Direct and Indirect Combustion Noise in an Idealised Combustor

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1 Abstract

We examine travelling of acoustic, entropy and vorticity perturbations created by a flame zone through a thin annular choked nozzle, where a normal shock wave is present in the divergent section of the nozzle. The supersonic region between throat and normal shock is assumed to be acoustically compact. Exact solutions are developed for the acoustic, entropy and vorticity waves at the outlet of the nozzle. For simplicity a low Mach number approximation is made upstream and downstream of the nozzle. It is found that the pressure and entropy perturbations at the nozzle outlet can be obtained directly from the density perturbation at the nozzle inlet. Thus, there is no need to model either the linear waves or the mean flow within the nozzle including the supersonic region. For an idealised combustor model, compared to the direct acoustic noise, the entropy indirect noise is found to be the main source of noise at the combustor outlet, while the indirect vorticity noise has negligible contribution.

2 Introduction

Combustion noise is becoming increasingly important as a major noise source in gas turbines. This is partially because advances in design have reduced the other noise sources, and partially because next generation combustion modes burn more unsteadily. Two mechanisms of combustion noise generation can be identified: direct noise, which is due to the acoustic waves, and indirect noise, produces when entropy waves and vorticity waves are accelerated through the turbine blade rows [1]. Combustion noise has been found to be important at low frequencies up to 1 kHz [1]. Previous studies tried to analyse the passage of acoustic or entropy perturbations through a nozzle. Marble and Candel [2] for choked compact nozzles and using linear analysis derived analytical expressions for the relationship between the amplitude of the acoustic waves generated and the temperature fluctuations of the incident entropy waves. Stow et al. [3] considered an effective nozzle length through an asymptotic expansion of the linearised Euler equations for non-compact nozzles. Using a simple quasi 1-D choked nozzle Leyko et al. [4] quantified both direct and indirect noise in a cold flame and found that the ratio of indirect-to-direct noise is small for laboratory experiments but large in most real aeroengines. Currently there is, in general, little agreement on the importance of indirect noise to the problem of combustion noise [1]. In this study we consider a highly idealised combustor geometry. The effect of the fluctuation in the heat release rate on the magnitude of the direct and indirect noise in a choked combustor with a shock wave at the divergent section is studied. For short annular combustors,

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circumferential modes are important and their transmission and the contribution of vorticity waves have not been well clarified before. Hence, the present work tries to determine the relative contribution of acoustic, entropy and vorticity perturbations created by the flame zone in the generation of the sound at the outlet of an idealised combustor.

3 Linear waves in an annular duct

We consider the form of perturbations that occur in the gap between two concentric cylinders. The mean flow is assumed to be axial. The flow is assumed to be inviscid, with the velocity field denoted by $\vec{u}=(u,v,w)$ and the duct has mean inner and outer radii r_i and r_o, respectively. In annular combustor, the radial gap r_o - r_i is typically much shorter than the circumference. Thus, we may approximate any variation of pressure in the radial direction negligible with then the radial velocity, v, to be zero [3]. This flow is taken to be composed of a steady axial mean flow denoted by bars and a small perturbations denoted by primes, e.g. $p=\bar{p}(x)+p'(x,\theta,t)$. The disturbances have frequency ω with the angular dependence of the form $e^{in\theta}$ with circumferential wavenumber n. Hence, we consider the disturbances to be of the form $p'=\mathrm{Re}[\hat{p}(x)e^{i\omega t+in\theta}]$ etc. Since, $\bar{v}=\bar{w}=0$, the perturbations are written as [1]

$$p'=(A_+e^{ik_+x}+A_-e^{ik_-x})e^{i\omega t+in\theta}, \tag{1}$$

$$\rho'=\frac{1}{\bar{c}^2}(A_+e^{ik_+x}+A_-e^{ik_-x}-A_Ee^{ik_0x})e^{i\omega t+in\theta}. \tag{2}$$

$$u'=(\frac{-k_+}{\bar{\rho}\alpha_+}A_+e^{ik_+x}+\frac{-k_-}{\bar{\rho}\alpha_-}A_-e^{ik_-x}+\frac{n}{\bar{\rho}\bar{c}}A_Ve^{ik_0x})e^{i\omega t+in\theta}, \tag{3}$$

$$w'=(\frac{-n}{r\bar{\rho}\alpha_+}A_+e^{ik_+x}+\frac{-n}{r\bar{\rho}\alpha_-}A_-e^{ik_-x}-\frac{k_0r}{\bar{\rho}\bar{c}}A_Ve^{ik_0x})e^{i\omega t+in\theta}. \tag{4}$$

$$\text{vorticity perturbation: } \zeta'=\frac{i}{\bar{\rho}\bar{c}r}[n^2+(k_0r)^2]A_Ve^{i\omega t+in\theta+ik_0x}, \tag{5}$$

$$\text{entropy perturbation: } S'=\frac{c_p}{\bar{\rho}}(\frac{p'}{\bar{c}^2}-\rho'). \tag{6}$$

r is the mean radius, $\alpha_\pm=\omega+\bar{u}k_\pm$ and the axial wavenumbers for the acoustic waves are

$$\bar{c}k_\pm=\left(\bar{M}\omega\mp[\omega^2-\omega^2_{cut-off}]^{1/2}\right)\Big/\left(1-\bar{M}^2\right), \tag{7}$$

where $\bar{c}$ is the mean speed of sound, $\omega_{cut\text{-}off}$ is the cut-off frequency of the duct $(n\bar{c}/r)(1-\bar{M}^2)^{1/2}$ and $\bar{M}$ is the mean Mach number. Entropy and vortical disturbances convect with the mean flow and so their space-time dependence is $e^{i\omega t+in\theta+ik_0x}$, where $k_0=-\omega/\bar{u}$. Writing $X=x/L$, the flow perturbations can be written in non-dimensional form as: $\frac{p'}{\gamma\bar{p}}=\hat{p}(X)e^{i\omega t+in\theta}$, $\frac{\rho'}{\bar{\rho}}=\hat{\rho}(X)e^{i\omega t+in\theta}$, $\frac{u'}{\bar{u}}=\hat{u}(X)e^{i\omega t+in\theta}$, and $\frac{w'}{\bar{c}_{in}}=\hat{w}(X)e^{i\omega t+in\theta}$.

4 Analysis for nozzle with shock in divergent section

We assume that the nozzle is thin and annular (Fig. 1), and that the cross-sectional area of the nozzle decreases to a throat before increasing again, where a normal shock is present in the divergent section.

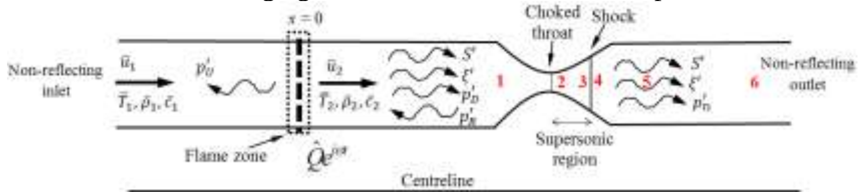


Figure 1. One-dimensional disturbances in a choked nozzle.

We consider an idealised combustor with non-reflecting boundary conditions at inlet and exit. Upstream of the flame there is then only an upstream propagating acoustic wave (p_U'). Downstream of the flame, acoustic (p_D'), entropy (S') and vorticity (ξ') waves produced by the flame, travel downstream toward the nozzle contraction, where they are partially reflected to give an upstream propagating wave (p'_R). We wish to obtain the downstream-travelling acoustic, vorticity and entropy waves in the parallel section of the nozzle outlet point 5. To end this we first determine the perturbations upstream and downstream of the moving shock at points 3 and 4, respectively. The linearised interaction between the flow perturbations and the moving shock wave has been studied previously [3]. It involves applying the Rankine-Hugoniot equations of conservation of mass, momentum and energy flux in a frame of reference in which the shock is at rest. Thus, we have

$$\hat{u}=\frac{u'_4}{\bar{u}_4}=\frac{-x'_s}{\bar{M}_3[1+\frac{1}{2}(\gamma-1)\bar{M}_3^2]}\times\left[(1+\frac{\bar{M}_3^2-1}{\bar{M}_4^2-1})\frac{d\bar{M}_3}{dx}-(1+\bar{M}_3^2)\frac{i\omega}{\bar{c}_3}\right]+\frac{u'_3}{\bar{u}_3}-\frac{4}{2+(\gamma-1)\bar{M}_3^2}\frac{M'_3}{\bar{M}_3}. \tag{8}$$

$$\hat{\rho}=\frac{\rho'_4}{\bar{\rho}_4}=\frac{x'_s}{\bar{M}_3[1+\frac{1}{2}(\gamma-1)\bar{M}_3^2]}\times\left[(2-\bar{M}_3^2)\frac{d\bar{M}_3}{dx}-2\frac{i\omega}{\bar{c}_3}+\bar{M}_4^2\frac{\bar{M}_3^2-1}{\bar{M}_4^2-1}\frac{d\bar{M}_3}{dx}\right]+\frac{\rho'_3}{\bar{\rho}_3}+\frac{4}{2+(\gamma-1)\bar{M}_3^2}\frac{M'_3}{\bar{M}_3}. \tag{9}$$

$$\hat{p}=\frac{p'_4}{\gamma\bar{p}_4}=\frac{x'_s}{\bar{M}_3[1+\frac{1}{2}(\gamma-1)\bar{M}_3^2]}\times\left[\left(\bar{M}_3^2\frac{3+\gamma-2\bar{M}_3^2}{2\gamma\bar{M}_3^2-(\gamma-1)}+\bar{M}_4^2\frac{\bar{M}_3^2-1}{\bar{M}_4^2-1}\right)\frac{d\bar{M}_3}{dx}-\frac{2\bar{M}_3^2[2+(\gamma-1)\bar{M}_3^2]}{2\gamma\bar{M}_3^2-(\gamma-1)}\frac{i\omega}{\bar{c}_3}\right]$$
$$+\frac{4\bar{M}_3^2}{2\gamma\bar{M}_3^2-(\gamma-1)\bar{M}_3^2}\frac{M'_3}{\bar{M}_3}+\frac{p'_3}{\gamma\bar{p}_3}. \tag{10}$$

Furthermore, the circumferential velocity is as $w'_3=w'_4$. x'_s is perturbation in the shock position caused by the downstream acoustic wave. In the following we obtain perturbations in the mass flux $m=a\rho u$, angular momentum flux $f_\theta=rmw$ and energy flux $e=a\gamma pu/(\gamma-1)+m(\frac{1}{2}u^2+\frac{1}{2}w^2)$ downstream of the shock. Thus, one can write the perturbations in the mass flux $m'=\bar{m}(\hat{\rho}+\hat{u})$, angular momentum flux $f'_\theta=r\bar{m}w'$ and energy flux $\frac{(\gamma-1)}{\bar{c}_0^2}e'=m'+\bar{m}(\gamma\hat{p}-\hat{\rho}+(\gamma-1)\bar{M}^2\hat{u})/[1+\frac{1}{2}(\gamma-1)\bar{M}^2]$. We will assume that the supersonic region is compact and hence ignore the $\omega/\bar{c}_3$ terms in (8)-(10). Substituting for $\hat{\rho}$ and $\hat{u}$ from (8) and (9) we obtain $m'_3=m'_4$, $f'_{\theta3}=f'_{\theta4}$ and $e'_4=e'_3$. Thus, we can write that

$$m'_1=m'_2=m'_3=m'_4=m'_5,\ f'_{\theta1}=f'_{\theta2}=f'_{\theta3}=f'_{\theta4}=f'_{\theta5},\ e'_1=e'_2=e'_3=e'_4=e'_5. \tag{11a}$$

It should be pointed out that the axial momentum is not conserved from 1 to 3 or from 4 to 5, as there is area change in the nozzle which leads to an axial force. Similarly for the mean flow we have

$$\bar{m}_1=\bar{m}_2=\bar{m}_3=\bar{m}_4=\bar{m}_5,\ \bar{e}_1=\bar{e}_2=\bar{e}_3=\bar{e}_4=\bar{e}_5 \text{ and } \bar{f}_\theta=0 \text{ since } \bar{w}=0. \tag{11b}$$

The outlet boundary condition (Fig. 1) is non-reflecting, i.e. $A_-=0$. Substituting (1)-(3) into (11a) gives

$$A_+=\left[\frac{\bar{\rho}_5\bar{M}_5\bar{c}_5^2}{1+(\gamma-1)\bar{M}_5^2}(\psi1+\psi2)\right]/\left(\frac{\gamma\bar{M}_5}{1+(\gamma-1)\bar{M}_5^2}-\frac{\bar{c}_5k_+}{\alpha_+}-\frac{n^2\bar{c}_5}{k_0r^2\alpha_+}\right). \tag{12.a}$$

$$A_E=-\frac{(\gamma-1)(1-\bar{M}_5^2)}{1+(\gamma-1)\bar{M}_5^2}A_++\frac{\bar{\rho}_5\bar{c}_5^2}{1+(\gamma-1)\bar{M}_5^2}\left(\psi1-(\gamma-1)\bar{M}_5^2\psi2\right). \tag{12.b}$$

$$A_V=-\frac{n\bar{c}_5}{k_0r^2}(\frac{\bar{\rho}_5r}{n}\psi3+\frac{A_+}{\alpha_+}). \tag{12.c}$$

Using (12.a) and substituting into (1) we have the acoustic perturbations as

$$p'_5=\frac{\bar{\rho}_5\bar{M}_5\bar{c}_5^2(\psi1+\psi2)}{\gamma\bar{M}_5-\left(\frac{\bar{c}_5k_{+5}}{\alpha_{+5}}+\frac{n^2\bar{c}_5}{k_{05}r^2\alpha_{+5}}\right)\left[1+(\gamma-1)\bar{M}_5^2\right]}e^{ik_{+5}x}. \tag{13}$$

where $\psi1$, $\psi2$ and $\psi3$ are the known properties downstream of the flame zone at point 1 as

$$\psi1=\left[(1+\frac{1}{2}(\gamma-1)\bar{M}_2^2)/(1+\frac{1}{2}(\gamma-1)\bar{M}_1^2)\right]\left[\gamma\hat{p}_1-\hat{\rho}_1+(\gamma-1)\bar{M}_1^2\hat{u}_1\right],\ \ \psi2=\hat{\rho}_1+\hat{u}_1,\ \ \psi3=\hat{w}_1. \tag{14}$$

For low Mach number flow downstream of the nozzle we neglect the high order term of mean Mach number compared to 1 hence $\left[1+(\gamma-1)\bar{M}_5^2\right]\to1$. Furthermore, in the limit, $1/k_0$ tends to zero and hence $n^2\bar{c}_5/(k_{05}r^2\alpha_{+5})\to0$ and $\psi1+\psi2=p'_1/\bar{p}_1+u'_1/\bar{u}_1$ thus,

$$\frac{p'_5}{\gamma\bar{p}_5}=\left(\bar{M}_5\Big/\left(\gamma\bar{M}_5-\frac{\bar{c}_5k_{+5}}{\alpha_{+5}}\right)\right)\times\left(\frac{p'_1}{\bar{p}_1}+\frac{u'_1}{\bar{u}_1}\right)e^{ik_{+5}x}. \tag{15}$$

Substituting for the waves p'_1 and u'_1 form (1) and (3) leads to

$$\frac{p'_5}{\gamma\bar{p}_5}=\left(\bar{M}_5\Big/\left(\gamma\bar{M}_5-\frac{\bar{c}_5k_{+5}}{\alpha_{+5}}\right)\right)\times\left(Ae^{ik_{+1}L}\left(\frac{1}{\bar{p}_1}-\frac{k_{+1}}{\bar{\rho}_1\bar{u}_1\alpha_{+1}}\right)+A_Re^{ik_{-1}L}\left(\frac{1}{\bar{p}_1}-\frac{k_{-1}}{\bar{\rho}_1\bar{u}_1\alpha_{+1}}\right)\right)e^{ik_{+5}x}, \tag{16}$$

where L denotes the distance between the flame zone and choked throat. The downstream propagating wave A (p'_D in Fig. 1) interacts with the choked nozzle and produce a reflected upstream propagating acoustic wave towards the flame, A_R (p'_R in Fig. 1). Dowling and Mahmoudi [1] showed that at the choked throat the vorticity noise is negligible and determined the reflected waves A_R as

$$A_Re^{ik_{-1}L}=-\left(\frac{\bar{u}_1\alpha_{-1}}{2k_{-1}\bar{c}_1^2}A_Ee^{ik_{01}L}+\frac{\alpha_{-1}k_{+1}}{\alpha_{+1}k_{-1}}Ae^{ik_{+1}L}\right). \tag{17}$$

Substituting (17) into (16) results in

$$\frac{p'_5}{\gamma\bar{p}_5}=\left(\bar{M}_5\Big/\left(\gamma\bar{M}_5-\frac{\bar{c}_5k_{+5}}{\alpha_{+5}}\right)\right)\times\left(Ae^{ik_{+1}L}\left[\frac{1}{\bar{p}_1}\left(1-\frac{\alpha_{-1}k_{+1}}{\alpha_{+1}k_{-1}}\right)\right]+A_Ee^{ik_{01}L}\left[-\left(\frac{1}{\bar{p}_1}-\frac{k_{-1}}{\bar{\rho}_1\bar{u}_1\alpha_{-1}}\right)\left(\frac{\bar{u}_1\alpha_{-1}}{2k_{-1}\bar{c}_1^2}\right)\right]\right)e^{ik_{+5}x}. \tag{18}$$

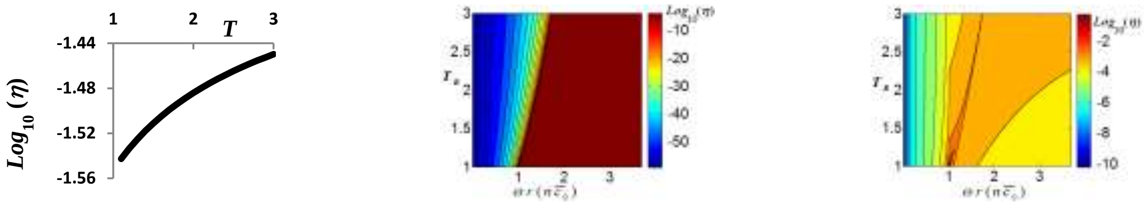
A and A_E are the magnitude of the acoustic and entropy perturbations generated by the flame at point 1. Dowling and Mahmoudi [1] obtained the magnitude of these waves analytically in a non-reflecting duct as $A_E=\bar{c}_1^2\left(-k_{+1}+k_{-0}\right)A/\bar{u}_1\omega$, in which k_{-0} is the axial wavenumber upstream of the flame. Thus, the ratio of the direct and the indirect entropy noise at the outlet, point 5 is determined by

$$\eta=\left|\frac{\bar{u}_1\omega}{\left(-k_{+1}+k_{-0}\right)}\frac{2k_{-1}\left(1-\dfrac{\alpha_{-1}k_{+1}}{\alpha_{+1}k_{-1}}\right)}{\alpha_{-1}\left(\bar{u}-\dfrac{k_{-1}}{\alpha_{-1}}\dfrac{\bar{p}_1}{\bar{\rho}_1}\right)}e^{ik_{+1}L}\right|. \tag{19}$$

In the low Mach number limit as $\bar{u}_1,\bar{M}\to 0$ (19) simplifies as

$$\eta=\left|\frac{2\gamma\bar{M}_0}{\bar{c}_0}\frac{\omega(k_{-1}-k_{+1})}{k_{-1}\left(k_{-0}-k_{+1}\right)}e^{ik_{+1}L}\right|. \tag{20}$$

This equation involves the mean flow Mach number upstream of the flame, $\bar{M}_0$. For plane waves, n = 0, $k_{-1}=\omega/\bar{c}_1$, $k_{+1}=-\omega/\bar{c}_1$ and $k_{-0}=\omega/\bar{c}_0$, where $\bar{c}_0$ is the sound speed upstream of the flame. Therefore, this ratio is equal to $\eta|_{n=0}=4\gamma\bar{M}_0/\left(1+\bar{c}_0/\bar{c}_1\right)=4\gamma\bar{M}_0/(1+T_R^{-1/2})$, where T_R is the mean temperature ratio across the flame. This ratio with $\bar{M}_0=0.01$ is plotted in Fig. 2(a). It is seen that the magnitude of the direct noise is more than an order of magnitude smaller than that of the entropy noise. For circumferential waves, the ratio in (19) for a fixed inlet Mach number is a function of frequency and mode number as well as mean temperature ratio. From inspection of k_{-1} and k_{+2}, the ratio depends on frequency in the combination $\omega r/n\bar{c}_1$ and on the non-dimensional length L as Ln/r. At frequencies below the k_{+2} cut-off, the acoustic wave is exponentially small by the time it reaches the outlet of the nozzle. Therefore, the indirect component of the total noise is more than the direct. The contours of the logarithmic ratio of the magnitude of direct to indirect entropy noise for circumferential modes with Ln/r = 50 and 1 are plotted in Figs. 2(b) and 2(c), respectively. For a large value of Ln/r Fig. 2(b), there are two distinct regions for the values of the ratio are visible. For frequencies below the cut-off (based on $\bar{c}_1$), the ratio is very small. Indicating that the direct noise has negligible role in generation of the noise. For frequency higher than cut-off as the frequency increases the ratio almost remains unchanged. For a short nozzle, $Ln/r=1$, Fig. 2(c) shows three different regions for the logarithmic ratio of the magnitude of direct to entropy noise. For frequencies $\omega<\omega_{cut-off}$ (based on $\bar{c}_1$) the ratio in Fig. 2(c) behaves like that in Fig. 2(b). At the cut-off frequency based on $\bar{c}_2$, $k_{-1}=k_{+1}=0$ and from (20) that the ratio tends to maximum value of $2\gamma\bar{M}_0/(1-T_R^{-2})^{-1/2}$. For frequencies close to the cut-off, the decay due to the exponential $e^{ik_{+2}L}$ is modest, because the non-dimensional length Ln/r is small. Hence the magnitude of the ratio in (20) has a maximum value when $k_{-1}=k_{+1}=0$ and $\omega r/(n\bar{c}_1)=1$.



(a) $n = 0$ (b) $Ln/r = 50$ (c) $Ln/r = 1$

Figure 2: Ratio of the magnitude of direct acoustic to indirect (entropy) noise obtained using (20) with $\bar{M}_1 = 0.01$ for a) plane wave with $n = 0$, and for circumferential waves with Ln/r equal to b) 50 and c) 1.

Referring again to (15) and using the boundary condition of Marble and Candel [2] at point 1, we obtain $\psi 1 + \psi 2 = p'_1/\bar{p}_1 + (1/2)\left(p'_1/\bar{p}_1 - \rho'_1/\bar{\rho}_1\right)$. In the limit as $\bar{u} \to 0$, $p'_1/\bar{p}_1$ is negligible compared to $u'_1/\bar{u}_1$ and $\rho'_1/\bar{\rho}_1$. Thus,

$$\frac{p'_5}{\gamma \bar{p}_5} = -\frac{1}{2}\frac{\bar{M}_5}{\bar{c}_5 k_{+5}/\alpha_{+5}} \times \frac{\rho'_1}{\bar{\rho}_1} e^{ik_{+5}x}. \tag{21}$$

In the limit, $\bar{M} \to 0$, the strength of ρ'_1 tends to infinity, hence we need to keep the product $\bar{M}\rho'$ [1]. Thus, in low Mach number flows the pressure perturbation at the nozzle outlet can be obtained directly from the density perturbation at the nozzle inlet. The entropy perturbation at the outlet is obtained by substituting (12) into (6)

$$\frac{S'_5}{c_p} = \frac{1}{\left[1+(\gamma-1)\bar{M}_5^2\right]} \times \left(\frac{-(\gamma-1)(1-\bar{M}_5^2)\bar{M}_5(\psi 1 + \psi 2)}{\gamma \bar{M}_5 - \left(\frac{\bar{c}_5 k_{+5}}{\alpha_{+5}} + \frac{n^2 \bar{c}_5}{k_{05} r^2 \alpha_{+5}}\right)\left[1+(\gamma-1)\bar{M}_5^2\right]} + \psi 1 - (\gamma-1)\bar{M}_5^2 \psi 2 \right) e^{ik_{05}x}. \tag{22}$$

In the low Mach limit when $\bar{u}, \bar{M} \to 0$, the third term in the right hand side of (22), tends to zero and $\psi 1$ simplifies to $\gamma \hat{p}_1 - \hat{\rho}_1$. In the limit as $\bar{u} \to 0$, neglecting $p'_1/\bar{p}_1$ compared to $\rho'_1/\bar{\rho}_1$ and $u'_1/\bar{u}_1$ leads to

$$\frac{S'_5}{c_p} = \left[\frac{(1/2)(\gamma-1)\bar{M}_5}{\bar{c}_5 k_{+5}/\alpha_{+5}} - 1\right] \times \frac{\rho'_1}{\bar{\rho}_1} e^{ik_{05}x}, \xrightarrow{\bar{M}_5 \to 0} \frac{S'_5}{c_p} = -\frac{\rho'_1}{\bar{\rho}_1}. \tag{23}$$

Equations (23) represents that the entropy perturbation at the outlet can be obtained directly from the density perturbation at the nozzle inlet. Comparison of (23) and (21) shows that since the acoustic wave scales with $\bar{M}_5$, the acoustic perturbation is negligible compared to the entropy perturbation. Furthermore, according to (6) the entropy perturbation at point 1 is written as

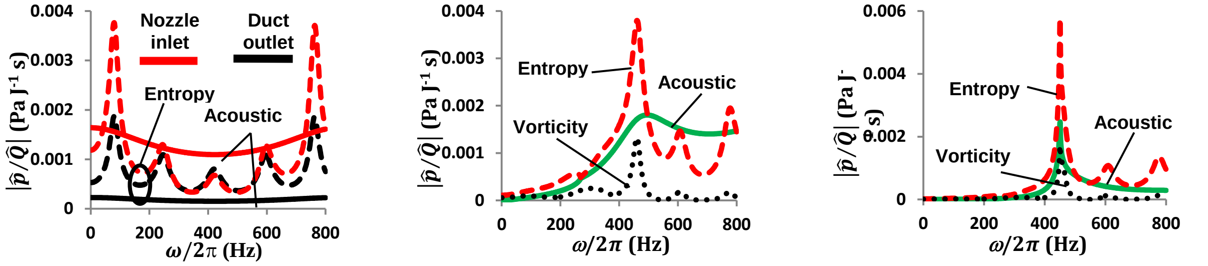
$$\frac{S'_1}{c_p} = \frac{p'_1}{\gamma \bar{p}_1} - \frac{\rho'_1}{\bar{\rho}_1}, \xrightarrow{\bar{M} \to 0} \frac{S'_1}{c_p} = -\frac{\rho'_1}{\bar{\rho}_1}. \tag{24}$$

Comparison of (24) and (23) shows that in low Mach number the entropy perturbation at the nozzle outlet is equal to that at the choked throat. In a similar way using (5) the vorticity perturbation is

$$\zeta'_5 = -ik_0\left(w'_1 + \frac{n\bar{u}_5}{2rk_+}\frac{\rho'_1}{\bar{\rho}_1}\right). \tag{25}$$

5 Low order modelling of the combustion noise

We have used a computer program LOTAN (Low-Order Thermo-Acoustic Network model) [5] to obtain the perturbations. The results are obtained by imposing $\hat{Q}e^{i\omega t}$ as a source term, with no feedback from the acoustics. To calculate the direct sound, we calculate the entropy and vorticity waves generated at the flame but ignore their contribution when applying the downstream boundary condition. The direct sound field is obtained by adding the downstream and upstream propagating acoustic waves. The indirect noise due to the entropy and vorticity waves can be determined by subtracting the direct noise from the full sound field calculated. The transfer function $\left|\hat{p}/\hat{Q}\right|$ obtained using LOTAN is shown as a function of frequency in Fig. 3 at the nozzle inlet and at the outlet of the duct with an inlet Mach number $\bar{M}_0 = 0.06$ and mean temperature ratio of $\bar{T}_{02}/\bar{T}_{01}$ =3.



(a) plane wave (n = 0) (b) (n = 1) nozzle inlet (c) (n = 1) duct outlet

Figure 3: Magnitude of the indirect noise and direct noise as function of frequency for (a) plane wave (n = 0) and circumferential wave with n =1 (b) and (c) at the nozzle inlet and at the end of the duct.

It is seen that compared to the direct noise, the indirect entropy and vorticity noise have more complicated dependencies on frequency due to the resonance [1]. The resonant is due to the multiple reflection of p'_R with the flame (see Fig. 1). For plane wave at the nozzle inlet except for the resonant frequencies, the direct noise is higher than that of the indirect entropy noise. Conversely, in the outlet duct the indirect entropy noise is higher than that of the direct noise. This is in agreement with analytical results shown in Fig. (2). Furthermore, it is seen that except for the resonant frequency, the indirect noise at the outlet is almost similar to that at the nozzle inlet. This in agreement with the analysis presented in (23) and (24). We have neglected any diffusion of the entropy wave which may attenuate it at higher frequencies in practice. For circumferential wavenumber n = 1, Figs. 3(a) and (b) show that for frequencies below the cut-off frequency (~450 Hz) the acoustics waves are cut-off and thus their magnitude is exponentially small. At the cut-off frequency the magnitude of the vorticity noise is nearly comparable to that of the direct acoustic noise. For frequencies higher than cut-off the acoustic waves are propagating and thus their magnitude increases. However, the indirect entropy noise is still high enough to be the most significant contributor to noise in the downstream duct, although at high frequencies our predictions are likely to be an over estimate because they do not account for turbulent diffusion of entropy or vorticity. In general, the indirect vorticity noise is one order of magnitude smaller than the direct acoustic and indirect entropy noise.

6 Concluding remarks

We studied the transmission of the acoustic, entropic and vortical waves and the relative contributions of direct and indirect noise generated by a flame zone for a thin annular compact choked nozzle where a normal shock wave is present downstream of the divergence section of the nozzle. For low a Mach number flow, it was found that the perturbations in pressure and entropy at the nozzle outlet can be obtained directly from the density perturbation at the nozzle inlet.

Acknowledgments

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