Sensitivity of Scaling Exponents for Turbulent Burning Velocity to Evaluation Method: A Numerical Study

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1 Introduction

For decades, the focus of experimental and theoretical research into premixed turbulent combustion was placed on flame speed S_t, i.e. the speed of a mean flame surface with respect to the unburned gas, and burning velocity U_t, i.e. the mass rate of product creation per unit area of a mean flame surface, divided by the density of the unburned gas. Nevertheless, obtained results are still contradictory and the values of S_t and U_t measured by various groups are strongly scattered, e.g. see Fig. 4.11 in Ref. [1], even if the basic conditions of the experiments were similar, i.e. the fuel, equivalence ratio, pressure, and temperature were the same, while rms turbulent velocities and turbulence length scales were comparable. As shown in many studies, e.g. see Refs. [2-5], the quantitative scatter of available experimental data on S_t and U_t results in particular from the strong sensitivity of measured values of turbulent flame speeds or burning velocities to a method used to process experimental data. However, as far as qualitative trends in the behavior of S_t or U_t are concerned, sensitivity of such trends to evaluation method still requires a target-directed study. In particular, it is not yet clear whether or not the scaling exponent for dependence of S_t or U_t on the rms turbulent velocity u', length scale L, or laminar flame speed S_L is substantially sensitive to a method used to evaluate S_t or U_t. The present study aims at filling this gap.

2 Method of Research

For this purpose, we (i) performed RANS simulations of statistically stationary, confined, conical, cylindrically symmetrical premixed turbulent flames stabilized by recirculation zones due to abrupt expansion of the PSI burner [6,7], (ii) varied the inlet value of u', L, or S_L in a wide range, (iii) evaluated S_t and U_t invoking several methods available in the literature, and (iv) compared obtained dependencies of differently determined S_t and U_t on the aforementioned inlet values. The variations in u', L, or S_L were performed with respect to a single reference case, i.e. a preheated (the temperature T_u of unburned flow was equal to 673 K), lean (the air-fuel ratio λ=1.8) methane-air flame stabilized in the flow with the inlet bulk velocity of 40 m/s, u'=2 m/s, and L=3.1 mm under elevated pressure of 0.5 MPa. Under these conditions, S_L=0.325 m/s and the density ratio is equal to 2.8 [8]. Such a reference

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flame was selected, because it was experimentally investigated by Siewert [6] and was recently simulated by us [8]. Wide use of conical premixed turbulent flames in previous measurements of S_t and U_t was another reason for the present choice of the reference flame. Moreover, such a flame offers an opportunity to address three major effects, i.e. divergence of the mean flow, curvature of the mean flame brush, and turbulent flame development (growth of the mean flame brush thickness) with distance from flame holder, that contribute to the significant scatter of available data on S_t and U_t [1].

RANS simulations were performed invoking the so-called TFC model [9] of the influence of turbulence on premixed combustion. As discussed in detail elsewhere [1], the TFC model deals with a single balance equation for the Favre-averaged combustion progress variable $\tilde{c}$

$$\partial_t(\overline{\rho}\tilde{c}) + \nabla\cdot(\overline{\rho}\tilde{\mathbf{u}}\tilde{c}) = \nabla\cdot(\overline{\rho}D_t\nabla\tilde{c}) + \rho_u U_{TFC}\left|\nabla\tilde{c}\right|, \qquad U_{TFC} = 0.5u'^{3/4}L^{1/4}S_L^{1/2}\kappa_u^{-1/4}$$

where **u** is the flow velocity vector, ρ is the density, D_t is turbulent diffusivity, κ is the molecular heat diffusivity, $\overline{q}$ and $\tilde{q} = \overline{\rho q}/\overline{\rho}$ are the Reynolds- and Favre-averaged values of a quantity q, respectively, and subscripts u and b designate unburned and burned gas, respectively.

The TFC model was selected not only because it had been validated by several independent research groups against various experimental data obtained from a wide set of substantially different premixed turbulent flames, as reviewed elsewhere [1], including the present reference PSI flame [8], but also and mainly for the following two basic reasons.

First, if the above balance equation is applied to a statistically planar, one-dimensional flame that propagates in statistically stationary, homogeneous turbulence not affected by combustion, then, the turbulent burning velocity yielded by the TFC model is simply equal to U_{TFC}. Therefore, the U_t-scaling exponents are equal to 0.75, 0.25, or 0.50 for u', L, or S_L, respectively, i.e. $U_t \sim u'^{3/4}$, $U_t \sim L^{1/4}$, or $U_t \sim S_L^{1/2}$. Accordingly, this theoretical scaling can be used (i) to assess scaling yielded by another method for evaluating S_t or U_t and (ii) to gain an insight into the influence of flame configuration on the scaling.

Second, within the framework of the TFC model, turbulent burning velocity is a physically meaningful quantity, which is equal to U_{TFC} and is controlled by mixture and turbulence characteristics. Therefore, if both the magnitudes and scaling exponents of S_t and U_t evaluated by applying various methods to processing numerical data yielded by the TFC model are sensitive to the evaluation method, then, such a result would imply that the significant scatter of available measured data on S_t and U_t does not rule out that turbulent burning velocity is a valuable basic characteristic of premixed turbulent combustion. Furthermore, if the magnitude and scaling exponents of S_t or U_t evaluated by invoking a particular method are close to the magnitude and scaling exponents of the theoretical U_{TFC}, then, such a result would highlight that method and imply that sensitivity of other S_t and U_t to flame configuration does not impede considering turbulent flame speed or burning velocity to be a physically meaningful quantity controlled mainly by mixture and turbulence characteristics.

It is worth also noting that the use of the TFC model allows us to reveal the following physical mechanism that contributes to the scatter of data on S_t. In the aforementioned statistically planar one-dimensional case, the mean flame brush thickness yielded by the TFC model grows with time. Accordingly, the speed of any iso-scalar surface $\overline{c}(x,t) = \text{const}$ differs from the speed of another iso-scalar surface.

In the present work, statistically stationary, two-dimensional, cylindrically symmetrical fields of $\tilde{c}(x,r)$ and $\tilde{u}(x,r)$, computed using the TFC model, as discussed in detail elsewhere [8], were processed and the following turbulent flame speeds and burning velocities were evaluated.

Global turbulent burning velocity

$$U_t^G(c_r) = U_B\,\pi R_0^2 / A_f\,,$$

where U_B is the bulk flow velocity at the inlet, R_0 is the inlet radius, and A_f is the total area of the mean flame surface. This area and the global burning velocity depend on the choice of a reference value c_r of the mean combustion progress variable, which is associated with the surface, i.e. $\overline{c}(x, r = R_f) = c_r$.

Local turbulent flame speed

$$s_t(x,\bar{c}) = -\overline{\rho}\widetilde{\mathbf{u}}\cdot\mathbf{n}/\rho_u, \qquad \mathbf{n} = -\nabla\tilde{c}/|\nabla\tilde{c}|.$$

In a statistically stationary, planar, one-dimensional flame, $\overline{\rho}\widetilde{u}/\rho_u$ is independent of $\tilde{c}$ and is equal to the magnitude of the mean velocity of unburned gas. In a general case, however, $\overline{\rho}\widetilde{\mathbf{u}}\cdot\mathbf{n}$ can significantly vary within the mean flame brush due to divergence of the mean flow [2,3].

Leading edge flame speed

$$s_{le}(x,c_r) = \left.\frac{\nabla\cdot(\overline{\rho}\widetilde{\mathbf{u}}\tilde{c})}{\overline{\rho}|\nabla\tilde{c}|}\right|_{\tilde{c}=c_r}$$

introduced by Dunstan et al. [10] who proposed to evaluate it at a low reference value c_r.

Extrapolated flame speed [11,12]

$$s_u(x) = |\overline{\mathbf{u}}_u(x, l\to 0)\cdot\mathbf{n}(x)|,$$

where l is distance from the nearest point characterized by $\bar{c}$ = 0.5, the mean field $\overline{\mathbf{u}}_u(x,l)$ is extracted upstream of the mean flame brush and is subsequently extrapolated to the mean flame surface $\bar{c}(x,r)$ = 0.5 along the local normal to it. The BML state equation $\rho_b\bar{c} = \overline{\rho}\widetilde{c} = \rho_u\tilde{c}/(1+\tau\tilde{c})$ [13] was invoked to determine the mean flame surface. Here, $\tau=\rho_u/\rho_b$-1 is a heat release factor.

Local burning velocity

$$u_t(x,c_r) = \frac{2}{R_f}\int_{-\infty}^{\infty} U_{TFC}|\nabla\tilde{c}|dl,$$

where the mean flame radius R_f is determined from $\bar{c}(x, r = R_f) = c_r$ and integration is performed along the local normal to the surface $\bar{c}(x,r)$ = 0.5. The local burning velocity depends substantially on the choice of the mean flame surface, i.e. on the reference value c_r.

Finally, it is worth noting that, while the RNG k-ε turbulence model [14] was invoked by us [8] in order to validate the TFC combustion model against the PSI data [6,7], results reported in the following section were obtained in the case of statistically stationary, homogeneous turbulence not affected by the flame. The reasons for such a simplification are as follows. First, as our goal was to investigate whether or not various evaluation methods could yield substantially different scaling exponents for S_t or U_t vs. u', L, or S_L, the simplified case of homogeneous turbulence appears to be more challenging. Indeed, it is associated with underestimated variations in the scaling exponents, because flame-induced spatial inhomogeneity of local turbulence characteristics should increase differences between differently determined S_t and/or U_t. Second, in the case of homogeneous turbulence, there is a reference scaling given by the theoretical expression for U_{TFC}. Third, predictive capabilities of available models of the influence of premixed combustion on turbulence are still poor, as reviewed elsewhere [1,15], see also Ref. [8].

3 Results and Discussion

Figure 1 shows that magnitude of S_t or U_t depends not only on the expression used to evaluate it, cf. different symbols, but also on the reference value c_r, cf. Figs. 1a, 1b, and 1c, and the axial distance x from the nozzle. Even if the x-dependence is less pronounced, it is worth remembering that variations in x were limited, i.e. $x\leq$45 mm, in order for $\bar{c}(x, r = 0) << 1$ in all cases. Note that the fuel nozzle

diameter for the PSI combustion chamber was 25 mm. Moreover, the global burning velocity, see red diamonds, is independent of x by definition, as well as U_{TFC}, see black circles.

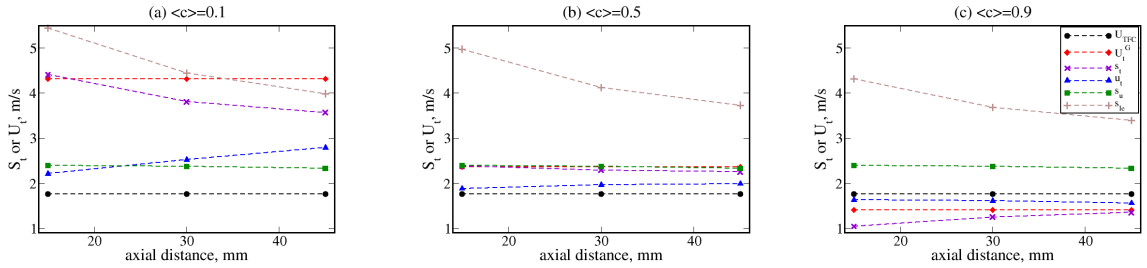

Figure 1. Variations of S_t or U_t with the axial distance x, computed in the reference case at (a) $\overline{c}$ = 0.1, (b) $\overline{c}$ = 0.5, and (c) $\overline{c}$ = 0.9. The leading edge flame speed was computed at $\widetilde{c}$ = 0.01, 0.02, and 0.05, respectively.

As far as dependence of S_t or U_t on c_r is concerned, it is very well pronounced. In particular, both the local burning velocity, see blue triangles, and, especially, the global burning velocity, see red diamonds, are reduced when c_r is increased, because the mean flame radius R_f and area A_f are increased by c_r. The significant decrease in the local flame speed, see violet crosses, with an increase in c_r is controlled by divergence of the mean flow [2,3]. It is worth also noting that if c_r=0.5, then, all evaluated S_t or U_t, with exception of s_{le}, see brown pluses, are close to each other, see Fig. 1b, with u_t being very close to U_{TFC}, cf. black circles and blue triangles, while s_t, s_u, U_t^G being very close to each other, cf. violet crosses, green squares, and red circles, respectively.
Symbols in Fig. 2 show S_t or U_t evaluated using various aforementioned expressions and reference values c_r at x=30 mm. Significant quantitative differences between various S_t (and U_t) are confirmed.

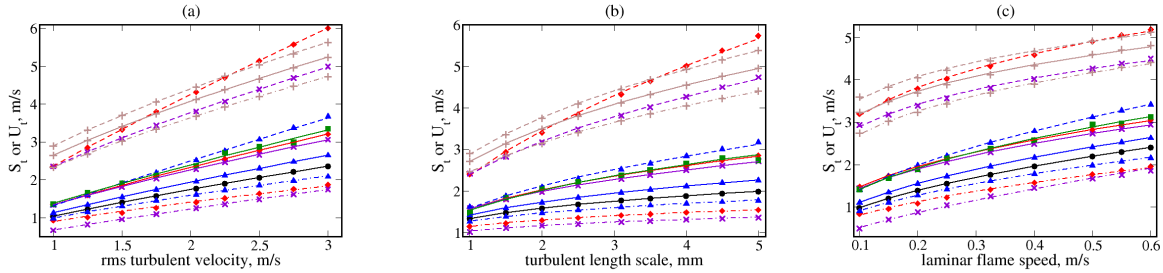

Figure 2. Dependencies of various S_t and U_t on (a) u', (b) L, and (c) S_L. Symbols show results computed at x=30 mm and are explained in legends in Fig. 1. Dashed, solid, and dotted–dashed lines show power fits to S_t-curves computed at $\overline{c}$ = 0.1, 0.5, and 0.9, respectively, or s_{le} computed at $\widetilde{c}$ = 0.01, 0.02, and 0.05, respectively.

Lines show power fits $f=ay^b$ to the computed dependencies $f(y)$, with the scaling exponents b being reported in Tables 1-3. If a S_t or U_t depends on a reference value c_r of the Reynolds- or Favre-averaged combustion progress variable, then, the scaling exponents were determined at c_r=0.1, 0.5, and 0.9, with exception of the leading edge flame speed s_{le}, which was evaluated at low c_r, see right columns in Tables 1-3, following Dunstan et al. [10] who introduced that flame speed. In order to gain insight into spatial variations of the scaling exponents, they were also evaluated at x=15 and 45 mm.
For dependence of S_t or U_t on u', see Table 1, all addressed flame speeds and burning velocities, including U_{TFC}, but with exception of s_{le}, are characterized by a similar scaling provided that they are evaluated at c_r=0.5. However, the scaling exponents depend substantially on c_r and even on x if $c_r \neq 0.5$. The leading edge flame speed s_{le} is associated with lower scaling exponents when compared to other S_t and U_t, including U_{TFC}.

Table 1: Scaling exponents for S_t or U_t vs. u'

x , mm	c_r	Scaling exponents for various flame speeds					c_r
		s_t	s_u	u_t	U_t^G	s_{le}	
15	0.1	0.65		0.87	0.85	0.59	0.01
30		0.69		0.94		0.60	
45		0.72		1.07		0.62	
15	0.5	0.74	0.79	0.78	0.79	0.61	0.02
30		0.75	0.82	0.78		0.62	
45		0.76	0.67	0.79		0.64	
15	0.9	0.93		0.72	0.66	0.61	0.05
30		0.87		0.68		0.63	
45		0.84		0.67		0.65	

Table 2: Scaling exponents for S_t or U_t vs. L

x , mm	c_r	Scaling exponents for various flame speeds					c_r
		s_t	s_u	u_t	U_t^G	s_{le}	
15	0.1	0.42		0.35	0.55	0.40	0.01
30		0.42		0.43		0.39	
45		0.44		0.52		0.38	
15	0.5	0.34	0.37	0.27	0.37	0.39	0.02
30		0.34	0.37	0.29		0.38	
45		0.34	0.29	0.31		0.36	
15	0.9	0.11		0.21	0.19	0.37	0.05
30		0.18		0.21		0.37	
45		0.20		0.20		0.35	

Table 3: Scaling exponents for S_t or U_t vs. S_L

x , mm	c_r	Scaling exponents for various flame speeds					c_r
		s_t	s_u	u_t	U_t^G	s_{le}	
15	0.1	0.21		0.51	0.27	0.17	0.01
30		0.24		0.50		0.19	
45		0.26		0.52		0.23	
15	0.5	0.39	0.42	0.51	0.41	0.20	0.02
30		0.40	0.42	0.48		0.23	
45		0.41	0.44	0.48		0.27	
15	0.9	0.86		0.51	0.50	0.22	0.05
30		0.70		0.48		0.25	
45		0.66		0.47		0.29	

For S_t or U_t as a function of L, see Table 2, dependence of the scaling exponent on the evaluation expression is well pronounced even at c_r=0.5, with the computed exponents being strongly (by a factor of 2-2.5) reduced with an increase in c_r. For the local consumption velocity evaluated at c_r=0.5 or 0.9, the scaling exponent is most close to the theoretical value, i.e. 0.25.

As far as variations in the laminar flame speed are concerned, again, the scaling exponent (i) depends substantially on the evaluation expression, (ii) is strongly sensitive to c_r, and (iii) is most close to the theoretical value, i.e. 0.5, for the local consumption velocity independently of c_r. Moreover, the scaling exponents obtained for s_t at c_r=0.5 or U_t^G at c_r=0.5 and 0.9 are sufficiently close to 0.5.

4 Conclusions

Results of the present simulations indicate that not only magnitudes, but even scaling exponents of differently evaluated turbulent flame speeds and burning velocities depend substantially on the evaluation method even in the case of homogeneous turbulence not affected by combustion. Among various S_t and U_t addressed by us, the local consumption velocity u_t shows the best agreement with the invoked theoretical expression for the turbulent burning velocity U_{TFC} of a statistically planar, one-dimensional flame, i.e. u_t is the least sensitive to flame shape in the case of homogeneous turbulence. However, in experiments, the sensitivity could be higher due to the influence of heat release and density variations on the local turbulence characteristics.

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