# **Dynamics of Secondary Breakup of Emulsified Fuel Drop**

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#### **1** Introduction

Diesel engines are used as the main source of driving power both for land and shipping transport, due to their simple and reliable design and high fuel economy [1]. However, the diesel engines were recognized as the main polluters of air in hubs. They kill three times more people, than HIV, malaria and tuberculosis together taken [2]. It is well-known that high temperature and the oxygen abundance in the combustion chamber are the main reasons for the nitrogen oxides formation [3]. It was shown experimentally that the emulsified fuels (EF) usage reduces the emissions. Presence of water vapors in the flame zone has an impact on physics and chemical kinetics of burning; the water vapors have also useful effect on the rate of energy release. In some experiments the reduction of fuel consumption was additionally revealed when using EF [3]. Also, EF is much safer in operation, especially at accidents, in aviation as well as in marine transport. More detailed data on EF studies can be found in [3].

The EF effect is connected with the mixing quality improvement within the combustion chamber. After the EF injection and jet breakup onto separate drops (*primary atomization*) the fast heating of these drops occurs. In a certain range of the combustor air temperature it leads to a rapid water boiling and fast conversion of water drop into a vapor bubble [4]. Then, the huge vapor pressure expands a bubble resulting in the fuel film breaking up into a large number of microdrops (*micro-explosion, or secondary atomization*). Reduction of harmful emissions is connected with a small size of these droplets, which gives the fast fuel evaporation and more uniform, leaner gas mixture of vapors with oxidizer [3]. Besides, water vapors decrease the flame temperature in the combustion chamber, change chemical composition of mixture, which dilutes zones of the rich fuel content and leads to higher concentration of radical *OH* controlling the rate of nitrogen oxides and soot formation. The EF advantages work only in diesel engines where the air temperature is high, so that it causes a powerful heat flux inside EF drops [4]. Thus, the EF drop micro-explosion proceeds as a key process, which launches mechanisms of the mixing improvement in the combustion chamber.

In order to govern the secondary atomization it is vital to study physical bases of micro-explosion and those EF properties, which affect it. The mechanism of micro-explosion hasn't been examined yet, despite the fact that it controls kinetics of all subsequent preparatory processes. It is generally accepted that the sizes of secondary droplets, their quantity and time of their generation determine quality of combustion. In turn, these values are established by the mechanism of micro-explosion, which is imposed by a system of forces during the vapor bubble expansion. Thus, the problem requires elaborating of mathematical model, which is able to describe quantitatively dynamics of a vapor bubble expansion, to find out the mechanism of micro-explosion and thus – kinetic parameters of the secondary dispersion.

## 2 The Macro-emulsion Drop

Dynamics of each type of EF (*macro-emulsions, micro-emulsions*) have their own specifics and ought to be considered separately. The mechanics of EF of the first type looks like a simpler one, therefore we will start our consideration with it. The EF fuel drop, which was formed as a result of primary fuel jet atomization, may contain one, two or more water drops. Focusing our study on the micro-explosion mechanism, we consider the simplest case of one water droplet inside EF drop.

After the primary breakup, the process of a vapor bubble formation depends on a regime of the fuel drop heating. Hot combustor air temperature  $T_3$  produces large heat flux to a drop. Analysis of water behavior inside an EF drop makes clear that changing of its thermodynamic state occurs in a completely different way than inside a free water drop. Having reached the boiling temperature  $T_{\rm s} = 263.9^{\circ}C \approx 0.83T_{\rm cr}$  at pressure in the combustion chamber  $p_3 = 0.5MPa$ , the state of water can't be changed further isobaric since this would require an increase of specific volume of water by  $\approx 30$ times to the state of dry steam and the related expansion of a fuel layer bounding the water drop. However, fuel mass in the layer has substantial mechanical inertia that doesn't permit even very small instant vapor expansion. Instead, the proceeding heat influx increases pressure and temperature at almost constant volume. For this reason the process develops close to isochoric one along curve 7 (fig. 1), between isotherm 4 and the saturated states curve 6. When the critical isotherm 3 is reached at some value  $p_{2a0}$ , water becomes a superheated liquid and the intense heat flux transfers water into a state of superheated vapor at  $T > T_{cr}$ . Figure 1 shows that the pressure maximum,  $p_{2a0}$ , can exceed critical value  $p_{cr}=22.1MPa$  by 4-5 times. The real, complicated development of the process, which is roughly described by curve 7, we will approximate by idealized one, when water reaches a superheated state at  $T=T_{cr}$ ,  $p=p_{2a0}$  instantly, at t=0, and there is an adiabatic vapor expansion after that.

As a result, an EF drop at t=0 contains a *water vapor bubble* of radius  $a_0$ , bounded by *spherical fuel layer* (SFL) of radius  $b_0$  (fig. 2). The motion of this system, which we will call *fuel globule*, in fact differs from oscillations of both a thin-walled (soap) bubble and a vapor bubble in unconfined liquid when its dynamics is described by well-known Rayleigh–Lamb equation [5]. The differential equation of a fuel globule dynamics is derived below, which shows essential difference in the two solutions.



Figure 1. Isotherms of water; 7-real course of process.

Figure 2. Water-in oil fuel globule.

# **3** Equation of Radial Motion of a Fuel Globule

Assuming potentiality of a flow in SFL [5], we apply Cauchy – Lagrange integral. Eliminating the constant, we obtain *the differential equation of motion of a spherical vapor bubble of a fuel globule*:

$$a\ddot{a} = \dot{a}^{2} \left[ \frac{1}{2} \left( 1 + \frac{a}{b} \right) \left( 1 + \frac{a^{2}}{b^{2}} \right) - 2 \right] - \frac{1}{\rho_{1}} \left[ 4\mu_{1} \left( 1 + \frac{a}{b} + \frac{a^{2}}{b^{2}} \right) \frac{\dot{a}}{a} + \frac{2b}{b-a} \left( \frac{\Sigma_{b}}{b} + \frac{\Sigma_{a}}{a} \right) + \frac{b}{b-a} \left( p_{3} - p_{2a} \right) \right].$$
(1)

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Here p – pressure;  $\Sigma$  – surface tension coefficient;  $\rho, \mu$  – density and dynamic viscosity; sub-indices "*1*", "2", "3" refer to liquid fuel, vapor, air in the combustion chamber; "*a*", "*b*" – values are taken at internal or external SFL surfaces. At  $b_0 \rightarrow \infty$  eq. (1) transforms into the classical Rayleigh – Lamb equation [5]. External radius b(t) can be determined from the mass conservation law of fuel in SFL:



Figure 3. SFL Accelerations.

be determined from the mass conservation law of rule in SFL.  $b^{3}(t) - a^{3}(t) = b_{0}^{3} - a_{0}^{3}$ . It is possible to consider vapor state in the adiabatically expanding bubble as homobaric [5]; it is thus only determined by a degree of expansion:  $p_{2a}(t)/p_{2a0} = (a_{0}/a(t))^{3\gamma_{v}}$ with vapor adiabatic exponent  $\gamma_{v}$ . Runge – Kutta procedure of IV order of accuracy is applied for numerical integration of equation (1). Figure 3 gives typical variation of accelerations ( $g_{a,b}$ ·10<sup>-9</sup>,  $m/\sec^{2}$ ) of internal (curve 1) and external (curve 2) surfaces of SFL in time for  $a_{0}=30\mu m$ ,  $p_{2a0}/p_{3}=20$ ; the time is in  $\mu$ sec.

Some approximate expressions can be useful for studying the mechanism of SFL breakup. Getting eq. (1) a dimensionless form, we find the existence of following criteria of similarity, which determine the process of a fuel globule expansion:  $\Pi_p = p_{2a0}/p_3$ ,  $\Pi_a = a_0/b_0 = \alpha_{20}^{1/3}$ ,  $\Pi_\mu = \mu_1/(a_0\rho_1p_3)$ ,  $\Pi_{\Sigma} = \Sigma_b/a_0p_3$ . The big or small order of the values makes it possible to simplify some of the relations. For values of parameters in diesel engines these criteria are ranging as follows:  $\Pi_p = (10-30)$ ,  $\Pi_a = (0.4-0.7)$ ,  $\Pi_\mu = (3 \cdot 10^{-2} - 3 \cdot 10^{-1})$ ,  $\Pi_{\Sigma} = (10^{-4} - 10^{-3})$  pointing out that  $\Pi_p$  may be thought of as a main criterion. Herewith, *contribution of viscosity and surface tension can be neglected*, though the latter continue to affect the wavelength of unstable disturbance  $\lambda_m(t) = 2\pi \sqrt{6\Sigma/(1-\alpha)\rho_1g(t)}$ . Linearizing eq. (1) in the vicinity of the equilibrium location "e", we obtain equation of SFL small oscillations, which shows that SFL makes *damped oscillations* with period  $\tau_{os} \approx 2\pi a_e \sqrt{(b_e - a_e)\rho_1/3\gamma_v b_e p_3}$ . This formula gives the estimation of a period of fuel globule expansion:  $\tau \approx (10^{-7} - 10^{-6})$  sec.

To find parameters in the fuel globule position of maximum expansion we apply to SFL *the theorem* of kinetic energy changing of a mechanical system, which leads to equation for the maximal bubble radius  $a_{\rm m}$ :  $1+q(a_{\rm m}/a_0)^{3\gamma_{\rm v}}=(1+q)(a_{\rm m}/a_0)^{3(\gamma_{\rm v}-1)}$ . Using method of asymptotic expansion in series by small parameter  $q=(\gamma_{\rm v}-1)/\Pi_{\rm p}$ , we find the approximate solution  $a_{\rm m}\approx a_0(1-1/(81\Pi_{\rm p})^{1/3})(3\Pi_{\rm p})^{1/3}$  for  $\gamma_{\rm v}=1.3$ . We thus conclude that *the maximal expansion of a fuel globule is determined only by the ratio*  $\Pi_{\rm p}$  of *initial pressure in a vapor bubble to pressure in the combustion chamber*. The last formula gives the following values:  $a_{\rm m}=2.88a_0$  – at  $\Pi_{\rm p}=10$ ;  $a_{\rm m}=4.28a_0$  – at  $\Pi_{\rm p}=30$ . Moreover, it yields the important expression for SFL thickness at the moment of the fuel globule maximum expansion, which determines eventually the sizes of the secondary fuel droplets:

$$h_{\rm m} = a_{\rm m} - b_{\rm m} = \left(a_{\rm m}^3 + a_0^3(1 - \alpha_{20})/\alpha_{20}\right)^{1/3} - a_{\rm m} \,. \tag{2}$$

## 4 Mechanism of the Fuel Globule Breakup

It follows from the obtained estimations that SFL is affected by the huge forces of inertia caused by its acceleration. Average SFL velocity has an order of  $\langle w \rangle = b_0/0.5\tau_{os} = 300m/\text{sec}$ ; the average acceleration is then:  $\langle g \rangle = \langle w \rangle / 0.5\tau_{os} = 3 \cdot 10^9 m/\text{sec}^2$ . These accelerations are almost one billion times greater than the gravity acceleration, and the corresponding inertia forces – force of terrestrial gravitation. At such acceleration 1kg of fuel mass is affected by the inertia force  $\approx (10^8 - 10^9)N/kg$ . When small corrugations appear on a SFL surface this force is able to tear a liquid film asunder almost instantly.

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This raises a question about instability with respect to small disturbances of Rayleigh – Taylor type on both, internal and external SFL surfaces. Obtained estimation allows formulating a hypothesis regarding *instability as a possible mechanism of SFL breaking up due to the action of huge forces of inertia caused by SFL acceleration*. Indeed, it was shown in [6,7] that in the accelerated motion of a deformed drop, which has a shape of thin liquid layer, the inertia forces are the main reason of the layer breakup. These forces perforate the layer and form eventually an aerosol cloud of small droplets. In the fuel globule case the hypothesis gives the opportunity to find key parameters of EF secondary breakup in diesel engines – the size, quantity and moment of formation of secondary droplets. The verification of the *necessary conditions*, which have to be fulfilled in the unstable disturbance performance during SFL expansion, is done below to confirm the formulated hypothesis. The analysis is carried out on the basis of studying of the motion regularities, which are obtained via numerical integration of the equation of radial SFL oscillations (1). Then, the *model of SFL perforation* is

elaborated on the basis of analogy of SFL dynamics and breakup of accelerating drop; thus, the

approximate formulae for the secondary droplets quantity and their sizes are found below.

## 5 Necessary Conditions for the Instability Realization

**5.1. Internal SFL surface**. Nevertheless, a great value of acceleration doesn't mean yet that the surface is unstable; for the latter some conditions having the necessary character must have been fulfilled. It is well-known for the Rayleigh – Taylor instability that the acceleration must be directed from less dense medium to the more dense one. The analysis has shown that the density of expanding vapor is less than the fuel one in time interval  $1.95 \cdot 10^{-8} < t < 6.18 \cdot 10^{-7}$  sec. At the same time the graph in fig. 3 testifies that the acceleration is directed from fuel to vapors within interval  $8.55 \cdot 10^{-8} < t < 5.52 \cdot 10^{-7}$  sec. Hence, at the internal surface the necessary Rayleigh – Taylor condition is satisfied within intervals  $1.95 \cdot 10^{-8} < t < 8.55 \cdot 10^{-7}$  sec.

The second necessary condition was formulated in [6] when studying the mechanism of "bag" and "claviform" modes of drop breakup. It was postulated there that for the instability performance in non-stationary flow it is necessary that the characteristic time of disturbance growth  $\tau_{in} = E/(dE/dt) \equiv \omega^{-1}$  [ $E = E_0 \exp(ihz + \omega t)$  is a disturbance amplitude] is quite small,  $\tau_{in} < \tau_{st}$ , so that the period of existence of stationary drop acceleration by a stream,  $\tau_{st} = g/(dg/dt)$ , would give the possibility for the disturbance to grow sufficiently before the stream parameters will change and thereby will alter the disturbance parameters which depend on acceleration g. The analysis of calculated in present paper dependencies  $\tau_{in}(t)$ ,  $\tau_{st}(t)$  has shown that the necessary condition  $\tau_{in} < \tau_{st}$  is satisfied during period 1.19·10<sup>-7</sup> < t < 5.19·10<sup>-7</sup> sec. Thus, uniting the two necessary conditions we conclude that *instability realization is impossible at internal SFL surface*.

**5.2. External SFL surface.** For instability at the external surface, where  $\rho_3 \ll \rho_1$ , it is sufficient that acceleration is directed to the center of a globule (i.e.  $g_b$  is negative). Figure 3 shows that this requirement is satisfied within interval  $1.31 \cdot 10^{-7} < t < 5.07 \cdot 10^{-7}$  sec. At the same time the second necessary condition,  $\tau_{in} < \tau_{st}$ , is fulfilled at  $1.61 \cdot 10^{-7} < t < 4.76 \cdot 10^{-7}$  sec. Thus, simultaneous performance of two necessary conditions is possible at  $1.61 \cdot 10^{-7} < t < 4.76 \cdot 10^{-7}$  sec.

Even formulation of sufficient conditions is complicated. Apparently, in quasi-stationary environment the proper growth of unstable disturbance is sufficient for its performance. Let us introduce the *amplifier factor*  $n = \tau_{in}/\tau_{am}$ , which shows the number of  $e^n$ -fold growth of the disturbance amplitude during the period  $\tau_{am}$  when the condition  $\tau_{in} < \tau_{st}$  fulfills. Obviously, the amplifier factor essentially must be n > 1, but the sufficient value, when exponential rate of the disturbance enlargement is accounted for, can make n = (7-10). In the presented example, value of induction time of disturbance is so small,  $\tau_{in} \approx 1.6 \cdot 10^{-8} \sec(n \approx 20)$ , that it provides  $\approx e^{20} \approx 5 \cdot 10^8$ -fold growth of the disturbance amplitude. Thus, it is obvious that *at an external SFL surface the mechanism of instability has high*  opportunity of the realization at the time moment close to the greatest SFL expansion, when its acceleration and the corresponding inertia forces are maximal.

Dependency  $n(\alpha_{20})$  is given in fig. 4. The graph shows that at  $\Pi_p=10$  and for EF with  $\alpha_{20}<0.05$  the condition  $\tau_{in} < \tau_{st}$  is satisfied in a too narrow time interval,  $\tau_{am} < \tau_{in}$ , therefore the micro-explosions are impossible. The same is valid for EF with  $\alpha_{20}<0.10$  at  $\Pi_p=20$  and with  $\alpha_{20}<0.01$  at  $\Pi_p=30$ .



Figure 4. Amplifier factor  $n(\alpha_{20};\Pi_p)$ .



Figure 5. SFL perforation by aperiodic instability.

## 6 The Model of SFL Perforation and the Secondary Dispersion

Since the experimental data about secondary dispersion are absent, we will use the analogy of the SFL bursting with "claviform" mode of drop breakup. It is known that the drop in air stream initially deforms into a thin flat disk and then the action of the inertia forces of acceleration launches up the instability mechanism: the air channel is being blown in the direction of disk acceleration, while liquid stem is being pulled out in the opposite direction. SFL plays the role of a disk in the case of a fuel globule, so at a stage of deceleration huge inertia forces pull the liquid stems out in outward radial direction making a "hedgehog" form (fig. 5). The "opposite" phase of unstable disturbance forms wide air bubbles in inward counter-direction at air penetration through SFL. Action of inertia forces is similar to the action of gravity and tends to exchange places the liquid in SFL and air out of a globule. These processes will continue until the thinning fuel film bursts. The decrease of SFL thickness will go on from value  $h_{\rm m}$  [eq. (2)] at  $t=t_{\rm m}$  to a value  $h_{\rm bs}$  at the moment of bursting. In order to find sizes of droplets, which are formed as a result of SFL perforation, we use the estimations obtained within the model of perforation of a thin film of "claviform" [8]: we put  $h_{\rm bs} \approx 0.1 h_{\rm m}$ ; then the droplet diameter makes  $d_f \approx (2-4)h_{bs} \approx 0.3h_m$ . Calculated dependences  $\lambda_{bm}(\alpha_{20};\Pi_p)$ ,  $h_m(\alpha_{20};\Pi_p)$  are given in fig. 6. Thus, at SFL perforation a fuel globule surface is divided by unstable disturbances into a system of axi-symmetric "cells"; each cell consists of a liquid stem and surrounding air bubble (fig. 5). When

disturbance grows, the liquid flows from film into the stem, so, the former becomes thinner and the latter is lengthening. Thus, these cell elements form droplets of various sizes at bursting, depending on stem diameter and film thickness. The total droplet amount at a micro-explosion is determined also by



Figure 6. From left to right: dependencies  $\lambda_{bm}(\alpha_{20};\Pi_p)$ ,  $h_m(\alpha_{20};\Pi_p)$ ,  $N_s(\alpha_{20};\Pi_p)$ ,  $N_f(\alpha_{20};\Pi_p)$  for  $k_s = 0.4$ .

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the cell quantity, which is equal to the ratio of SFL surface at the bursting moment to the single cell area:  $N_c = 4\pi b_m^2 / \lambda_{bm}^2$ ;  $\lambda_{bm}$  being the wavelength of a destroying disturbance at external SFL surface. Let  $k_s$  be a stem part of a total cell volume  $V_0 = h_m \lambda_{bm}^2$ . Then the quantity of the droplets formed by one stem is  $n_s = k_s h_m \lambda_{bm}^2 / (\pi/6d_s^3) \approx 71k_s h_m / \lambda_{bm}$ ; their diameter is  $d_s = 0.3\lambda_{bm}$ . As well, the film forms  $n_f = (1-k_s)h_m \lambda_{bm}^2 / (\pi/6d_f^3) \approx 71(1-k_s)\lambda_{bm}^2 / h_m^2$  droplets of diameter  $d_f = 0.3h_m$ . Now we can estimate the total number N of the droplets formed at a fuel globule micro-explosion:

an estimate the total number *W* of the droplets formed at a fuel globule inicio-explosion.

$$N = N_{\rm s} + N_{\rm f} \,. \tag{3}$$

Here  $N_{\rm s} = n_{\rm s}N_{\rm c} = 71k_{\rm s}h_{\rm m}/\lambda_{bm}\cdot 4\pi b_{\rm m}^2/\lambda_{bm}^2 = 284k_{\rm s}\pi h_{\rm m}b_{\rm m}^2/\lambda_{bm}^3$ ,  $N_{\rm f} = n_{\rm f}N_{\rm c} = 71(1-k_{\rm s})\lambda_{bm}^2/h_{\rm m}^2\cdot 4\pi b_{\rm m}^2/\lambda_{bm}^2 = 284(1-k_{\rm s})\pi b_{\rm m}^2/h_{\rm m}^2$ . Dependences  $N_{\rm s}(\alpha_{20})$ ,  $N_{\rm f}(\alpha_{20})$  for various  $\Pi_{\rm p}$  and  $k_{\rm s} = 0.4$  are given in fig. 6.

#### 7 Conclusions

The mathematical model of the emulsified fuel drop micro-explosion is elaborated for the case, when the fuel globule contains one water drop. Hydrodynamic instability is validated as a mechanism of destruction. The enclosed model of spherical fuel layer perforation is elaborated based on analogy of SFL dynamics and breakup of a deformed accelerating drop. Approximate formulae for the secondary droplets' number and sizes are found [eq. (3)]. These are the main quantities for calculation of further processes of homogeneous gaseous mixture preparation in the combustion chamber of a diesel engine. Micro-explosion occurs at the moment close of the fuel globule maximum expansion. It means that the formed droplets of secondary dispersion have a small initial velocity in the combustion chamber. Small value of characteristic time of the process  $t_m$  means that micro-explosion is limited by a period

of a fuel globule heating up to the boiling temperature of water in the engine combustion chamber. At  $\Pi_p$  and  $\alpha_{20}$  decreasing, the efficiency of micro-explosion decreases, together with the number of secondary droplets, which is due to lessening of the specific internal explosive energy of the vapor phase. Figure 4 shows that in EF with  $0.01 < \alpha_{20} < 0.05$  at  $\Pi_p = 10$  and with  $0.01 < \alpha_{20} < 0.02$  at  $\Pi_p = 20,30$  the micro-explosions may be unsuccessful.

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