

Effects of Equivalence Ratio on Shock-Induced Combustion around Hypervelocity Blunt Projectiles in Hydrogen/Air Mixture

Kazushige Maeda and Akiko Matsuo
Department of Mechanical Engineering, Keio University,
Hiyoshi 3-14-1, Kohoku-ku, Yokohama 223-8522, Japan

1 Introduction

Numerous studies on a next-generation propulsion system such as the SCRAM jet engine have been conducted in the past few decades. Such engines involve disadvantage in terms of weight derived from combustor length to ignite supersonic combustion. Shock-Induced Combustion (SIC), which can rapidly induce supersonic combustion, has attracted interests to reduce the combustor length.

For the merit mentioned above, SIC around hypersonic projectiles has been studied by the experiment using a ballistic range and a numerical calculation. In many studies, stoichiometric hydrogen/air mixture or hydrogen/oxygen mixture were used. In a certain condition, the wave interactions between the bow shock and the reaction front can be seen in front of the projectile. These phenomena generate periodic oscillations of the reaction front and the bow shock wave. There are two kinds of oscillation regime: One is High Frequency regime (HF) in which the non-dimensional oscillation period (τ/t_{ind}) is about 1, and the other is Low Frequency regime (LF) in which the range of τ/t_{ind} is more than 3, where τ and t_{ind} represent the oscillation period and the induction time on the stagnation line, respectively. There is also Steady State regime (SS) in which the reaction front and the bow shock wave do not oscillate.

As a prediction method of an oscillation regime, Matsuo and Fujii [1] suggested the approach using the first Damköhler number (D_1) defined as the ratio of the fluid characteristic timescale (t_f) to the chemical characteristic timescale (t_c). Here, t_f and t_c are defined as

$$D_1 = t_f/t_c, t_f = D/a_2, t_c = T_2/(dT/dt)_{\max}$$

where D , a_2 , T_2 , and $(dT/dt)_{\max}$ indicate the projectile diameter, the speed of sound and the temperature immediately behind the shock wave, and the maximum value of the temperature increase per unit time obtained by time integration of the species equations in zero dimension under the constant volume assumption, respectively. Matsuo and Fujii stated that the critical first Damköhler number distinguishing HF from LF was 80 for stoichiometric H₂-air condition, and there were no attempts at applying the other equivalence ratio.

In this paper, we conducted numerical investigations on SIC around hypersonic blunt projectiles in H₂-air mixture of which the equivalence ratio equals two. The purpose of this study is to compare the flow

Table 1 Calculation conditions and results.

Case	$\phi = 2$									$\phi = 1$ [1]			
	M_0	p_0 atm	D m	τ/t_{ind}	regime	t_f μs	t_c μs	D_1	Drag J/m	t_f μs	t_c μs	D_1	regime
(1)	4.781	0.421	0.010	0.918	HF	10.04	0.2348	42.77	50.4	11.79	0.2285	51.61	HF
(2)	4.781	0.421	0.012	0.866	HF	12.05	0.2348	51.32	72.6	-	-	-	-
(3)	4.781	0.421	0.013	0.944	HF	13.05	0.2348	55.60	85.2	-	-	-	-
(4)	4.781	0.421	0.014	7.161	LF	14.06	0.2348	59.88	98.8	-	-	-	-
(5)	4.781	0.421	0.015	6.445	LF	15.06	0.2348	64.15	113.3	17.69	0.2285	77.42	HF
(6)	4.781	0.421	0.020	5.784	LF	20.08	0.2348	85.54	201.5	23.59	0.2285	103.2	LF
(7)	4.172	0.421	0.015	0.516	HF	16.58	0.2624	63.18	85.7	19.39	0.2497	77.64	HF
(8)	4.804	0.500	0.005	0.648	HF	4.953	0.1862	26.59	15.4	5.819	0.1821	31.96	SS
(9)	4.804	0.500	0.009	0.700	HF	8.915	0.1862	47.87	49.9	-	-	-	-
(10)	4.804	0.500	0.010	3.554	LF	9.906	0.1862	53.19	61.6	11.64	0.1821	63.91	HF
(11)	4.804	0.500	0.012	7.192	LF	11.89	0.1862	63.86	88.7	-	-	-	-
(12)	4.804	0.500	0.015	7.129	LF	14.86	0.1862	79.78	138.6	17.46	0.1821	95.87	LF
(13)	4.304	0.500	0.010	0.302	HF	10.71	0.2035	52.61	48.1	12.54	0.1959	64.03	HF
(14)	4.304	0.500	0.015	2.395	LF	16.06	0.2035	78.91	108.2	18.81	0.1959	96.05	LF

Projectile velocity for $\phi = 2$ mixture: (1-6) 2168 m/s, (7) 1892 m/s, (8-12) 2204 m/s, (13-14) 1975 m/s.

Projectile velocity for $\phi = 1$ mixture: (1-6) 1931 m/s, (7) 1685 m/s, (8-12) 1963 m/s, (13-14) 1758 m/s.

The initial temperature: (1-7) 293 K, (8-14) 300 K.

features for $\phi = 2$ mixture with those for stoichiometric H_2 -air condition, try to predict an oscillation regime by the method using first Damköhler number and newly suggested method using the pressure drag, and discuss the effect of the equivalence ratio on SIC.

2 Computational Setup

Our computational target is a hypersonic blunt body projectile which flies into hydrogen/air mixture. The equivalence ratio (ϕ) of the fuel gas equals two. Table 1 shows initial conditions carried out in the present study for $\phi = 2$ mixture and the flow characteristics for the predictions. Some of flow characteristics for $\phi = 1$ mixture [1] are also listed in Table 1. In this table, M_0 , p_0 , and D indicate the projectile Mach number, the initial pressure, and the projectile diameter, respectively. M_0 , p_0 , and the initial temperature (T_0) for $\phi = 2$ mixture are the same values for $\phi = 1$ mixture. The projectile velocity and the initial density are not the same between $\phi = 1$ and $\phi = 2$ mixtures. Figure 1 shows the grid and the boundary conditions used for our calculations. The number of the grid points is 601×601 , which are equally distributed in each direction.

In this paper, calculations are conducted under axisymmetric, inviscid flow, and ideal gas assumption. The governing system of equations is the two-dimensional axisymmetric Euler equation with detailed kinetics. Yee's non-MUSCL type total variation diminishing (TVD) explicit scheme [2] is employed to discrete flux vectors. Chemical source terms are treated by the point implicit method. To calculate the values of species-thermochemical properties, NASA Thermochemical Polynomials [3] are used. The hydrogen/oxygen combustion mechanisms used in this study is the modified Jachimowski's model [4], which considers 8 species and 19 reactions.

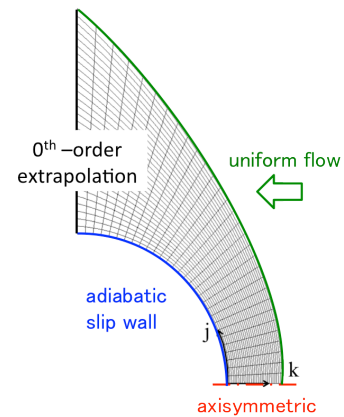


Figure 1. The grid and boundary conditions.

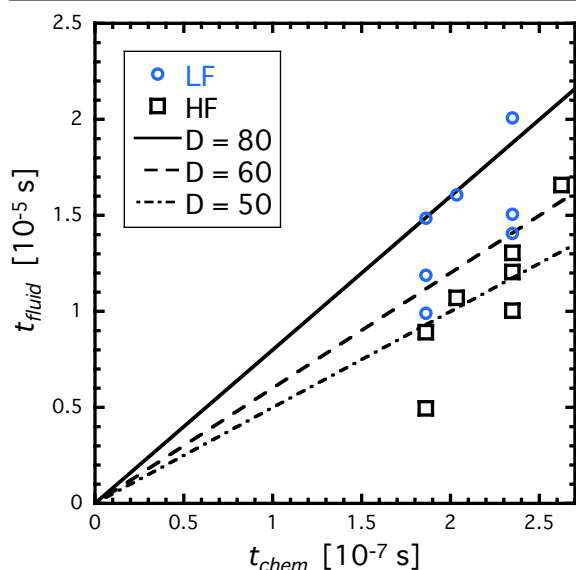


Figure 2. The prediction of a regime using D_1 for $\phi = 2$ mixture.

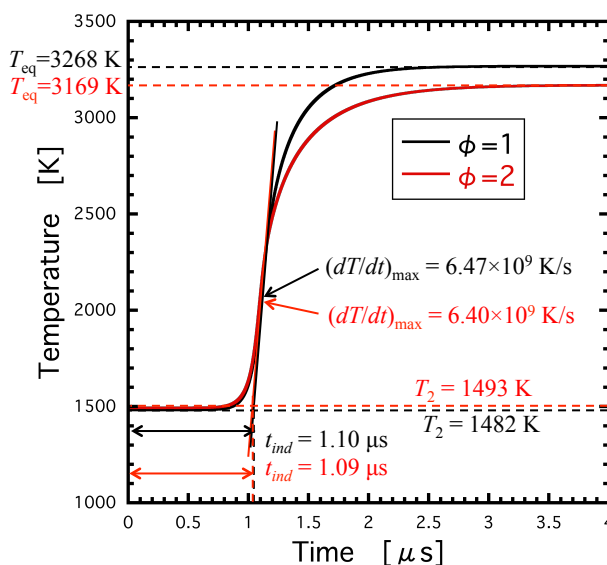


Figure 3. The time-evolving temperature profile.

3 Results and Discussion

Numerical Results for $\phi = 2$ Mixture

The oscillation regimes and non-dimensional oscillation periods obtained by our calculations are shown in Table 1. According to the results, HF changes into LF with increasing the diameter. This tendency is the same as the cases for $\phi = 1$ mixture. In comparison with the results of $\phi = 1$ mixture under the same initial conditions, LF is observed in smaller diameter. Therefore, SIC becomes unstable when the equivalence ratio is rich.

The Prediction of a Regime Using the First Damköhler Number

The result of the prediction of a regime using the first Damköhler number for $\phi = 2$ mixture is shown in Fig. 2. The solid line, the broken line, and the chain line written in Fig. 2 indicate the line for $D_1 = 80$ which distinguishes HF from LF in stoichiometric condition, the line for $D_1 = 60$, and the line for $D_1 = 50$, respectively. In Fig. 2, the criterion of critical first Damköhler number appears between the broken line and the chain line, and the solid line does not distinguish. Therefore, it is clear that the critical number which predicts a regime for stoichiometric H_2 -air mixture can not apply to the other equivalence ratio condition. The reason why the critical numbers for each mixture are different is as follow. The numbers of t_c for $\phi = 2$ mixture are almost the same as those for $\phi = 1$ mixture as seen in Table 1. Figure 3 shows the results of time integration of the species equations in zero dimension under the constant volume assumption for $\phi = 1$ and 2 mixtures of which initial conditions are $(M_0, p_0, T_0) = (4.781, 0.421 \text{ atm}, 293 \text{ K})$. In Fig. 3, the values of t_{ind} , $(dT/dt)_{max}$, and T_2 for $\phi = 2$ mixture, which are utilized for the estimating t_c , are certainly almost the same as those for $\phi = 1$ mixture. In this case, the difference of the equivalence ratio does not affect the time-evolving temperature profile. On the other hand, there is about 20 % difference between t_f for $\phi = 1$ and $\phi = 2$ mixtures as seen in Table 1. The speeds of sounds immediately behind the shock wave (a_2) for $\phi = 1$ and $\phi = 2$ mixtures, which are utilized for the estimating t_f , are 882 m/s and 996 m/s, respectively. This difference effects on the value of t_f , and the critical first Damköhler number changes.

The Prediction of a Regime Using the Pressure Drag

In the oscillatory mechanism of LF [5], the superdetonation has intermittently generated in each oscillation period. Vasiljev [6] and Lee [7] independently stated that to generate a detonation around the projectile, the drag on the projectile surface needed to exceed the critical energy required to initiate a cylindrical detonation:

$$E = 10\gamma p_0 M_{CJ}^2 \lambda^2 \text{ [J/m]}$$

where, γ , M_{CJ} , and λ indicate the ratio of specific heat, CJ Mach number, and the detonation cell size, respectively. Considering the drag might be useful to predict a regime, because the superdetonation has intermittently generated in each oscillation period of LF, as previously mentioned. So the prediction of a regime using the pressure drag in a non-reacting flow is conducted.

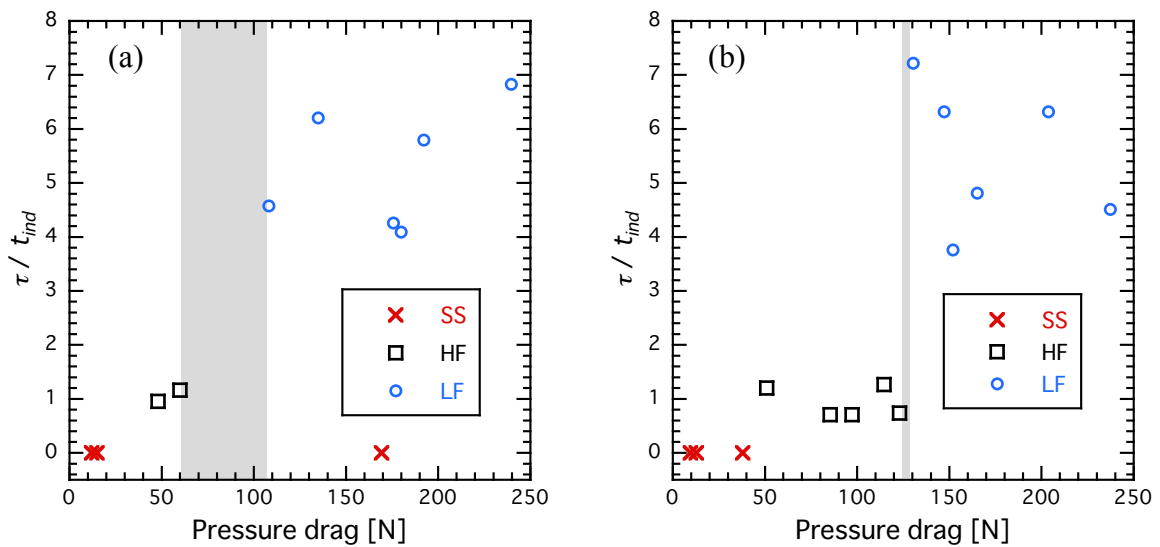


Figure 4. The prediction of a regime using the pressure drag for $\phi = 1$ mixture. (a): Initial condition is $(p_0, T_0) = (0.500 \text{ atm}, 300 \text{ K})$. (b): Initial condition is $(p_0, T_0) = (0.421 \text{ atm}, 293 \text{ K})$.

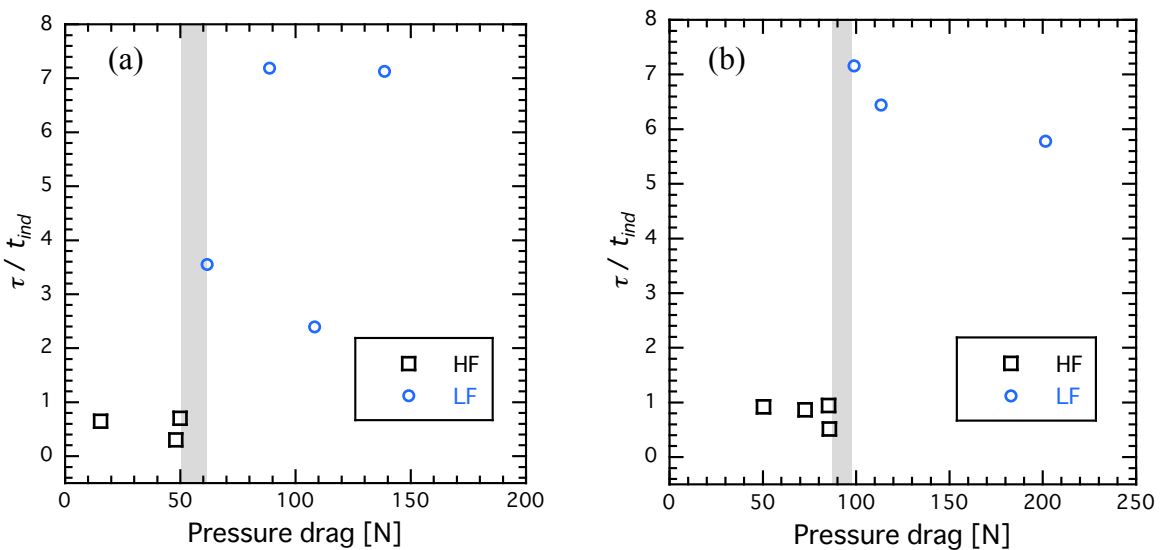


Figure 5. The prediction of a regime using the pressure drag for $\phi = 2$ mixture. (a): Initial condition is $(p_0, T_0) = (0.500 \text{ atm}, 300 \text{ K})$. (b): Initial condition is $(p_0, T_0) = (0.421 \text{ atm}, 293 \text{ K})$.

First, we examine this new approach for the calculation results of Matsuo and Fujii [1] which calculated for various conditions of the projectile velocity and the projectile diameter for $\phi = 1$ mixture. The results are shown in Fig. 4. SIC has a tendency to become unstable as the pressure drag increases. This method can evaluate the effect of the diameter and the projectile velocity on SIC for each initial condition. The range of the critical pressure drag distinguishing HF from LF is about 60 - 108 N for $(p_0, T_0) = (0.500 \text{ atm}, 300 \text{ K})$, on the other hand, that is about 125 N for $(p_0, T_0) = (0.421 \text{ atm}, 293 \text{ K})$. Generally, the detonation cell size has a tendency to become small as the initial temperature or pressure rises [8]. The critical energies required to initiate a cylindrical detonation for $(p_0, T_0) = (0.500 \text{ atm}, 300 \text{ K})$ and $(p_0, T_0) = (0.421 \text{ atm}, 293 \text{ K})$ are about 3700 J/m and 4500 J/m, respectively. The critical energy for $(p_0, T_0) = (0.421 \text{ atm}, 293 \text{ K})$ is higher than that for $(p_0, T_0) = (0.500 \text{ atm}, 300 \text{ K})$. Therefore, the critical pressure drags might be different. We nondimensionalize the critical pressure drag for each initial condition by each critical energy. The results are about 0.016-0.029 and 0.028, respectively.

In Fig. 4(a), there is one point of SS located in the LF area. This point also can not be applied for the prediction method using the first Damköhler number [1]. Matsuo and Fujii stated that the induction length of this case was long because the projectile Mach number of this case was significantly lower ($M_0 = 4.038$) than the other cases. Therefore, this case was out of range of the prediction method using the first Damköhler number. This matter is also seen in the prediction method using the pressure drag, and the work range of this approach has a limit.

Second, we conduct the prediction using the pressure drag for our calculation results. The results are shown in Fig. 5. This method can also evaluate the effect of the diameter on SIC for each initial condition same as the case for $\phi = 1$ mixture. The range of critical pressure drag are about 50 - 61 N for $(p_0, T_0) = (0.500 \text{ atm}, 300 \text{ K})$, 86 - 98 N for $(p_0, T_0) = (0.421 \text{ atm}, 293 \text{ K})$. In the case of $\phi = 2$ mixture, the critical number for $(p_0, T_0) = (0.421 \text{ atm}, 293 \text{ K})$ is also higher than that for $(p_0, T_0) = (0.500 \text{ atm}, 300 \text{ K})$ same as the case for $\phi = 1$ mixture. The critical pressure drag for $\phi = 2$ mixture are lower than that for $\phi = 1$ mixture.

Generally, the detonation cell size for $\phi = 1$ mixture is smaller than that for $\phi = 2$ mixture [8]. Therefore, the critical energy required to initiate a cylindrical detonation for $\phi = 2$ mixture is higher than that for $\phi = 1$ mixture. This fact could not explain the difference of the critical pressure drag between $\phi = 1$ and $\phi = 2$ mixtures. We suggest that the difference of the energies of the reaction shock between $\phi = 1$ and $\phi = 2$ mixtures is one of the reason why the critical pressure drags for $\phi = 2$ mixture are lower than that for $\phi = 1$ mixture. We calculate the energies under the following

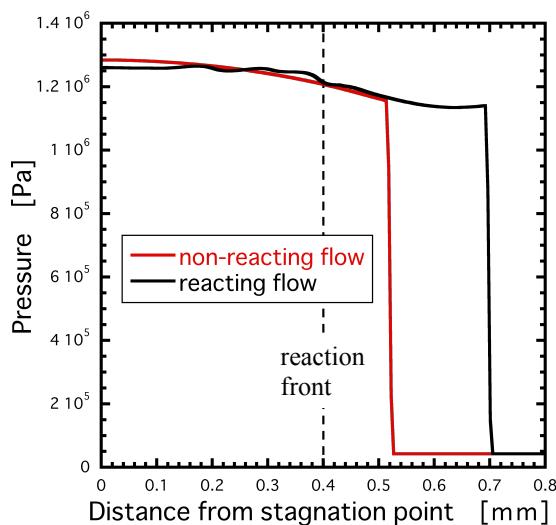


Figure. 6 Pressure distribution on the stagnation line: Initial condition is $(M_0, p_0, D, \phi) = (4.781, 0.421 \text{ atm}, 0.005 \text{ m}, 1)$.

assumptions. Figure 6 shows the pressure distribution on the stagnation line for SS and that for non-reacting flow. The broken line written in Fig. 6 indicates the location of the reaction front. In Fig. 6, the pressure profile for the reaction flow does not change significantly before and behind the reaction front. Therefore, we calculate the energies under constant-pressure combustion assumption. In constant-pressure combustion, the work of gas for unit mass (l) is calculated by the following equation:

$$l = \int_1^2 p dv = p(v_2 - v_1) = R_2 T_2 - R_1 T_1$$

where, 1 and 2 indicate the properties before reaction and those after reaction, respectively.

The properties are calculated by using time integration of the species equations in zero dimension under the constant-pressure combustion assumption. We consider the work as the energy of the reaction shock. The results are that the energies of reaction shock for $\phi = 1$ and $\phi = 2$ mixtures of which initial conditions are $(M_0, p_0, T_0) = (4.781, 0.421 \text{ atm}, 293 \text{ K})$ are $4.90 \times 10^5 \text{ J/kg}$, $5.74 \times 10^5 \text{ J/kg}$, respectively. The energy of reaction shock for $\phi = 2$ mixture is higher than that for $\phi = 1$ mixture. This is because the gas constant for $\phi = 2$ mixture is higher than that for $\phi = 1$ mixture. The difference of the energy of reaction shock might effects on the oscillation regime and critical pressure drag. The prediction method using the pressure drag can distinguish HF from LF by considering the effect of the projectile diameter and velocity. The value of critical pressure drag depends on initial conditions such as the initial pressure, the initial temperature, and the equivalence ratio. An additional modification should be done to reach a unique criterion for the prediction.

4 Conclusion

Numerical investigations on shock-induced combustion around hypervelocity blunt projectiles in hydrogen/air mixture of which the equivalence ratio equals two were carried out.

First, comparison of oscillation regime between $\phi = 1$ and $\phi = 2$ mixtures under the same initial conditions were conducted. The results show that $\phi = 2$ mixture generates LF easier than $\phi = 1$ mixture.

Second, the prediction of a regime using the first Damköhler number were carried out for $\phi = 2$ mixture. The result shows that $D_1 = 50\text{-}60$ could distinguish HF from LF, instead of $D_1 = 80$ distinguished HF from LF for $\phi = 1$ mixture.

Third, prediction method using the pressure drag was applied. This approach can distinguish HF from LF for each initial condition by considering the effect of diameter on SIC.

References

- [1] Matsuo, A., and Fujii, K., "Prediction Method of Unsteady Combustion Around Hypersonic Projectile in Stoichiometric Hydrogen-Air," *AIAA Journal*, Vol. 36, No.10, 1998, pp.1834-1841.
- [2] Yee, H. C., "Upwind and Symmetric Shock Capturing Schemes," NASA TM-89464, 1987.
- [3] Wilson, G. J., "Computation of Steady and Unsteady Shock-Induced Combustion Over Hypervelocity Blunt Bodies," Ph.D. Thesis, Dept. of Aeronautics and Astronautics, Stanford Univ., Stanford, CA, Dec. 1991.
- [4] Matsuo, A., and Fujii, K., "Detailed Mechanism of the Unsteady Combustion Around Hypersonic Projectile," *AIAA Journal*, Vol. 34, No. 10, 1996, pp. 2082-2089.
- [5] Matsuo, A., and Fujii, K., "Computational Study of Large-Disturbance Oscillations in Unsteady Supersonic Combustion Around Projectiles," *AIAA Journal*, Vol. 33, No. 10, 1995, pp. 1828-1835.
- [6] Vasiljev, A. A., "Initiation of gaseous detonation by a high speed body," *Shock Waves*, Vol. 3, 1994, pp. 21-326.
- [7] Lee, J. H. S., "Initiation of Detonation by a Hypervelocity Projectile," *Prog. Astronaut. Aeronaut.*, Vol. 173, 1997, pp. 293-310.
- [8] GALCIT Explosion Dynamics Laboratory Detonation Database, http://www.galcit.caltech.edu/detn_db/html/db.html