

# Diffusive-Thermal Instability of Low-Lewis-Number Premixed Flames in Stretched Flow of Two Slot Burners

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## 1 Introduction

Fundamental knowledge on flammability limits and near-limit flame characteristics are of interest for both laminar flame theory and advanced combustion technologies, such as mild combustion. Investigations of stretched premixed flames were proved to be usable for studies of extinction limits and ultra-lean flame behavior. The extinction limits of counterflow premixed flames with near unity Lewis numbers have been intensively studied experimentally [1], numerically [1-3] and theoretically [4, 5]. It was found that in equivalence ratio / stretch rate plane, the extinction limit is represented by C-shape curve. Numerical simulations [1, 2] reveal that radiation heat loss is the dominant mechanism of flame extinction at low stretch rates. Although, for near unity Lewis numbers the flame front is planar and one-dimensional simulations describe flame structure well, for low Lewis numbers the flame can be influenced by diffusive-thermal instability. Recent [6] microgravity experiments and three-dimensional numerical simulations show that ultra-lean low-Lewis-number stretched premixed flames appear as a set of separate cup-like fragments in a state of chaotic motion and splitting at low stretch rates. The extension of the flammability limits of one-dimensional flame, related with non-planar flame structure was detected both experimentally and numerically. Good qualitative agreement between numerical and experimental results [6] justifies the application of reduced thermal-diffusion model with one-step global kinetics for numerical simulations.

In all the above mentioned studies, the conventional axisymmetric counterflow burners were applied. In this configuration extensional strain occurs in both coordinate directions parallel to the flame surface. In [7], the diffusive-thermal instabilities of low Lewis number flames were studied experimentally using counterflow slot-jet apparatus which provides extensional strain only in the direction that is orthogonal to the plane of the slots. The experimental observations [7] demonstrate that stretched flames can exist beyond the extinction limits of planar flame, in the form of flame tubes. Such spatial flame structure is significantly different from the sporadic combustion wave structure observed in [6] for conventional counterflow configuration. Multiple flame tubes of ultra-lean mixtures exist throughout the entire range from low to high stretch rates. In contrast, in conventional counterflow burner, the sporadic combustion regime existing beyond the extinction limits of planar flame can be observed only at low stretch rates [8]. At the same time, there is a lack of theoretical investigations of the diffusive-thermal instability of counterflow flames in slot-jet configuration.

In the present study, the characteristics of lean low-Lewis-number flames in counterflow slot-jet burner are studied numerically and theoretically. The regime diagram in the equivalence ratio /

stretch rate plane is plotted and compared with available experimental data [7]. The differences in combustion regimes for conventional and slot-jet counterflow burners are discussed.

## 2 Mathematical model

Three-dimensional thermal-diffusion model [6, 8] of premixed flames in the counterflow slot-jet configuration is considered. The air–fuel mixture is supplied from two opposite slot-jet burners located at positions  $y=\pm L_y$ , forming two flames near the stagnation plane  $y=0$ . The set of governing equations can be written in the following dimensionless form [6]

$$T_t + \bar{\nabla} \nabla T = \nabla^2 T - h(T^4 - \sigma^4) + (1 - \sigma)W(T, C) \quad (1)$$

$$C_t + \bar{\nabla} \nabla C = Le^{-1} \nabla^2 T - W(T, C) \quad (2)$$

Here  $T$  is the scaled temperature in units of  $T_b$ , the adiabatic temperature of combustion products;  $C$  is the scaled concentration of the deficient reactant in units of  $C_0$ , its value in the fresh mixture. The velocity is measured in units of the velocity of a planar adiabatic flame  $U_b$ , the distance in the units of the thermal width of flame  $l_{th}=D_{th}/U_b$ , where  $D_{th}$  is the thermal diffusivity of the mixture and the time is measured in units  $D_{th}/U_b^2$ .  $Le=D_{th}/D_{mol}$  is the Lewis number where  $D_{mol}$  is the fuel molecular diffusivity. Non dimensional initial temperature is defined as  $\sigma=T_0/T_b$  where  $T_0$  is the fresh mixture

temperature;  $W(T, C) = \frac{C}{2Le} (1 - \sigma)^2 N^2 \exp(N(1 - 1/T))$  is the dimensionless chemical reaction rate

where  $N=T_a/T_b=\sigma T_a/T_0$  is the ratio of activation temperature  $T_a$  and adiabatic flame temperature  $T_b$ ;  $h=A(1/\sigma-1)\exp(\sigma T_a/T_0)$  is the non-dimensional radiative heat loss intensity which depends on dimensionless fuel concentration  $1/\sigma-1$ . For the counterflow slot-jet burners, the velocity vector has only two non-zero components and may be written as  $\bar{V}=(ax, -ay, 0)$  where  $a$  is the non-dimensional stretch rate which is equal to the ratio of flow velocity at burner outlet to the half of separation distance between the burners. Notice that for conventional axisymmetric counterflow burners, the nondimensional velocity vector is  $\bar{V}=(a/2x, -ay, a/2z)$ .

Equations (1)-(2) are subjected by the following boundary conditions:

$$\text{at inlet } (y=\pm L_y): T=\sigma, C=1 \quad (3)$$

$$\text{at } z=\pm L_x: T=\sigma, C=0 \quad (4)$$

$$\text{at } x=\pm L_z: \text{periodic boundary conditions} \quad (5)$$

The set of governing equations (1)-(2) with boundary conditions (3)-(5) were solved numerically by explicit finite-difference scheme. Convergence of the numerical scheme was checked by the simulations on a set of gradually refining grids.

## 3 Results and discussion

Three-dimensional simulations in the framework of the model (1)-(5) revealed that in almost all range of stretch rates and dimensionless fuel concentrations defined as  $1/\sigma-1$ , the temperature and concentration fields are independent on  $z$ -coordinate. Therefore, the further numerical simulations were performed in the frame of two dimensional problem statement formulated in  $x$ - $y$  plane. Notice that for conventional axisymmetric counterflow burners the structure of low-Lewis-number lean flames is essentially three-dimensional [6, 8].

Depends on stretch rate and mixture content, the following regimes of combustion were identified: planar flame, wrinkled flame and flame tubes. In order to investigate the placement of these combustion regimes in fuel concentration /stretch rate plane the following numerical procedure was applied. At first, the flame was ignited for the initial values of stretch rate and fuel concentration  $1/\sigma-1$  which are chosen somewhere inside the C-shape curve (see Fig. 1) which bound the region of existence of 1D counterflow flames. Once the combustion regime is established, the parameter  $1/\sigma-1$  or/and stretch rate are varied linearly with time approaching to the C-shape curve in  $1/\sigma-1 / a$  plane.

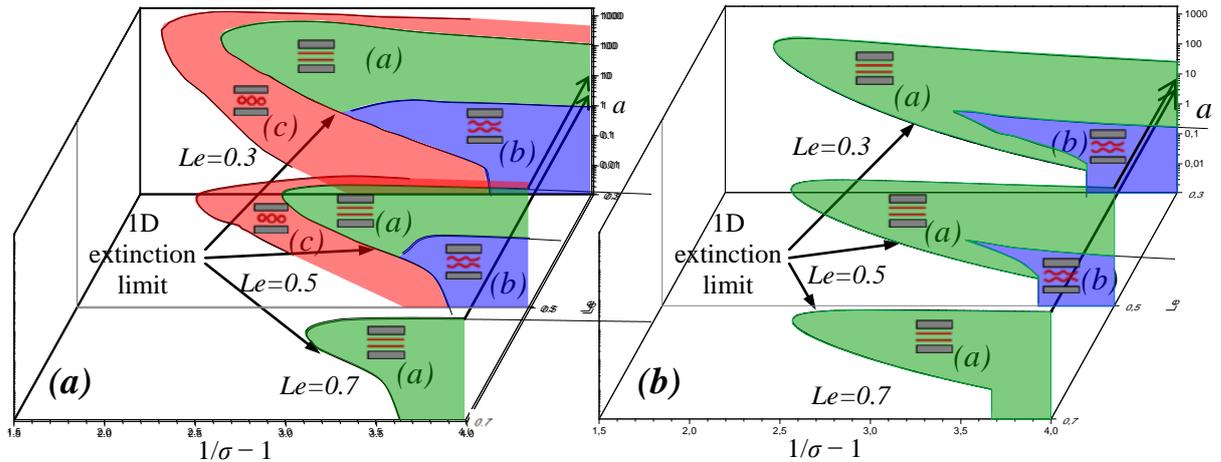


Figure 1. Non-dimensional regime diagrams in fuel concentration / stretch rate plane obtained by numerical simulations (Fig. 1a) and by linear stability analysis of one-dimensional stationary solutions (Fig. 1b) for different Lewis numbers. Regions of existence of planar, cellular and tubes-like flame structure are denoted as (a), (b) and (c), correspondingly

Using this procedure the placement of combustion regimes along the line  $(1/\sigma(t)-1, a(t))$  can be determined.

Figure 1a shows non-dimensional regime diagram in fuel concentration / stretch rate plane calculated for different Lewis numbers. Inside the C-shape curve corresponding to extinction limit of planar counterflow flames the twin planar and continuous wrinkled flames exist. In the range of high stretch rates (region (a) in Fig. 1a) the planar flames are observed. At relatively small stretch rates (region (b) in Fig. 1a) the flames have a cellular structure. Temperature distribution typical for wrinkled flames is shown in Fig 2a. Besides, numerical simulations the linear stability analysis of one-dimensional stationary solutions were performed. Theoretical results were obtained in the frame of the model (1)-(2) with additional assumptions. Namely, the linear dependency of radiative heat losses on temperature  $h(T-\sigma)$  and infinitely thin reaction zone were assumed [5]. At the first step of theoretical study, the stationary one-dimensional problem formulated along y-axis was solved. Under the applied assumptions this problem is described by the set of linear ODEs with non-linear boundary conditions at the flame front surface which position and temperature is unknown [5]. By solving this problem we obtained the set of non-linear algebraic equations which allow to determine stationary flame front position and temperature as functions of problem parameters, i.e.  $(1/\sigma-1)$  and  $a$ . At the second step, the linear stability analysis [5] of stationary solutions with respect to two-dimensional perturbations were performed. As a result, the dispersion relation allowing to obtain dependency of perturbation's growth rate  $\Omega$  on perturbation's wave number  $k$  for each stationary solution in  $(1/\sigma-1) / a$  plane was derived. By knowing  $\Omega(k)$  dependency for the specific non-dimensional fuel concentration and stretch rate the conclusion on flame structure at these parameters can be done. Namely, if  $\Omega(k) < 0$  for all  $k$  the one-dimensional solution describing planar flame is stable, if  $\Omega(k)$  dependency has a positive maximum at some  $k_{max}$  the cellular flame structure is expected. Regime diagrams obtained by means of linear stability analysis are shown in Fig. 1b. As it is seen from Fig.1 the numerical and theoretical results obtained for the parameters  $1/\sigma-1$  and  $a$  inside the C-shape curve coincides well. It may be concluded that theoretical analysis is capable to predict the placement of different combustion regimes inside the extinction limit curve as well as boundary separating planar and wrinkled flame regimes in fuel concentration / stretch rate plane. Moreover the results of linear stability analysis allow us to estimate the typical cell size as  $\lambda=2\pi/k_{max}$  where  $k_{max}$  is the wave number of the most growing perturbation. Numerical and theoretical results are consistent with the conclusion that characteristic cell size of wrinkled flames increases with increase of Lewis number while it is almost independent on non-dimensional fuel concentration and stretch rate.

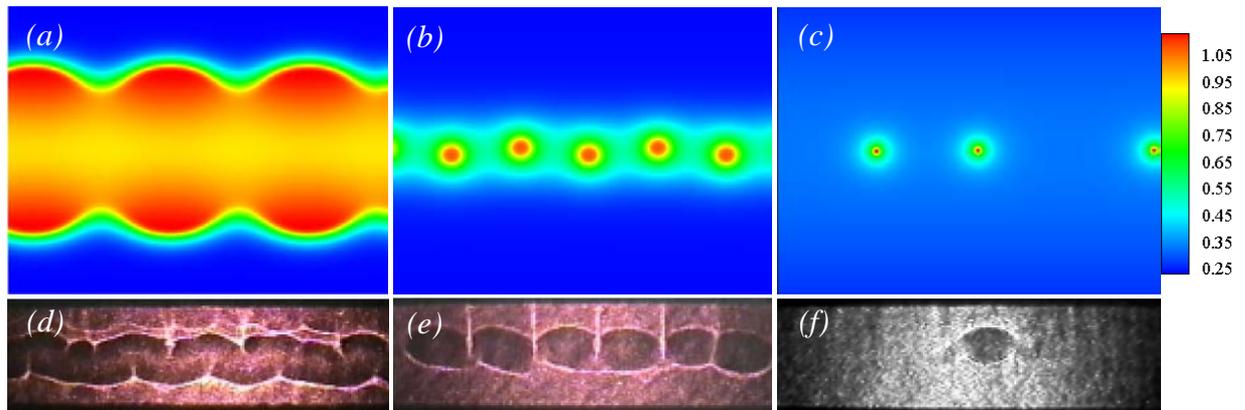


Figure 2. Calculated temperature distributions typical for cellular twin flames (a), flame tubes (b) and isolated flame tubes (c). Fig. 2d-e show shadowgraph images of wrinkled flames (d), moving flame tubes (e) and single tube (f) obtained in experimental work [7].

Numerical simulations show that ultra-lean counterflow flames with non-dimensional fuel concentration less than extinction limit of 1D stretched flames appear throughout the entire range of stretch rates (region (c) in Fig. 1a) as a set of flame tubes (see Fig. 2b). It is interesting to note that for conventional axisymmetric counterflow flames, the extension of extinction limits is observed only in the range of small stretch rates [8]. As can be seen from Fig. 1, the extension of flammability limits of low-Lewis-number flames occurs due to the possibility of existence of non-planar flame structure. In the narrow region near the extended extinction limits the separated flame tubes (Fig. 2c) are observed. It is necessary to notice, that these results can not be obtained in the frame of our theoretical approach because the one-dimensional stationary problem has no solutions for the parameters beyond the extinction limits of planar counterflow flames.

Both numerical simulations and theoretical analysis show that regime diagrams for  $Le=0.5$  and  $Le=0.3$  are qualitatively the same (Fig.1). At the same time, the ranges of stretch rates in which the counterflow flames can exist dramatically depends on Lewis number. This range for  $Le=0.5$  is much narrower than for  $Le=0.3$ . For  $Le \geq 0.7$  the flame tubes do not observed and the only combustion regime is the planar flame which exists inside the extinction limits curve. Therefore, formation of flame tubes may be associated with the effect of diffusive-thermal instability.

Figure 3 shows dimensional regime diagram calculated for  $Le=0.3$  and experimental results obtained in [7] for lean hydrogen-air mixtures. Inside the C-shape curve the wrinkled flames (Fig. 2d) were experimentally observed at moderate and high stretch rates. Notice that in experiments the

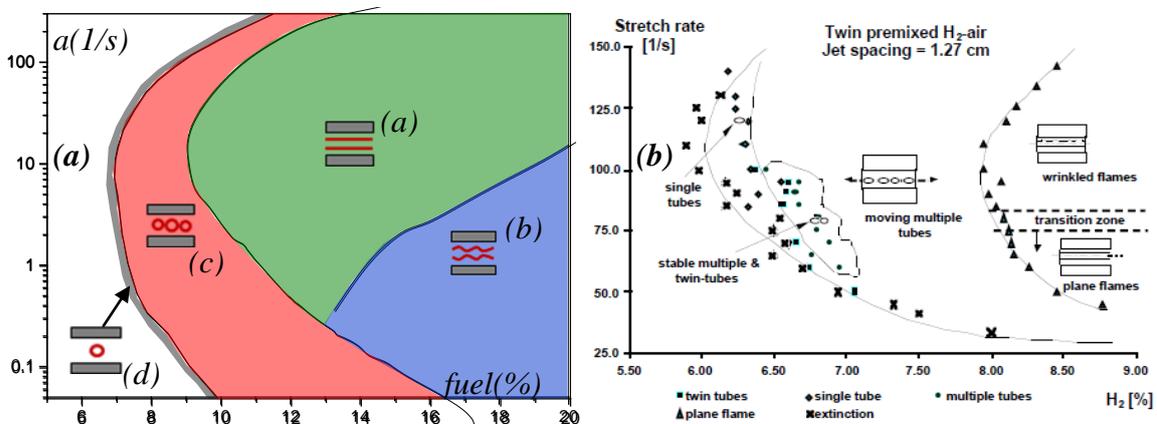


Figure 3. Dimensional regime diagram in fuel concentration / stretch rate plane calculated for  $Le=0.3$  (Fig. 3a) and obtained in experiments by Kaiser et.al. [7] (Fig. 3b).

hydrodynamic effects and heat losses in the burner rims affect the flame structure at high and low stretch rates, correspondingly [7]. These effects, as well as buoyancy effect are neglected in reduced thermal-diffusion model (1)-(2) that can explain the discrepancy between numerical and experimental results. Experimental results [7] shows that beyond the extinction curve of planar twin flames the tubes-like flame structure (Fig. 2e) is realized at all stretch rates (Fig. 3b). Near the extended extinction curve the single (Fig. 2f) and twin flame tubes were experimentally observed. This combustion regime seems to correspond to the isolated tubes regime (Fig. 2c) obtained in numerical simulations at near extinction parameters (region (d) in Fig. 3a). Therefore the numerical results are in good agreement with existing experimental data [7] on behavior of ultra-lean counterflow flames with low Lewis number. Experimental [7] and numerical investigations consistent with the conclusion of the extension of low-Lewis-number counterflow flames flammability limits related with the non-planar tube-like structure of the flame.

## 4 Conclusions

Characteristics of lean premixed flames in stretched flow of two counterflow slot-jet burners were studied numerically and theoretically in the frame of thermal-diffusion model. In contrast to the low-Lewis-number flames in conventional axisymmetric counterflow configuration which demonstrates essentially three-dimensional structure and non-stationary behavior [6, 8], in slot-jet configuration the flame structure is two-dimensional and time-independent in almost all range of parameters. A wide variety of flame structures were detected in numerical simulations and regime diagrams summarizing the placement of different combustion regimes in fuel concentration / stretch rate plane were plotted. Comparison of numerical and theoretical results shows that inside the C-shape curve representing extinction limit of one-dimensional counterflow flames the flame structure can be described well on the basis of linear stability analysis of one-dimensional stationary solutions. Results of such analysis allow to predict the regions of existence of planar and wrinkled flames which agree with numerical results. Numerical simulations show that beyond the C-shape curve, the ultra-lean counterflow flames appear throughout the entire range of stretch rates as a set of flame tubes. Therefore, the extension of flammability limits of low-Lewis-number stretched premixed flames is observed due to the possibility of existence of non-planar flame structure related with diffusive-thermal instability. This result is consistent with existing experimental data [7].

The effect of Lewis number was studied numerically and theoretically. It was found that cellular and tubes-like non-planar flame structures appear at  $Le < 0.6$  while at higher Lewis numbers the only planar flame structure is observed. Numerical and theoretical results show that with the decrease of Lewis number, the range of stretch rates and fuel concentrations in which the counterflow flames can exist becomes wider. At the same time, relative disposition of different combustion regimes remains the same.

## 5 Acknowledgements

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