Propagation Velocity of Flame Balls Array in Low Lewis Number Gas Premixture

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1 Introduction

The most distinctive feature of combustion waves is its ability to assume the form of a self-sustained reaction wave propagating at a well-defined speed. A flame front or surface with maximal heat liberation located within zone of chemical reaction of combustion wave is considered generally as continuous surface although it can possesses cellular structure in some cases. Formation of non-planar cellular structure of flame front is result of development of thermo-diffusive instability. The instability is most prominent in weak near-limit low-Lewis-number premixtures sensitive to radiative heat losses. In such systems, the cellular flames often break up into separate cap-like fragments which sometimes close upon themselves to form seemingly spherical structures called flame-balls [1, 2].

The combustion wave in this case represents an array of separate flame-ball like objects in the state of permanent chaotic motion. Such combustion wave may be termed as “sporadic combustion wave” to distinguish its special characteristics differing from conventional continuous flame features. One of the unusual features of sporadic combustion wave is incomplete burning of fuel which remains in the combustion products [3]. This incompleteness is caused by fuel leakage through the gaps among the ball-like flames. The uncertainty in evaluation of total heat release related with incompleteness of combustion as well as complex spatial-temporary structure of reaction zone create difficulties in estimation of sporadic combustion wave propagation velocity. The present study is an attempt to estimate propagation velocity and to distinguish general parameters determining dynamics of sporadic combustion wave.

To make problem tractable we approximate sporadic combustion wave by planar array of ball-like flames (see Fig.1.). At the next stage of simplification we assume planar hexagonal array of flame balls. In this case due to hexagonal symmetry, the problem may be reduced to the description of single flame ball propagation in the tube with hexagonal cross section. Then the tube with hexagonal cross

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section is approximated by round tube for simplicity. We suppose that propagation velocity of a single flame ball in the tube with radius R_0 is close to velocity of flame balls array with average distance between neighboring flame balls equals to $2R_0$. The propagation velocity of flame ball arrays in turn depends on flame ball density that is inversely proportional to R_0^2 . Lets assume existence of an “optimal” density of flame balls (optimal R_0) corresponding to maximal propagation velocity and attribute this value to propagation velocity of sporadic combustion wave. This supposition stems from the possibility of self-organization of the flame balls collective. Being detached from advanced group, the trailing flame balls extinguished because of fuel deficiency and they are replaced by new ones appearing as results of leading flame balls fission.

We assume that propagation velocity of sporadic combustion wave is close to maximal velocity of a single ball-like flame propagating in the tube. A theoretical estimations of single flame ball propagating in the tube are given in the next section. We conducted also 2D numerical simulations of gas combustion in the rectangular channel with different diameters to estimate maximal propagation velocity of ball-like flame.

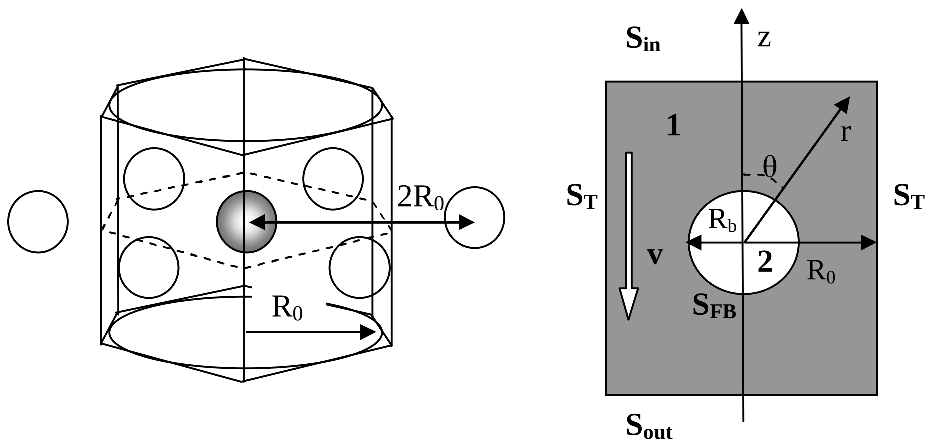

Figure 1. Left: Scheme of flame balls planar hexagonal array. Right: Scheme of flame ball propagating with velocity **v** in tube with radius R_0. (Shaded region is unburned mixture).

2 Mathematical Model

A conventional reaction-diffusion model is employed similar to that applied in paper [4]

$$-vLeC_{1z} = \Delta C_1, \tag{1}$$

$$-v(T_{1,2})_z = \Delta T_{1,2} - h(T_{1,2} - \sigma) \tag{2}$$

Here T is the scaled temperature in units of T_b, the adiabatic temperature of combustion products; C is the scaled concentration of the deficient reactant in units of C_0, its value in the fresh mixture; $\sigma = T_0/T_b$ where T_0 is the fresh mixture temperature; h is nondimensional radiative heat loss parameter and Le is the Lewis number. The symbols (z,ρ) denote cylindrical coordinates and symbols (r,θ) are radius and angle in spherical coordinates, correspondingly. The spatial coordinate is measured in units D_{th}/U_b^2 and the propagation velocity v is normalized by adiabatic flame speed U_b , where D_{th} is thermal diffusion coefficient. Indexes 1,2 correspond to unburned and burned mixtures. $\mathbf{S_{FB}}$ in Fig.1 is the flame ball surface that is assumed spherical ($r=R_b$). $\mathbf{S_T}$ is the tube walls surface ($\rho=R_0$); $\mathbf{S_{in}}$ and $\mathbf{S_{out}}$ are correspondingly the tube cross section. Equations (1) and (2) are considered in the cylinder $-\infty < z < +\infty$, $0 \le \rho \le R_0$ and subject to the following boundary conditions

Inlet $\mathbf{S_{in}}$ $(z\to+\infty)$: $T_1=\sigma$; $C_1=1$ (3)

Outlet $\mathbf{S_{out}}$ $(z\to-\infty)$: $T_1=\sigma$; $\partial C_1/\partial\rho=0$ (4)

Tube side $\mathbf{S_T}$ $(\rho=R_0)$: $\partial C_1/\partial\rho=0$, $\partial T_1/\partial\rho=0$ (5)

Flame ball surface $\mathbf{S_{FB}}$ $(r=R_b)$:

$$\frac{\partial T_2}{\partial r}-\frac{\partial T_1}{\partial r}=\frac{(1-\sigma)}{Le}\frac{\partial C_1}{\partial r};\quad \frac{1}{Le}\frac{\partial C_1}{\partial r}=exp\left(\frac{N}{2}\left(1-\frac{1}{T_F}\right)\right);\ T_1=T_2=T_F;\ C_1=0 \tag{6}$$

Here N is dimensionless activation energy of the chemical reaction.
The equations of (1), (2) can be rewritten in the following equivalent forms:

$$div(\vec{v}\,LeC-\nabla C)=0 \quad \text{or} \quad div(\exp(vLe\,z)\vec{\nabla}C)=0 \tag{7}$$

$$\Delta(T_{1,2}e^{\lambda_{\pm}z})\mp 2\kappa\,\partial\left(T_{1,2}e^{\lambda_{\pm}z}\right)/\partial z=0 \tag{8}$$

Here $\lambda_{\pm}=v/2\pm\kappa$, $\kappa=(v^2/4+h)^{1/2}$. The solutions of the equations (1), (2) and equivalent equations (7), (8) can be written in the form $C_1=C_b+C_w$, $T_1=T_{1b}+T_{1w}$, where C_b , T_b correspond to a flame ball propagating with constant velocity in free space and T_w, C_w are solutions describing distortion of temperature and concentration fields C_{1b} and T_{1b} caused by tube walls. Note that T_w, C_w have to satisfy of Eqs. (1), (2) with boundary conditions $\partial C_{1w}/\partial\rho=-\partial C_{1b}/\partial\rho$,$\partial T_{1w}/\partial\rho=-\partial T_{1b}/\partial\rho$ at the tube side $\rho=R_0$, $C_{1w}=0$ as $z\to+\infty$ and $C_{1w}=C_\infty$ as $z\to-\infty$. C_∞ is the concentration of the fuel in the downstream far field. Integrating over cylindrical volume of the four equations (7), (8) and taking into account boundary conditions (6) at the flame ball surface one can obtain the following relations:

$$\oint_{S_{FB}} exp\left(\frac{N}{2}\left(1-\frac{1}{T_F(\theta)}\right)\right)dS_{FB}=\pi R_0^2\int_{-\infty}^{+\infty}\left.\frac{\partial C_{1b}}{\partial\rho}\right|_{\rho=R_0}dz=\pi R_0^2 v(1-C_\infty) \tag{9}$$

$$\oint_{S_{FB}} exp\left(\frac{N}{2}\left(1-\frac{1}{T_F(\theta)}\right)\right)e^{vLeR_b\cos(\theta)}dS_{FB}=\pi R_0^2\int_{-\infty}^{+\infty}e^{vLe\,z}\left.\frac{\partial C_{1b}}{\partial\rho}\right|_{\rho=R_0}dz \tag{10}$$

$$\oint_{S_{FB}} exp\left(\frac{N}{2}\left(1-\frac{1}{T_F(\theta)}\right)\right)e^{\lambda_{\pm}R_b\cos(\theta)}dS_{FB}=-\pi(1-\sigma)R_0^2\int_{-\infty}^{+\infty}e^{\lambda_{\pm}z}\left.\frac{\partial T_{1b}}{\partial\rho}\right|_{\rho=R_0}dz \tag{11, 12}$$

Supposing that flame surface temperature T_F can be approximated by formula $T_F=T_B+\tau\cos(\theta)$, $(T_B-\tau<\sigma)$, the temperature and concentration distributions in the far field $r>>R_b$ corresponding to the flame ball propagating in free space assume the form [4,5]:

$$C_{1b}=1-\frac{R_b}{r}\exp\left(\frac{-vLe}{2}(r-R_b)(1+\cos(\theta))\right), \tag{13}$$

$$T_{1b}=1-\frac{R_b(T_B+\tau\cos(\theta))}{r}\exp\left(\frac{-v}{2}(r-R_b)(1+\cos(\theta))\right) \tag{14}$$

Substituting expressions (13), (14) in equations (9)-(12) one can get the system of four implicit equations that bound four variable v, R_b, T_B and τ . The solutions of these equations are discussed in the next section. Notice that in the limits $R_0 \to \infty$, $v \to 0$, $\tau \to 0$ the solutions of Eqs.(9)-(12) are converted in two equations describing motionless flame ball in free space:

$$T_F - \sigma = \frac{(1-\sigma)}{2Le}\frac{(1-\exp(-2\sqrt{h}R_b))}{\sqrt{h}R_b}, \qquad \frac{1}{LeR_b} = \exp\left(\frac{N}{2}\left(1-\frac{1}{T_F}\right)\right) \tag{15}$$

The solution of Eq.(15) yield dependencies of stationary flame ball radius R_b on heat losses parameter h evaluated at N=10, σ = 0.2 that are shown in left side of figure 2.

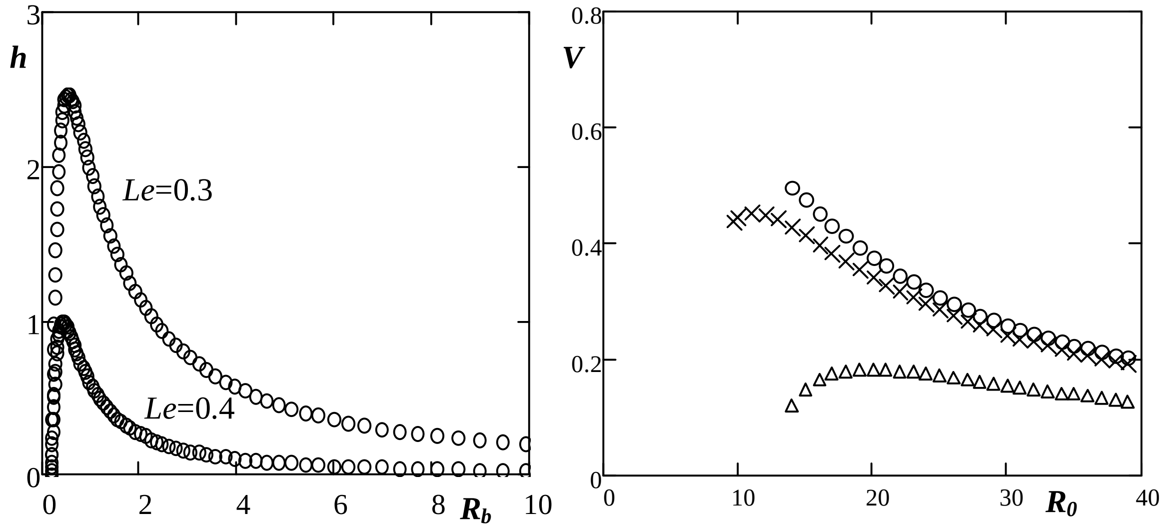

Figure 2. Left: Heat loss h dependencies on flame radius R_b, evaluated at N=10, σ = 0.2, Right: Dependencies of flame ball propagation velocity $\boldsymbol{v}$ on tube radius R_0, evaluated at Le=0.3, h=0.3 (circles), Le=0.3, h=0.5 (crests) and Le= 0.4, h=0.3 (triangles).

3 Flame ball propagation velocity

In calculations of Eq.(9-12) we set N=10 and σ =0.2. The numerical simulations revealed existence of maximal flame ball propagation velocity that is attained at a critical tube radius R_0. In the right side of figure 2 the dependencies of flame ball propagation velocity v on tube radius R_0 are presented. The left ends of the curves $v(R_0)$ correspond to the zeros residual fuel concentration C_∞ corresponding to the complete combustion. In the case Le=0.3 and h=0.3, the maximal propagation velocity is attained in the tube with minimal tube radius determined by condition C_∞ = 0. In other two cases given in figure 2 the maximal flame ball propagation velocities are attained at the points where the residual concentrations were nonzero. The dependencies of residual concentration C_∞ on tube radius R_0 are given in the left side of figure 3. In the cases Le=0.4, h= 0.3 and Le= 0.3, h=0.5 , the flame ball propagation velocities were maximal at R_0 =20 and R_0 =11, correspondingly. In this cases the residual fuel concentrations were respectively C_∞ =0.598 and C_∞ = 0.199. In the right side of figure 3 the dependencies of flame ball radius on the tube radius R_0 are shown. The results of calculations indicate that in large tubes the increase of tube diameter leads to the decreasing of the flame ball propagation velocity and to increasing of the flame ball radius. In the limit of R_0 >>1 the flame ball radius tend to the radius of the stationary flame ball in the free space.

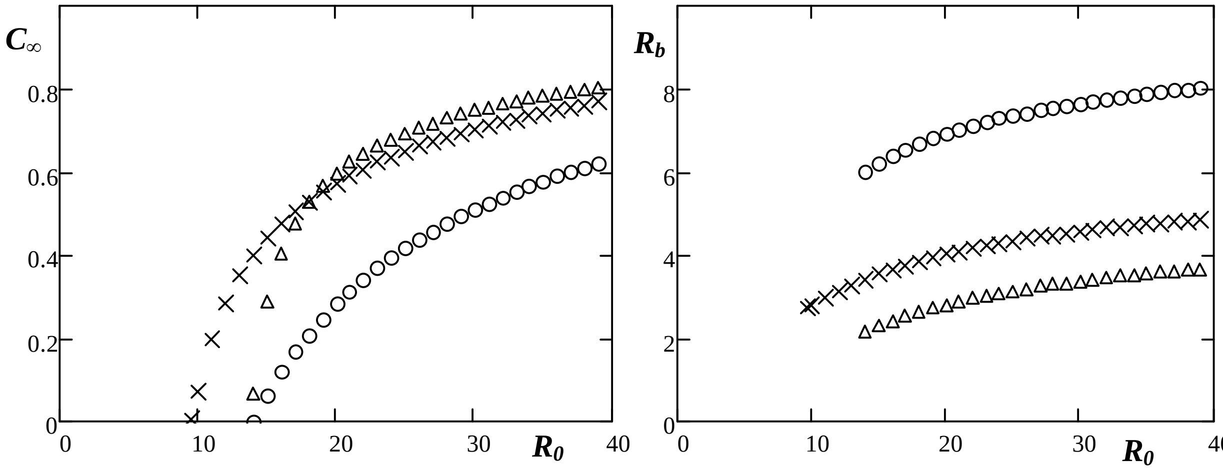

Figure 3. Left: Dependencies of residual fuel concentration $\boldsymbol{C_\infty}$ on tube radius $R_{0;}$
Right: Dependencies of flame ball radius $\boldsymbol{R_b}$ on tube radius $\boldsymbol{R_0}$, evaluated at *Le*=0.3, *h*=0.3 (circles), *Le*=0.3, *h*=0.5 (crests) and *Le*= 0.4, *h*=0.3 (triangles).

4 Conclusions

The method of evaluation of propagation velocity of a sporadic combustion wave consisted of separate ball-like flames is proposed. Recent numerical investigations of low-Lewis-number flames propagating in divergent channel [3] and the presented results demonstrate that sporadic combustion regime can leads to combustion incompleteness. The propagation velocity of a sporadic combustion wave consisting of flame balls planar array was analytically estimated by means of developed analytical method. The comparison of the analytically obtained flame ball propagation velocities with data obtained in the course of direct numerical simulations of the flame ball propagating in the tubes with different diameters will be conducted in future.

5 Acknowlegments

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