conditions) should be wider than laminar flames because the energetic small-scale turbulent eddies increase the effective thermal diffusivity. This is generally known as the thin reaction zones regime. Most of the simulations in our parameter study are predicted to be in this regime, but measurements of the flame width tell a different story. As turbulent intensity increases, the flame actually becomes thinner (on average) than laminar value. This is highly unusual and suggests that the flame is being strongly strained, probably by the Rayleigh-Taylor instability. This suggests that RT unstable flames may not follow the traditional turbulent combustion regimes and that DDT scenarios that depend on the transition from flamelets to reaction zones may not apply to Type Ia supernovae.

As a basic test of the flame speed subgrid models used in Type Ia full-star simulations, we compared the time-averaged flame speed (s) and the time-averaged rms velocity (u') measured from the simulations to the predictions of three types of turbulent flame speed models that have been adapted from the traditional turbulent combustion community: simple linear models, scale invariant models and models that reproduce bending. In general, none of these models fit the data well for a simple reason: the simulation data, as plotted on a burning velocity diagram (s vs. u') are concave up, while all of the models are either linear or concave down. This suggests that RT unstable flames are fundamentally different from traditional turbulent flames. In addition, we found that models with no fitting parameters, or using the parameter values assumed in Type Ia simulations, overestimated the flame speed. We also tested the RT flame speed model. It fit the data well for low values of GL, but underestimated the flame speed at higher values of GL. Overall, it is possible that correctly-calibrated turbulent flame speed models and the RT model may be sufficient for Zone 2, but both types of models fail in Zone 3.

Overall, it is clear that Rayleigh-Taylor unstable flames behave differently than turbulent flames. As mentioned above, the flame moves faster than expected when the RT instability and turbulence are strong. One possible explanation for this behavior is that many regions of high curvature (cusps) form on the flame surface, changing the local burning rate and therefore the global flame speed [27]. If most of the regions of high curvature are negative, then the thermal focusing in these regions will cause the global flame speed to be higher than expected. Cusps can be formed either by turbulence or directly by the RT instability. From our simulations, it seems likely that, as long as turbulence and the RT instability are weak, flames propagate at the RT-predicted flame speed; but, when turbulence and the RT instability act energetically at the scale of the flame width, the flame propagates more quickly.

4 Future Directions

In the future, we plan to rigorously test the hypothesis that cusp formation, due to the RT instability and/or turbulence, increases the flame speed. To do this, we plan to develop an entirely new group of diagnostics for RT unstable flames. These new diagnostics are local in nature and will be used to link the global behavior of the flame to the local changes caused by curvature and stretch. We will measure quantities like the local flame width, the surface curvature and the local flow velocity at each point along the flame surface. These measurements will allow us to directly quantify all of the various contributions to the local flame stretch and its effect on the local burning rate. We will also use these local measurements to find global statistics for the flame surface, for example, quantifying how much of the flame surface is curved by a certain amount. We can then use those statistics to answer many key questions. Is the flame surface is more positively curved than negatively curved? How much of the surface area is very highly curved, and how does the local flame speed in those areas differ from the laminar flame speed? Do these regions of high curvature form because the Gibson scale or the fire-polishing scale is smaller than the flame width? How densely must the flame be covered in cusps before the global flame speed is affected? Overall, these local measurements, and their statistics on the flame surface, will help us to understand the real physical causes of the speed of RT unstable flames and will aid the formulation of new flame speed subgrid models. In addition, we plan to carry out new low Mach simulations using a more realistic flame model (the CF88 model, which is directly calibrated to match the structure of carbon burning flames) so that the results can quantifiably applied to Type Ia supernovae.

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