Turbulent flames in astrophysical combustion

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1 Turbulent combustion in astrophysical systems

Combustion processes drive the evolution of many astrophysical systems [1]. In particular, the evolution of stars is due to an interplay of gravitation and nuclear conversion processes. As stellar objects are on a macroscopic scale described by gas dynamics, their modeling is based on the corresponding balance laws of mass, momentum and energy, including (in the simplest case) source terms due to gravity and nuclear reactions. This constitues the description of combustion in stellar objects – depending on the scales under consideration possibly combined with "microphysical" models for energy transport phenomena.

Compared with terrestrial combustion setups, astrophysical systems comprise a much wider range of spatial and temporal scales. In particular, the huge spatial extent and the large velocities typically found in stellar plasma contrast the rather ordinary viscosity of the material, so that the Reynolds numbers of flows are huge (on the order of 10^{10} to 10^{14}), justifying the use of the Euler equations for modeling hydrodynamics and indicating the important role of turbulence in astrophysical combustion processes.

Another peculiarity of astrophysical combustion is due to the extreme conditions of matter encountered here – expressed in an appropriate astrophysical equation of state that may diverge significantly from an ideal gas law. An example is thermonuclear burning in white dwarf (WD) stars, compact objects that are stabilized against gravitational collapse by electron degeneracy. Here, the Lewis numbers are

$$Le \approx 10^7$$

and the Prandtl numbers are

$$Pr \approx 10^{-5},$$

[2] highlighting the extraordinary efficiency of energy transport by conduction in WD matter. The high degeneracy of the material leads to additional features. The density contrast between burnt and unburnt material is typically much smaller (characterized by Atwood numbers of $\lesssim 0.5$) than in terrestrial combustion systems. This is because the energy release only partially lifts the degeneracy of the material.

In astrophysical processes, energy conversion is due to nuclear reactions. Although these can be described in similar ways as chemical reactions, there are several distinct features. Most importantly, for nuclear reactions, no "oxidizer" is required. In this sense, it resembles premixed combustion. Nuclear

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reactions may be induced by conditions of high pressure of temperature. In the latter case, that we focus on here, the reaction rates are typically very sensitive to temperature. As an example, we consider again thermonuclear burning in WD material, consisting of carbon and oxygen. The rate of carbon-carbon fusion scales with T^{20}. This restricts burning to a thin layer of material and gives rise to the phenomenon of combustion waves.

The high sensitivity of thermonuclear reaction rates leads to extremely thin flame structures. Therefore, combustion waves can be described as sharp discontinuities between fuel and ash. In this model, the equations of hydrodynamics allow for the two well-known modes of propagation: as subsonic deflagration mediated by thermal conduction (due to the degenerate electron gas in case of thermonuclear burning in WDs) and supersonic detonations driven by shock waves. Here we will focus on the modeling of of astrophysical deflagration flames.

Of particular interest in astrophysical systems is the efficiency of burning. It decides on the success of a process to lead to an observable astrophysical explosion. Therefore, flame acceleration by interaction with turbulence has received much attention, in particular

2 Flame modeling

On scales of the stellar object, numerical simulations are not able to resolve the inner flame structure in the foreseeable future. This large separation of spatial scales makes it necessary to decide whether the aim of the modeling is to resolve the flame structure and the microphysical processes, or to capture the overall combustion dynamics on the large scales, that in the end is essential for the observable astronomical event.

Laminar thermonuclear flames have been studied in "microphysical" resolved simulations or analytical approaches, e.g. [2–4]. Also, simulations have been performed that test the response of these flames to turbulence [5–10].

These "small scale" flame models naturally are unable to capture the large-scale dynamics in the stellar burning process that affects the flame itself. Hydrodynamic instabilities and the interaction of flame propagation with self-generated turbulence are fundamental for the event's dynamics, the evolution of the physical phenomenon and the implications for astronomical observables.

Apart from the generation of turbulence on large scales, an important global effect in astrophysical explosions is the expansion of the material leading to a continuous decrease in the density of the fuel material. Depending on the astrophysical object under consideration, different regimes of turbulent combustion are encountered or a sequence of different regimes is traversed in the evolution.

Again, a well-known example for the latter case are deflagrations in thermonuclear supernovae [11]. For simulations of such events, detailed flame models have been developed: After determining flame properties in microphysical studies (e.g. [2, 9]), large-scale simulations require to describe flame-turbulence interaction in the flamelet regime in an efficient way. For burning in the flamelet regime, this is implemented in an approach that treats the flames as sharp discontinuities based on the level-set technique [12, 13], combined with a subgrid-scale model to describe turbulence effects [14]. Alternatives to this approach include an advection-diffusion-reaction model for flames [15] combined with a buoyancy model for determining the turbulent flame speed [5].

3 Numerical implementation and examples of application

A particular problem is the thermonuclear explosion of Type Ia supernovae as the result of combustion in carbon-oxygen white dwarf stars (for reviews see [16, 17]). Similar concepts apply to models for other

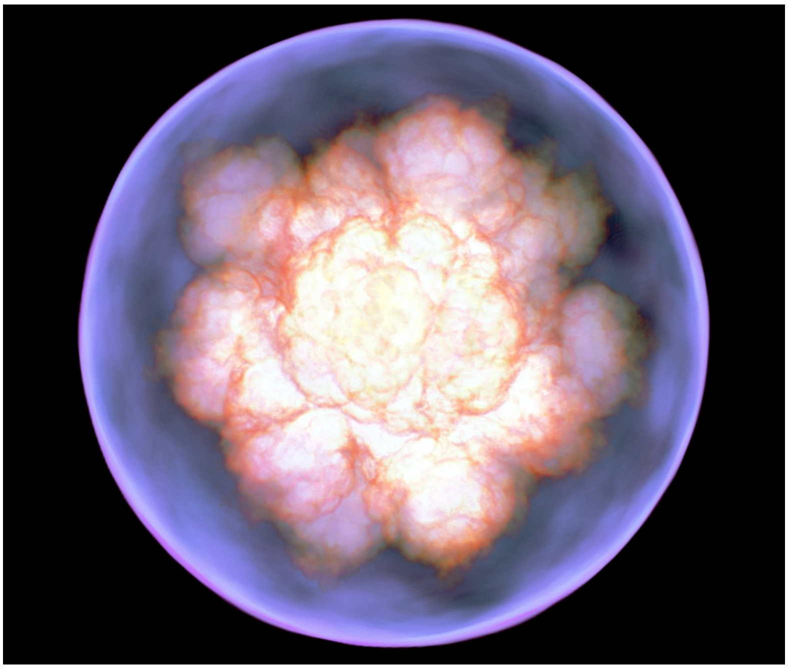

Figure 1: Deflagration in a Chandrasekhar-mass carbon-oxygen white dwarf star (for details see [22]).

astrophysical transients, but the field of Type Ia supernovae is of particular importance for astrophysics because of their prominent role in observational cosmology (facilitating the measurements that led to the discovery of the accelerated expansion of the Universe [18, 19], Nobel prize in Physics 2011) and in the chemical evolution of galaxies (main source of iron in the Universe).

Details of the astrophysical process are still uncertain because the progenitor system of Type Ia supernovae is still not identified by astronomical observations. This leads to the inconvenient situation of missing initial conditions for the modeling. Therefore, a goal is to construct synthetic observables from explosion simulations that assume a progenitor state and then validate this assumption from comparison with observations. This is possible only if modeling is as parameter-free as possible. Multi-dimensional hydrodynamical simulations of thermonuclear combustion in WD stars led to substantial progress in the field.

Multi-dimensional hydrodynamical simulations of turbulent thermonuclear deflagrations in WDs have been performed with the level-set/sub-grid scale turbulence model approach (e.g. [20–23], see Fig. 1 for an example) and with advection-diffusion-reaction approaches (e.g. [24–27]). Recent simulations indicate that pure turbulent deflagrations in carbon-oxygen white dwarfs cannot account for normal Type Ia supernovae but provide a model for the subclass of 2002cx-like events [28, 29].

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