# Expanding Statistically Spherical Premixed Turbulent Flames and Astrophysical Combustion

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## 1 Introduction

While certain basic features of astrophysical nuclear combustion, e.g. extremely wide range of scales, huge Reynolds and Lewis numbers, moderate density ratios, well-pronounced Rayleigh-Taylor (RT) instability due to strong gravitation, etc., are very peculiar when compared to terrestrial chemical premixed turbulent flames, modeling of the astrophysical burning is still strongly based on knowledge gained by investigating the chemical flames. Among various premixed turbulent flames studied in terrestrial laboratories, expanding statistically spherical flames appear to be the least different from deflagration waves associated with the astrophysical combustion. Moreover, the global features of the spherical flames were thoroughly investigated by many research groups in a wide range of substantially different conditions, with governing physical mechanisms of premixed turbulent burning manifesting themselves well in experiments with such flames. Accordingly, the goals of the present work are (i) to overview key effects documented in experiments with expanding statistically spherical premixed turbulent flames, with particular emphasis being placed on effects disregarded often in astrophysical applications, (ii) to discuss capabilities of available numerical models for predicting these effects, (iii) to highlight unresolved basic issues, and (iv) to contribute to bridging a gap between chemical and astrophysical combustion communities.

## 2 Turbulent Burning Velocity and Flame Speed

Over the past six decades, the focus of experimental investigations of statistically spherical premixed turbulent flames that propagated in homogeneous isotropic turbulence generated by fans in a combustion bomb was placed on studying dependencies of turbulent burning velocity $U_t$ or flame speed $S_t$ on the rms turbulent velocity $u'$ and mixture characteristics such as the laminar flame speed $S_L$, thickness $\delta_L$, density ratio $\sigma$, Lewis number Le, i.e. a ratio of the molecular mass diffusivity of the deficient reactant to the molecular heat diffusivity of the mixture, etc. A turbulent burning (or consumption) velocity is associated with mass burning rate that is normalized using the partial density of the consumed reactant and is evaluated per unit area of a mean flame surface. Turbulent burning velocities are commonly obtained by processing pressure curves recorded in a bomb during explosions [1]. A turbulent flame speed is equal to the speed of the propagation of a mean flame surface with respect to unburned gas and is commonly determined using Schlieren techniques [2]. Both the burning velocity and flame speed are sensitive to measurement method, e.g. to the choice of a mean flame surface, and differ quantitatively from one another, with a ratio of $S_t/U_t$ being sometimes as large as five [3]. Nevertheless, dependencies of $U_t$ and $S_t$ on basic turbulence and mixture characteristics exhibit qualitatively similar trends, which will be discussed later.

In experiments with turbulent flame kernels expanding in a fan-stirred bomb after spark ignition, the rms velocity $u'$ can be varied in a wide range by changing the fan speed, while the integral length scale $L$ of the fan-generated turbulence depends weakly on the speed. Accordingly, contrary to $u'$, effects of $L$ on $U_t$ or $S_t$ have not yet been investigated in a single bomb. The temperature $T$ and pressure $P$ in the bomb at the ignition instant can also be varied, with both $u'$ and $L$ being weakly sensitive to such variations in the ignition conditions. As far as the influence of mixture composition on $U_t$ or $S_t$ is concerned, the simplest and widely accepted method to characterize it consists of considering $U_t$ or $S_t$ to be a function of $S_L$. However, when fuel or equivalence ratio is varied, not only $S_L$, but also $\delta_L$ and $\sigma$ are varied simultaneously and it is very difficult to substantially change one of these three mixture characteristics by keeping two other characteristics roughly unchanged. To the contrary, the Lewis number can be varied independently of $S_L$ , $\delta_L$, or $\sigma$ by using various diluents such as He or Ar [1,4].

As reviewed elsewhere [5,6], the following trends are well documented in various experiments with the expanding statistically spherical premixed turbulent flames.

**First,** in moderately intense turbulence, both $U_t$ and $S_t$ are increased by $u'$. This effect is associated with an increase in the area of the instantaneous flame-front surface due to turbulent stretching of the front. At first glance, the effect is predicted by almost all available models of premixed turbulent combustion. However, inspection of dependencies of $U_t$ or $S_t$ on $u'$, measured in various mixtures, reveals another important effect, which challenges many models, e.g. models that yield $U_t=S_t=S_L+bu'$ in the statistically planar one-dimensional flame (note that $U_t=S_t$ in such a simple case). The point is that an increase in $S_L$, which is commonly accompanied by an increase in $\sigma$ and a decrease in $\delta_L$, results in not only increasing $U_t$ or $S_t$, in line with a typical model of premixed turbulent combustion, but also increasing positive slope $dU_t/du'$ or $dS_t/du'$, with the latter effect being well pronounced.

**Second,** both $U_t$ and $S_t$ are increased by the pressure $P$, whereas, for typical paraffins, $S_L$ is decreased when $P$ is increased. This trend appears to be of great importance, because it invalidates a model that yields $U_t=S_t=u'f(S_L/u')$, where $f(x)$ is an arbitrary increasing function. The opposite effects of $P$ on $S_L$ and $U_t$ or $S_t$ clearly show that the turbulent $U_t$ and $S_t$ should depend not only on the laminar flame speed, but also on the flame thickness $\delta_L$. More precisely, $U_t$ and $S_t$ should be increased when the thickness is decreased, because $\delta_L$ is reduced by the pressure for typical paraffins.

As reviewed elsewhere [5], there are a few models capable for properly predicting the response of $U_t$ or $S_t$ to variations in the rms velocity (in moderately intense turbulence), pressure, and mixture composition provided that the Lewis number is close to unity. In particular, the predictive capabilities of the following simple theoretical expression [7]

$$U_t = Au'\mathrm{Da}^{1/4} = Au'\left(\frac{\tau_t}{\tau_c}\right)^{1/4} = Au'\left(\frac{LS_L}{u'\delta_L}\right)^{1/4}$$

are well documented by various research groups under substantially different conditions, as reviewed elsewhere [5,6]. Here, $A$=0.5 is a constant, Da is Damköhler number, $\tau_t=L/u'$ and $\tau_c=\delta_L/S_L$ are turbulent and chemical time scales, respectively. It is worth stressing that this expression is capable of properly predicting the positive scaling exponent $q$ in $U_t$~$P^q$ [5] and the model was developed by Zimont [7] well before a seminal experimental study by Kobayashi et al. [8], which (i) drew attention of many researchers to the opposite effects of $P$ on $S_L$ and $U_t$ or $S_t$ and (ii) engendered a number of papers that attributed this finding to an important role played locally by Darrieus-Landau (DL) instability [9] of premixed flame front in a turbulent flow. However, the present author is not aware of a model that highlights the DL instability and is capable of predicting $q$ in $U_t$~$P^q$ for various fuels, whereas the aforementioned simple expression by Zimont [7] is able to do so [5].

**Third,** when turbulence is sufficiently intense and is further increased, $U_t$ or $S_t$ is reduced [1,2,4]. This phenomenon is often associated with local quenching of thin inherently laminar flame fronts (flamelets) by high turbulent stretch rates and is commonly modeled using results of theoretical, experimental, and numerical studies of stretched laminar flames. However, a predictive model of a decrease in $U_t$ or $S_t$ with an increase in $u'$ has not yet been developed and a similar trend can be simulated by ignoring the local combustion quenching, e.g. see Ref. [10].

**Fourth,** the values $u'_m$ of the rms turbulent velocities associated with the maxima of measured $U_t(u')$- or $S_t(u')$-curves depend strongly on Le. In particular, $u'_m$ is substantially higher for mixtures characterized by Le<1 when compared to mixtures characterized by Le>1, in line with the concept of local quenching of inherently laminar flamelets by high stretch rates.

Even in moderately intense (i.e. $S_L < u' < u'_m$) turbulence, the rate of an increase in $U_t$ or $S_t$ by $u'$ is significantly higher for the former mixtures, with all other things being equal [1,2,4]. As reviewed elsewhere [11], the effect is attributed to an increase (Le<1) or decrease (Le>1) in the local burning rate in curved and strained flamelets and is often modeled invoking results of investigations of weakly stretched laminar flames. In particular, the following relation $S_L = S_L^0 - \mathrm{Ma}\,\delta_L \dot{s}$, derived in the limit case of weak perturbations [12], i.e. $\delta_L \dot{s} << S_L^0$, is widely used in the turbulent combustion literature. Here, $S_L$ and $S_L^0$ are the speeds of stretched and unperturbed, respectively, laminar flames, $\dot{s}$ is the stretch rate, and Markstein number Ma is a physical-chemical characteristic of the burning mixture.

It is worth stressing, however, that experimental data obtained from lean hydrogen flames, which are characterized by a low Le, show a very strong increase in $U_t$ in such mixtures [1,4] when compared to values of $U_t$ expected at the same $u'$, $L$, $T$, $P$, $S_L$, $\delta_L$, and $\sigma$ in the case of Le=1. Such data imply that highly perturbed flamelets play an important role in premixed turbulent combustion. This issue will be further addressed when discussing the leading point concept of turbulent burning [9,11,13].

**Fifth**, both $U_t$ and $S_t$ obtained from expanding statistically spherical premixed turbulent flames increase as the flame kernel grows, with the reported dependencies of $S_t$ on the mean flame radius $R_f$ being almost linear [14] in the range of the radii addressed in a typical experiment (5 mm< $R_f$ <50 mm). Such measured data can be predicted by hypothesizing that the increase in $S_t$ is caused by weakening of the reduction effect of the curvature of the mean flame brush on the flame speed [15]. The reduction effect is controlled by a ratio $\delta_t/R_f$ of the mean flame brush thickness $\delta_t$ to the mean flame radius $R_f$ and this ratio is decreased when the flame kernel grows.

# 3 Governing Physical Mechanisms of Premixed Turbulent Combustion

**First,** an increase in the area of the instantaneous flamelet surface by turbulent eddies that stretch the flamelet is the most recognized physical mechanism of the influence of turbulence on premixed burning. This mechanism controls the increase in $U_t$ by $u'$ in moderately intense turbulence ($S_L < u' < u'_m$), but can also play a role both in weak ($u' < S_L$) and strong ($u' > u'_m$) turbulence.

**Second,** turbulent stretching not only increases the flamelet-surface area, but also affects the structure of inherently laminar flamelets, the local burning (or consumption) velocity $u_c$ per unit area of the flamelet surface, and can even extinguish combustion locally. The influence of turbulent stretching on $u_c$ is commonly considered to control (i) the effect of Le on $U_t$ in moderately intense turbulence, (ii) the decrease in $U_t$ by $u'$ in intense turbulence, and (iii) the effect of Le on $u'_m$.

**Third,** as discussed in detail elsewhere [6], recent experimental and DNS data indicate that heat release is localized to thin zones even in highly turbulent terrestrial premixed flames, thus, putting into question a classical hypothesis that small-scale eddies could increase burning rate by broadening reaction zones and enhancing heat and mass transfer inside them. Nevertheless, DNS data by Aspden et al. [16] imply that such a physical mechanism can play a role in astrophysical nuclear combustion.

**Fourth,** the DL instability of inherently laminar flamelets is hypothesized to be an important physical mechanism of turbulent burning [8,13,17]. In weakly turbulent flames, this mechanism is widely recognized, but, as discussed in detail elsewhere [11,18], the present author is not aware of data that cogently show a manifestation of the DL instability in moderately or highly turbulent combustion. It is worth remembering that (i) turbulent stretching of a flamelet surface efficiently damps its instability [11] and, (ii) even in sufficiently weak turbulence, stretch rates generated by small-scale eddies can be higher than the growth rate of the DL instability [19], thus, implying that the growth of flamelet-surface area due to turbulent stretching is of more importance when compared to the DL instability.

Contrary to the local DL instability of inherently laminar flamelets in a turbulent flow, which was addressed above, eventual global DL instability of entire turbulent flame brush was beyond the focus of mainstream discussions in the combustion literature. This gap is associated with the fact that a typical terrestrial turbulent flame brush occupies a significant part of a combustion chamber. Accordingly, the maximum perturbation length scale is comparable with $\delta_t$ and the global DL instability does not occur under such conditions. However, if a turbulent flame is unbounded, then, for any finite $\delta_t$, which may be as large as we want, there is a perturbation with a so large length scale $\lambda>>\delta_t$ that the influence of this perturbation on the thick turbulent flame brush can be described by the DL theory with $S_L$ being substituted by $S_t$. Accordingly, the flame brush will be subject to the global large-scale DL instability and both $S_t$ and $\delta_t$ will grow. Subsequently, the flame acceleration can trigger the RT instability also.

It is worth noting, however, that, in order for the DL theory to be applicable to a statistically spherical premixed turbulent flame, the mean flame brush thickness should be significantly smaller than the mean flame radius, which bounds perturbation length scales. Accordingly, in sufficiently small flame kernels characterized by moderate $\delta_t/R_f$, the growth of $S_t$ with increasing $R_f$ appears to be controlled by weakening of the reduction effect of the curvature of the mean flame brush on the flame speed [15], followed by the growth of $S_t$ due to the global DL instability when the kernel is large and $\delta_t/R_f<<1$.

Moreover, even if the local DL instability of inherently laminar flamelets is overwhelmed by turbulent stretching, the DL physical mechanism, i.e. acceleration of unburned gas flow by combustion-induced pressure perturbations, could still play a substantial role in turbulent burning, because the pressure perturbations are much stronger in turbulent flames when compared to laminar ones. For instance, the aforementioned DL mechanism can manifest itself in the growth of unburned mixture fingers that deeply intrude into combustion products and significantly increase the flamelet-surface area and, therefore, $U_t$ [20]. As a result, $U_t$, $S_t$, and $\delta_t$ exhibit significant oscillations in time [20,21].

**Fifth**, the physical mechanisms discussed above are mainly associated with the influence of turbulence on combustion, but are not relevant to the influence of combustion on turbulence, which manifests itself in the so-called flame-generated turbulence or countergradient transport, as reviewed elsewhere [18]. While substantial progress in understanding and simulating the latter phenomenon was obtained over the past years [18,22,23], capabilities of available models for predicting turbulence characteristics within a premixed flame brush are still poor and even proper characterization of turbulence in flames is an issue [24]. Moreover, it is not clear how changes of turbulent flow that are induced due to density drop within flamelets can affect the propagation of the flamelets with respect to upstream unburned gas. In order for combustion-induced flow perturbations to affect $S_t$, such perturbations should be generated upstream flamelets, but the influence of combustion on upstream turbulence in constant-density unburned mixture has yet been studied poorly. As far as the countergradient transport is concerned, an increase in its magnitude, with all other things being equal, results in decreasing $S_t$ in the case of a statistically planar one-dimensional fully-developed premixed turbulent flame [25].

**Sixth,** certain approaches [9,11,13,26] to modeling premixed turbulent combustion place the focus of consideration on the speed $S_{le}$ of the leading edge of the mean flame brush, because (i) $S_{le}$ should be equal to $U_t$ in the case of a statistically planar one-dimensional fully-developed burning wave, but (ii) $S_{le}>U_t$ in a more common case [5,6] of a developing flame with growing $\delta_t$. Moreover, from the purely mathematical perspective, the speed of the physically realizable travelling wave solution to a convection-diffusion-reaction equation associated with premixed turbulent flame propagation can be controlled by the behavior of the source term in that equation at the leading edge of the wave [25].

Accordingly, in order to well understand premixed turbulent burning, it is not sufficient to know physical mechanisms that control the increase in $U_t$ by $u'$, e.g. the growth of flamelet-surface area due to turbulent stretching in the case of Le=1 and $S_L < u' < u'_m$. It is also necessary to reveal physical mechanisms that control the appropriate increase in $S_{le}$ by $u'$. The latter task still challenges the combustion community. Rapid propagation of flamelets along the axes of elongated vortex filaments, which are well known to exist in turbulent flows [27], could be such a mechanism [6,11], but the issue definitely requires further study.

Nevertheless, approaches that highlight leading points has also advantages when compared to models that address turbulence-flame interaction within the entire flame brush. While the latter models have not yet succeeded in explaining enormously high $U_t$ obtained from very lean hydrogen flames [1,4] characterized by a low Le, the leading point concept offers an opportunity not only to explain the phenomenon, but also to simulate it with encouraging results [28] by invoking the following hypothesis [13]. Because the leading point cannot move deeper into unburned mixture (in the coordinate framework attached to the mean flame brush and in the case of a fully developed flame with a stationary $\delta_t$), the local conditions in the vicinity of such a point should be critical for flamelet propagation. Accordingly, it is tempting to associate the local structure of the leading flamelet with the local structure of highly perturbed laminar flame that is close to quenching. If Le is low and the flame is adiabatic, then, the local burning rate under such critical conditions can be much higher than $\rho_u S_L^0$ [11,26]. To the best of the present author knowledge, highly curved spherical flames (flame ball [9]) have the highest local burning rate among various geometrical configurations of adiabatic strongly perturbed laminar premixed flames with a low Le. Therefore, highly curved laminar flames were proposed to be considered to be a model of the structure of the leading points of a premixed turbulent flame brush [11]. Such highly curved flamelets could be associated with the tip of a combustion front that propagates along the axis of a vortex filament [11].

In the case of a large Lewis number relevant to astrophysical combustion, the situation is different. On the one hand, if Le>>1, then, even small positive stretch (strain and/or curvature) rates can locally extinguish burning. On the other hand, the local curvature of a leading flamelet cannot be negative for purely geometrical reasoning. Thus, it is tempting to assume that, in the case of Le>>1, the leading flamelets are locally planar and the local consumption velocity $u_c$ is equal to $S_L^0$. In other words, the leading point concept offers an opportunity to assume a weak dependence of turbulent flame speed on the Lewis number if Le is large. Accordingly, in LES of such a flame, the influence of unresolved eddies on the local characteristics of the leading flamelets and, hence, on $S_t$=$S_{le}$ can be neglected and the problem of evaluating $S_t$ is reduced to tracking the advection of the leading points by turbulent eddies. Therefore, the problem is substantially simplified especially as the highest advection velocities are induced by large-scale eddies resolved in LES. Such a hypothesis definitely requires further study.

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