Modeling of Non-stationary Gas Flow in Annular Nozzle

Vladimir A.Levin[1] Nataliya E.Afonina[1], Valeriy G.Gromov[1], Ivan S.Manuylovich[1],
Gennadiy D.Smekhov[1], Aleksander N.Khmelevsky[1], Vladimir V.Markov[2]
[1]Institute of mechanics of the Moscow State University, 119192, Moscow, Russia
[2]V.A.Steklov Mathematical Institute, RAS, 119991, Moscow, Russia

1 Introduction

Search for new nozzle devices that can compete in jet engines with traditional Laval nozzles, is one of the promising directions on a way of improvement of their overall-mass and specific characteristics. In this respect the certain interest are represent annular and dual-slotted nozzles with deflector in the form of a spherical segment. As have shown experiments [1, 2], there are various modes of gas flow in such nozzle devices. In steady-state operation, they belong to the class of nozzles with the central body [3, 4]. In non-stationary periodic pulsing operating modes specified nozzles represent high-frequency pulsed outlet devices. Such devices are considered to be promising for realization of pulsed detonation regime of fuels combustion [5]. In this paper, based on the Navier-Stokes equations, it is carried numerical parametric study of the influence of various factors on the development over time of the initial perturbations caused by the device start, blown by air, under a laminar flow model. In the calculations parameters of the numerical scheme and determinative conditions of problem were varied. As a result, in the calculation first discovered the flow regimes in which the starting perturbation accompanying the start of the annular nozzle, do not decay, and pass into the quasi-periodic regime. Determined the Fourier frequency spectrum of the pressure fluctuations in the center of the deflector thrust wall and the value of nozzle thrust. Showed typical pulsed pressure signals obtained in the computational model and registered in the experiments that are performed in a pulsed aerodynamic setup using as a working gas air and combustion products of stoichiometric air-acetylene mixture.

2 Gas phase model

Basic assumptions.

- The air is considered as an ideal one-temperature mixture of molecular oxygen and nitrogen with constant values of molar concentration of the mixture components $X_{O2} = 0.21$ and $X_{N2} = 0.79$.
- For the description of molecular transport the approximation Navier-Stokes is used.

The equation of state. The thermal equation of a condition has the form

$$p = \rho R_u T / m \quad ,$$

where R_u - a universal gas constant, m -average molecular air weight.

<u>Thermodynamic model</u>. Rotational and vibration energy modes of air components described by equilibrium model "rigid rotator-harmonic oscillator" with characteristic vibration temperatures $T_{v,O2}$= 2228 *K* and $T_{v,N2}$= 3336 *K.* In this case, the internal energy per unit mass of gas e and heat capacity c_p determined by the expressions

$$e = \frac{5}{2}\frac{R_u}{m}T + \frac{R_u}{m}\sum_k \frac{T_{v,k}X_k}{\exp(T_{v,k}/T) - 1}; \qquad c_p = \frac{7}{2}\frac{R_u}{m}T + \frac{R_u}{m}\sum_k \frac{(T_{v,k}/T)^2 \exp(T_{v,k}/T)X_k}{[\exp(T_{v,k}/T) - 1]^2};$$

<u>Model of molecular transport</u>. Molecular flow impulse tensor $\hat{\tau}$ associated with the strain rate tensor $\hat{\varepsilon}$ as ratio

$$\hat{\tau} = -\mu\,\hat{\varepsilon}\,,$$

where the tensor components $\hat{\varepsilon}$ are of the form

$$\varepsilon_{ij} = \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3}\frac{\partial u_k}{\partial x_k}\delta_{ij}\,,$$

and dependence of factor of molecular viscosity on temperature is approximated by a power-law formula $\mu = a_\mu T^{0.683}$.

Molecular heat flux $\vec{q}$ is defined by expression

$$\vec{q} = -\lambda\frac{\partial T}{\partial \vec{r}}\,,$$

where the coefficient of thermal conductivity λ is determined by μ, c_p, and molecular Prandtl number $\mathrm{Pr} = 0.7$:

$$\lambda = \frac{\mu c_p}{\mathrm{Pr}}$$

3 Basic equations and a numerical method

<u>Basic equations.</u> The Navier-Stokes equations in integral form describing unsteady axisymmetric gas flow in a cylindrical coordinate system $(x \geq 0, y, \varphi)$, have the form

$$\frac{\partial}{\partial t}\int_S \mathbf{U}x dS + \int_{\delta S}\vec{n}\cdot\vec{\mathbf{F}}x dl = \int_S \mathbf{\Omega} dS,$$

where *S* – the fixed control region in a meridian plane (*x, y*), δ*S* – border area, $\vec{n} = (n_x, n_y)$ - unit outward normal to δ*S*, **U** – a set of conservative variables per unit volume, $\vec{\mathbf{F}} = \vec{\mathbf{F}}^{inv} + \vec{\mathbf{F}}^{vis}$ - the sum of inviscid and viscid flows **U** through a unit area boundary area, $\mathbf{\Omega}$ consists of the source terms in the unit volume. For the considered gas-phase model there are

$$\vec{\mathbf{F}} = \begin{Bmatrix} \rho\vec{u} \\ \rho u\vec{u} + p\vec{n}n_x \\ \rho v\vec{u} + p\vec{n}n_y \\ \rho\vec{u}h_0 \end{Bmatrix} + \begin{Bmatrix} 0 \\ \vec{\tau}_x \\ \vec{\tau}_y \\ \vec{q}_h + u\vec{\tau}_x + v\vec{\tau}_y \end{Bmatrix}$$

$$\mathbf{U} = \{\rho, \rho u, \rho v, \rho e_0\}^{\mathrm{T}};$$

$$\mathbf{\Omega} = \{0, b_x, 0, 0\}^{\mathrm{T}}.$$

Here u, v - velocity vector components $\vec{u}$, $e_0 = e + 0.5(\vec{u} \cdot \vec{u})$ - total energy per unit of gas mass, $h_0 = e_0 + p/\rho$ - total enthalpy, $\vec{\tau}_x = (\tau_{xx}, \tau_{xy})$, $\vec{\tau}_y = (\tau_{yx}, \tau_{yy})$, $b_x = p + 2u/x - 2/3 div\vec{u}$.

Numerical method. Unsteady gas motion equations are solved numerically using an implicit difference scheme constructed by the finite volume method on a single block structured grid. In this approach, the system of difference equations consists of numerical analogs of the conservation equations for tetrahedral cells, covering a rated area, and difference approximations of boundary conditions. The equations are written with respect to the values of input variables $\mathbf{Z} = \{p, u, v, T\}$ in the cells centers and in the side cells centers lying on the body surface. Grid cells are formed by the intersection of two discrete families of curves. Inviscid fluxes $\vec{\mathbf{F}}_G^{inv}$ across cell boundaries are computed by solving the exact Riemann problem about the decomposition of an arbitrary discontinuity $\mathbf{Z}_G = \Re(\mathbf{Z}_G^L, \mathbf{Z}_G^R)$, where $\Re$ - is the task solution operator. In the scheme of the first order of accuracy with respect to the spatial variables left $\mathbf{Z}_G^L$ and right $\mathbf{Z}_G^R$ boundary values of input variables are set equal to their values at the center of the corresponding cell. In the scheme of the second order of accuracy $\mathbf{Z}_G^L$ and $\mathbf{Z}_G^R$ are determined using one-dimensional interpolation (extrapolation) of values $\mathbf{Z}$ at the centers of near-by cells on the given boundary between cells using a limiter *minmod*. Viscous flows through the cell faces are computed using the central or one-sided difference formulas of second-order accuracy. Time derivatives are approximated by one-sided two-point formulas of first-order accuracy or one-sided three-point formulas of second-order accuracy. At each time step difference equations are solved using a two-layer iterative scheme.

4 Results of calculations and experiments

The calculations are performed for the flow domain including:
- the deflector (thrust unit) with thrust wall in form of a spherical segment with radius of 36 mm and a height of 22 mm (base diameter d = 66.4 mm);
- input annular nozzle diameter d with a height of the critical section h = 4.4 mm;
- exhaust cone nozzle with half-angle 45° and a length of 15 mm;
- sufficiently large fly away region.

The calculations were performed on grids with the number of points of 100x316, 200x376 and 300x472. Grid nodes were concentrating near the surface of the thrust wall so that there were no less than 10 nodes in the boundary layer region. The boundaries of the computational domain and the distribution of the nodes of computational mesh 100x316 for early presented geometric nozzle parameters are shown in Figure 1. It was assumed that the blowing air through the annular nozzle occurs at the sound speed u_s at given constant stagnation pressure and temperature P_0 and T_0. The air expiration from the device through the exhaust conical nozzle occurs in the gas medium with a pressure p_e = 0.01 atm and temperature T_e = 300 K. For the problem closure at the outer boundary are used soft boundary conditions of extrapolation type. The surface of the thrust wall is assumed cooled to the temperature T_w = 300 K, or thermally isolated.

By hypothesis, the starting of the device, initially filled with stagnant air, occurs instantly, leading to the generation of intense unsteady gas dynamic processes and significant increase of pressure.

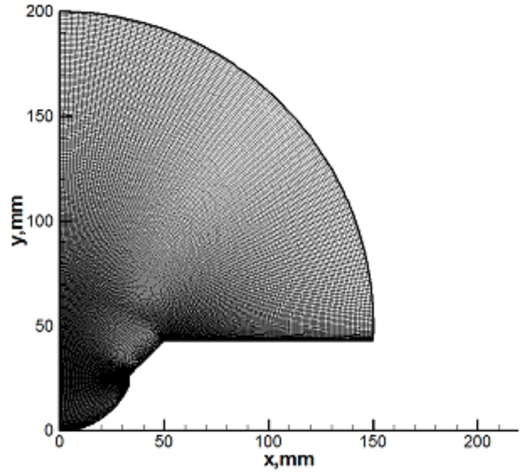

Figure 1. Configuration of rated domain and computational grid 100x316.

As the results of the calculations, the further development of the start perturbation depends significantly on both the numerical scheme, and the determining parameters of the problem.
The most radical influence on the solution has the order of the difference scheme accuracy in the spatial coordinates. For all considered difference grids using first-order schemes the initial perturbations are damped. The settling time of a stationary solution increases with increasing "density" of the grid. In the calculations due the scheme of the second order of accuracy the initial perturbations in most cases converted to quasi periodic regime. The solution in the calculations of this scheme was established only in the case in which the grid step near the surface of the thrust wall exceeds a certain critical value. Less influence on the solution has order accuracy of scheme on time, as well as the choice of values parameters *minmod* limiter. As an example, Figure 2 and Figure 3 show the time pressure dependence p_a in the central point of the thrust wall, and a force of the thrust $\boldsymbol{D}$ in the case of blowing cold air (P_0 = 19.8 atm, T_0 = 300K).

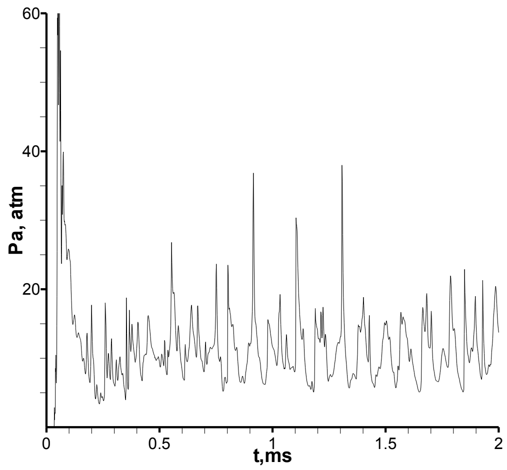

Figure 2. Dependence on time of pressure in the central point of a thrust wall at T_0 =300K.

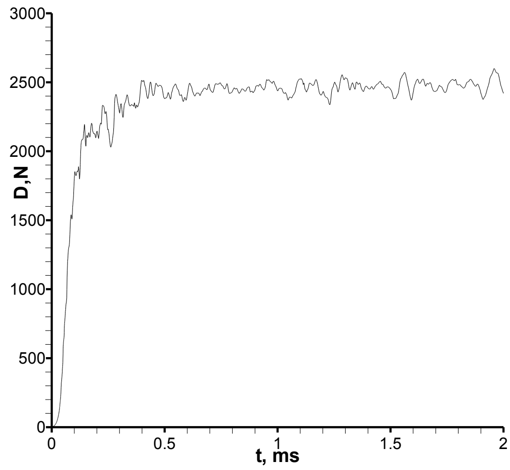

Figure 3. Dependence on time the thrust force at T_0 =300K.

The oscillation amplitude is significantly increased at blowing heated air, imitating by combustion products of stoichiometric air-acetylene mixture. On Figure 4 and 5 (similar to Figure 2 and 3) data for T_0 =3000K are presented.

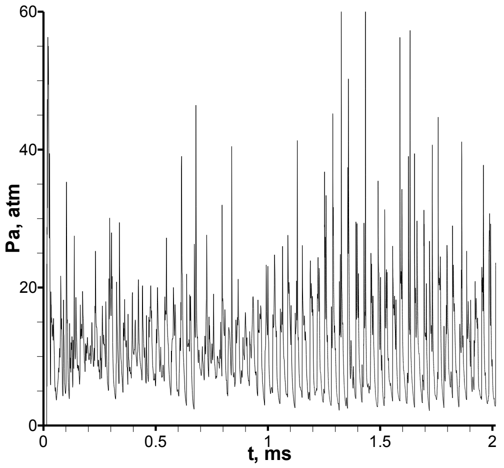

Figure 4. Dependence on time of pressure in the central point of a thrust wall at T_0 =3000K.

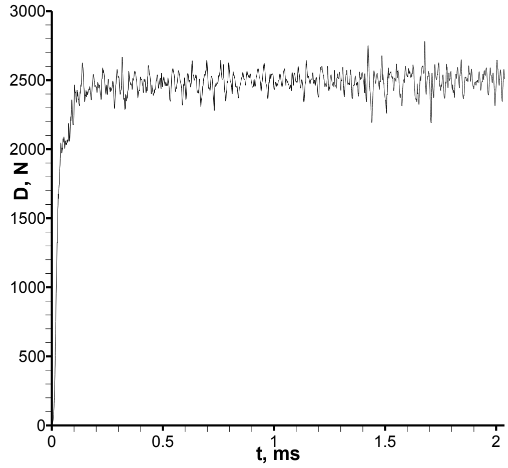

Figure 5. Dependence on time the thrust force at T_0 =3000K.

Fourier analysis of time series for a number of parameters of the numerical solution shows that the spectrum of the fundamental frequency at T_0 = 300 K is in the range of 2-30 kHz. That corresponds, firstly, to the experiments results in measuring the thrust and pressure, which are made in the aerodynamic pulse setup [1] and presented in Figure 4 and 5, and secondly, estimations based on the sound velocity and the radius of the annular nozzle.

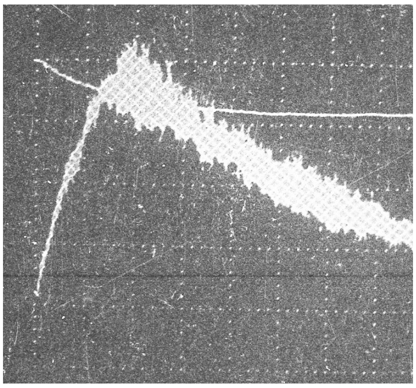

Figure 6. Dependence on time of pressure in the central point of a thrust wall (bottom beam) T_0=300K, P_0=7.1 atm. Sweep velocity 5 ms/div. Nozzle was blown by air.

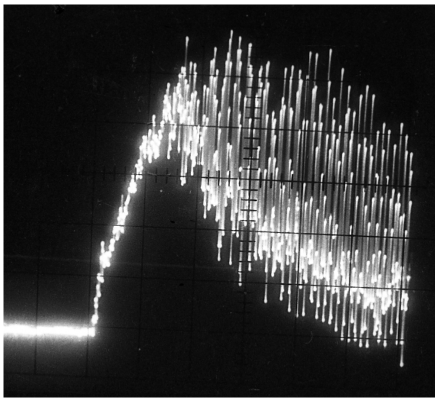

Figure 7. Dependence on time of pressure in the central point of a thrust wall at T_0=3000K, P_0=20.2 atm. Sweep velocity 2 ms/div. Nozzle was blown by C_2H_2 combustion products.

These frequencies are two orders of magnitude lower than the oscillation frequency of the scheme origin. When T_0 = 3000 K, this range is extended to 100 kHz. Parameters of difference grid and numerical scheme, as well as the order of approximation of the time derivatives are affected on the spectral characteristics of the solutions appreciably.

Variation of the numerical scheme parameters considerably smaller influence on average values of the device power characteristics. For comparison, the steady-state value p_a for the first order scheme for the spatial coordinates is equal of 10.28 atm, for the second order scheme on a grid 200x376 with a large step near the thrust wall surface - 11.21 atm, the mean value on the same grid with minimal step - 11.13 atm. The corresponding values forces of the thrust are 2496, 2513, and 2471 N.

Conclusions

As a result of a numerical study of the flow in the annular nozzle discovered undamped quasiperiodic flow regimes with significant pressure pulsations on the surface thrust wall, similar to those obtained in the experiments. Oscillation amplitude increased significantly with increasing the stagnation temperature of the blown gas. These modes are obtained in calculations using the schemes of second order accuracy in the spatial variables on the grid with sufficiently detailed definition of the boundary layer. Time scans pressure values depend strongly on the parameters of the numerical scheme. Significantly less affected by the variation of these parameters on the mean power characteristics of the annular nozzle.

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References

[1] Levin VA, Perezhogin VN, Khmelevsky AN. (1995). Feature of combustion products flow structure in spherical semi-open cavity. Combustion, Explosion and Shock Waves. 31: 30.

[2] Levin VA et al. (2012). Gas dynamic and thrust in the exhaust system of jet engine with an annular nozzle Combustion, Explosion, and Shock Waves. 48: 406.

[3] Levin VA et al. (2013). Experimental and Numerical Simulation of the Flow in a Driving Module with an Annular and Linear Double-Slot Nozzle. High Temperature. 51: 681.

[4] Levin VA et al. (2013). Investigation an annular nozzle on combustion products of hydrocarbon fuels. Thermophysics and Aeromechanics. 20: 265.

[5] Levin VA, Nechaev JN, Tarasov AI. (2001) A new approach to organizing operation cycles in pulsed detonation engines. In: High-Speed Deflagration and Detonation: Fundamentals and Control. Eds. G.D. Roy, S.M. Frolov, R.W. Netzer, and A.A. Borisov. Moscow, 2001, ELEX-KM Publisher, p. 223-238, (ISBN 5-93815-003-5).