Flame Disturbance Growth Induced by a Radial Flow

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1 INTRODUCTION

The turbulence at the flame front of a propagating premixed flame induces its acceleration. Since the intensity of a gas explosion depends mainly on the rate of pressure rise, which is caused by flame propagation, the prediction of turbulence development is indispensable for the evaluation of gas explosion effects or the assessment of gas explosion hazards. Many studies have been performed on the growth of the flame front turbulence at premixed flame fronts. Most of these studies are based on the concepts of the flame induced turbulence and/or flame-turbulent flow interaction. The results of studies[1-4] seem to indicate that the early stage growth of flame front turbulence is attributable mainly to interaction of flame front with an acoustic wave, or acceleration of gas flow normal to it. There are two mechanisms for the intrinsic instability of premixed flame, hydrodynamic and thermodiffusive ones[5-10]. The effect of hydrodynamic instability is counter-balanced by thermodiffusive instability that have a stabilizing effect that becomes stronger as the wave length of the disturbances is small. Computer simulations[11,12] of inviscid flow with the initial flame configuration seen in experiment lead to the formation of cusps and eventually hooks in convex parts of the flame. A model considering these instabilities is needed to determine the critical wavelength of the disturbance, the initial flame configuration. Flame front turbulence appears on the flame fronts propagating in orifice sections or periodically accelerating and decelerating flows where the flow of premixed mixture is not parallel[1-4]. To understand the initial turbulence inception, the flow field turbulence in premixed mixture needs to be examined. In a divergent flow, non-uniform flow appears above a critical velocity. As flame propagates in an orifice section, flame propagates in a converging and diverging flow. If the intensities of diverging flow were made very large as accelerated by a pressure wave in experiments[1-4,13], gas flow turbulence would appear due to the viscous stress of the fluid[14]. In purely divergent flow, viscous forces do not decrease indefinitely in relative magnitude as the Reynolds number tends to infinity and at no value of the Reynolds number are viscous forces negligible[14]. In purely divergent flow, vorticity is not convected towards the walls nor the vorticity generated at the wall is not permanently confined to a layer adjoining the wall whose thickness tends to zero as the Reynolds number tends to infinity[14]. In this study, a simple simulation of viscous flow was carried out to determine the flow field observed in experimental studies[1-4].

2 FLOW FIELD NEAR FLAME FRONT MODIFIED BY A FLOW

1) DIVERGENCE ANGLE OF GENERATED FLOW

Chemical reaction at flame front generates heat and combustion product from premixed mixture. Thermal expansion of gas generates a flow in premixed mixture. Assuming the angle α_0 of cylindrical flame front, the flow in premixed mixture with the potential-flow model has a uniform velocity of $|\boldsymbol{V_s}|$ as shown in figure 1. If this cylindrical flame front is propagating in the flow of velocity $\boldsymbol{U}$, the combined flow velocity is $\boldsymbol{V_s}+\boldsymbol{U}$ and the angle of the combined flow from the pseudo point source becomes α_P. Assuming radius of the flame front is r_0, the position at the edge of frame front is

$$(r_0 \cos\alpha_0 \,, r_0 \sin\alpha_0) \tag{1}$$

Assuming the source strength at (0, 0) is *I*, the flow velocity, $\boldsymbol{V_s}$ at the edge of frame front is

$$\left(\frac{I}{2\pi r_0}\cos\alpha_0 \,, \frac{I}{2\pi r_0}\sin\alpha_0\right) \tag{2}$$

The combined flow velocity, $\boldsymbol{V_s} + \boldsymbol{U}$ at the edge of flame front is

$$\left(\frac{I}{2\pi r_0}\cos\alpha_0 + U, \frac{I}{2\pi r_0}\sin\alpha_0\right) \tag{3}$$

The radius, r_p of the combined flow from the pseudo point source is $\left|\frac{\frac{I}{2\pi r_0}+|\boldsymbol{U}|}{\frac{I}{2\pi r_0}}\right| r_0$ for $\alpha_0 \ll 1$.

$$\tan\alpha_p = \frac{r_0 \sin\alpha_0}{\left|\frac{\frac{I}{2\pi r_0}+|\boldsymbol{U}|}{\frac{I}{2\pi r_0}}\right| r_0} = \left|\frac{\frac{I}{2\pi r_0}}{\frac{I}{2\pi r_0}+|\boldsymbol{U}|}\right| \sin\alpha_0 \tag{4}$$

For $\alpha_0 \ll 1$,

$$\alpha_p = \frac{\frac{I}{2\pi r_0}}{\left|\frac{I}{2\pi r_0}+|\boldsymbol{U}|\right|}\alpha_0 \tag{5}$$

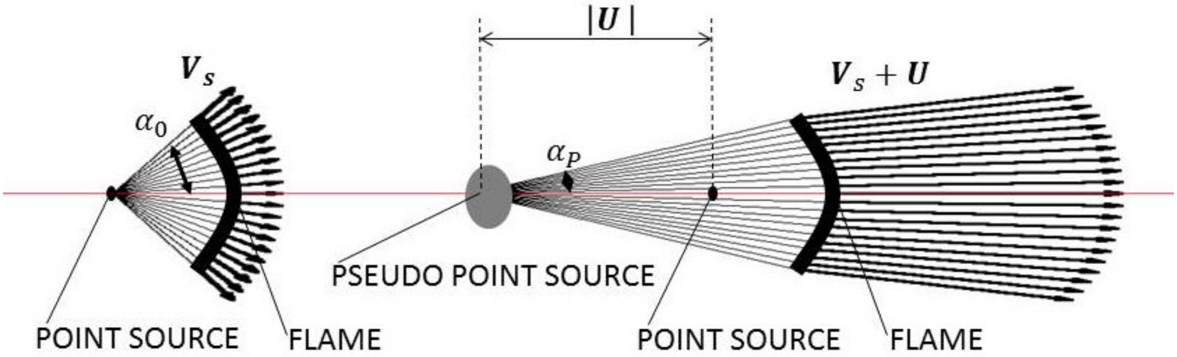

Figure 1 Flow from the pseudo point source

The flow velocity, $|\boldsymbol{V_s}|$ at the edge of flame increases from zero to the burned gas velocity, S_b depending on the flow condition. The combined flow velocity, $|\boldsymbol{V_s} + \boldsymbol{U}|$ equals flame propagation velocity, $\boldsymbol{V_f}$ subtracted by the laminar burning velocity S_u. As shown Eq.(4), the flame is cylindrical where $\alpha_0 \ll 1$. This paper investigates the flame front behavior where $\alpha_0 \ll 1$.

2) POSSIBLE FLOW OF EXPANDING VISCOUS FLUID

Assuming pseudo steady state after acceleration by the pressure wave, the continuity equation is formulated for a system with constant density. It is well known that the flow of a viscous incompressible fluid between two plane walls tilted by an angle of α with respect to each other is governed by the exact solution by Jeffery[15] and Hamel[16]. In cylindrical coordinates (r, θ, z), the equations for the velocity components (u, v, w) for the steady flow of an incompressible fluid are:

continuity[17],

$$\frac{1}{r}\frac{\partial(ru)}{\partial r} + \frac{1}{r}\frac{\partial v}{\partial \theta} + \frac{\partial w}{\partial z} = 0 \tag{6}$$

momentum,

$$u\frac{\partial u}{\partial r} + \frac{v}{r}\frac{\partial u}{\partial \theta} - \frac{v^2}{r} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \upsilon\left(\nabla^2 u - \frac{u}{r^2} - \frac{2}{r^2}\frac{\partial v}{\partial \theta}\right) \tag{7}$$

$$u\frac{\partial v}{\partial r} + \frac{v}{r}\frac{\partial v}{\partial \theta} + \frac{uv}{r} + w\frac{\partial v}{\partial z} = -\frac{1}{\rho r}\frac{\partial p}{\partial \theta} + \upsilon\left(\nabla^2 v - \frac{v}{r^2} + \frac{2}{r^2}\frac{\partial u}{\partial \theta}\right) \tag{8}$$

$$u\frac{\partial w}{\partial r} + \frac{v}{r}\frac{\partial w}{\partial \theta} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \upsilon(\nabla^2 w) \tag{9}$$

where

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}$$

p and ν denote the pressure and kinematic viscosity, respectively.

Assuming $u = u(r, \theta), v = 0, w = 0$,

$$u = \frac{F(\theta)}{r} \tag{10}$$

$$-\frac{F(\theta)^2}{r^3} = -\frac{1}{\rho}\frac{\partial P}{\partial r} + \frac{\nu}{r^3}\frac{\partial^2 F(\theta)}{\partial \theta^2} \tag{11}$$

$$0 = -\frac{1}{\rho r}\frac{\partial P}{\partial \theta} + \nu\frac{2}{r^3}\frac{\partial F(\theta)}{\partial \theta} \tag{12}$$

A partial differentiation of Eq. (12) with respect to r gives

$$-\frac{1}{\rho}\frac{\partial}{\partial r}\left(\frac{\partial P}{\partial \theta}\right)-\nu\frac{4}{r^3}\frac{\partial F(\theta)}{\partial \theta} = -\frac{1}{\rho}\frac{\partial}{\partial \theta}\left(\frac{\partial P}{\partial r}\right)-\nu\frac{4}{r^3}\frac{\partial F(\theta)}{\partial \theta} \tag{13}$$

where

$$\frac{\partial}{\partial r}\left(\frac{\partial P}{\partial \theta}\right)=\frac{\partial}{\partial \theta}\left(\frac{\partial P}{\partial r}\right)$$

A partial differentiation of Eq. (11) with respect to θ and substituting Eq. (13) give

$$2\,F(\theta)\frac{\partial F(\theta)}{\partial \theta}+4\nu\frac{\partial F(\theta)}{\partial \theta}+\nu\frac{\partial^3 F(\theta)}{\partial \theta^3}=0 \tag{14}$$

$$\theta = \alpha\eta \tag{15}$$

$$F(\theta) = F_0 f \tag{16}$$

where η and f denote the non-dimensional angle and velocity, respectively.

$$2R\alpha f f' + 4\alpha^2 f' + f''' = 0 \tag{17}$$

where

$$R = \frac{\alpha F_0}{\nu}$$

with the boundary conditions,

$$\eta = 1, \quad f = 0 \tag{18}$$

$$\eta = -1, \quad f = 0 \tag{19}$$

$$\eta = 0, \quad f = 1 \tag{20}$$

$$\eta = 0, \quad f' = 0 \tag{21}$$

$$\eta = 0, \quad f'' = F_2 \tag{22}$$

Thus, the partial differential equations of continuity and momentum have been reduced to an ordinary nonlinear differential equation of the third order whose solutions must satisfy in the physical problem under consideration two boundary conditions of vanishing velocity at the walls[17].

3 RESULTS

It is clear that the solution f of Eq, (17) is given by an elliptic integral. An evaluation is given by Hamel[16] in terms of the Weierstrassian functions[17]. Numerical integration of Eq. (17) from $\eta = 0$ to $\eta = 1$ gives f'', f', f with a range of F_2 whose satisfy two boundary conditions of vanishing velocity at the walls[17]. Numerical integration was carried out with 30000 elements along η axis. Figure 2 shows R and α those satisfy two boundary conditions of vanishing velocity at the walls for $F_2 > -10000$. Radial expanding flows are $R > 0$.

Assuming $F_0 = \left|\frac{I}{2\pi r_0} + |\boldsymbol{U}|\right| r_p \ [\mathrm{m}^2/\mathrm{s}], \nu = 1.6 \times 10^{-5} \ [\mathrm{m}^2/\mathrm{s}], \ \left|\frac{I}{2\pi r_0} + |\boldsymbol{U}|\right| = 2.7 \ [\mathrm{m}/\mathrm{s}], \ r_p = 1.0[\mathrm{m}], \alpha_0 = 0.2 \ [\mathrm{rad}]$,

$$R = \frac{\alpha_p F_0}{\nu} = \frac{\frac{I}{2\pi r_0}}{\left|\frac{I}{2\pi r_0} + |\boldsymbol{U}|\right|} \alpha_0 \frac{\left|\frac{I}{2\pi r_0} + |\boldsymbol{U}|\right| r_p}{\nu} = \frac{\alpha_0}{\nu}\left|\frac{I}{2\pi} + |\boldsymbol{U}| r_0\right| \tag{23}$$

Figure 3 shows the obtained non-dimensional velocity f from $\theta = 0$ to $\theta = \alpha$ for $R = 33000$. Four flows are obtained for positive and negative velocity gradients at the walls. The first minimum of non-dimensional velocity f appears near $\theta = 0.003$. The estimated scale of the flow turbulence on the flame of radius, $r_p =$ 1.0 m from the pseudo point source for the angle $2\theta = 0.006$ is $2\theta \ r_p$= 6.0 mm, which agrees with the scale of flame turbulence observed in experiments[1,4]. As seen in Fig. 3, non-dimensional velocity f becomes negative that indicates premixed mixture flows into the burned gas side. Flowing into the burned gas from premixed mixture could result in flame area increase.

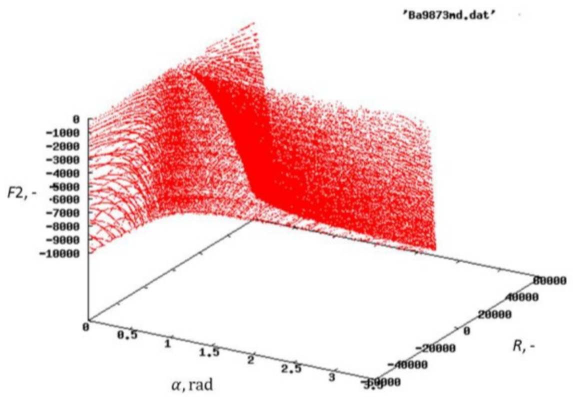

Figure 2 *F2* with *R* and α

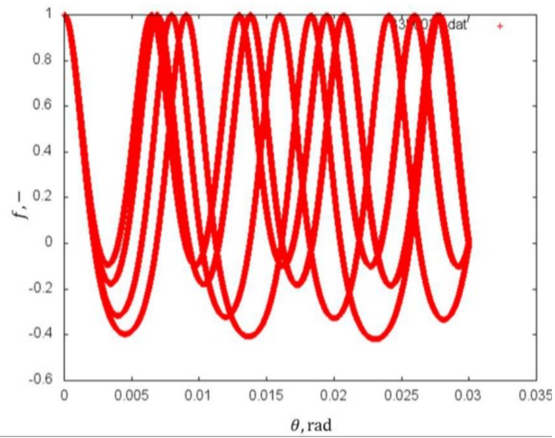

Figure 3 Non-dimensional velocity

4 DISCUSSION

The flow in premixed mixture is laminar along smooth flame front. Due to thermal expansion of burned gas, the laminar flow becomes a radial flow. The combined flow of this radial flow and the flow caused by pressure wave is another radial flow with large velocity and a small flow angle. This combined flow is not purely expanding flow at large velocity. A periodic flow appears in premixed mixture normal to the flame front. With a given f'' at $\eta = 0$, the periodic flow is possible with small angle α. This small angle α occurs when flame accelerates, or flame propagates in a long distance or a narrow channel. If f'' at $\eta = 0$ becomes much smaller, periodic flows are possible with larger angle α. The viscous stress determines the appearance of these periodic flows. The viscous stress increases as the channel diameter decreases or the channel wall roughness increases.

5 CONCLUSIONS

1. The combined flow velocity and angle in premixed mixture are approximated.

2. Periodic flows appear with the radial viscous flow model.

3. The estimated scale of the flow turbulence agrees with the scale of the flame front turbulence observed in experiments.

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