Flame Dynamics in Long Channels Open at Both Ends

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1 Introduction

In a recent publication [1] we examined the propagation of a premixed flame in a long narrow channel, open at both ends and exposed to atmospheric pressure. The gas expansion that results from the heat released by the chemical reactions produces a continuous flow of burned gas directed towards the ignition end and sets a pressure gradient that drives the fresh unburned gas towards the other end of the channel. As a result, the flame accelerates when traveling throughout the channel consistent with the early observations of Mason & Wheeler [2]. The adopted simplifications in [1] enabled extracting simple results about the flame position and the overall travel time within the channel. In this work the narrow channel assumption is removed and the two-dimensional problem is considered numerically in order to examine the effect of the channel's width on the propagation.

1 Formulation

A combustible mixture is contained in a channel of length L and width h, and ignited at time t = 0 at the left end of the channel; i.e., at x = 0. Upon ignition the diaphragms containing the mixture in the channel are simultaneously removed, and both ends remain open and exposed to a constant (atmospheric) pressure. Of particular interest is to examine the subsequent propagation in sufficiently long channels as a function of the aspect ratio. For simplicity, we consider here the adiabatic case only, leaving the influences of heat loss through the walls to future studies.



Figure 1: Sketch of the channel configuration, illustrating the various length scales associated with the flame propagation problem

The combustible mixture is considered to undergo a chemical reaction described by a global irreversible step of the form $F + O \rightarrow P$, where F denotes the fuel, O the oxidizer and P the products. Assuming the mixture is lean in fuel, the oxidizer mass fraction is nearly constant during combustion and the reaction rate depends solely on the fuel mass fraction. It is characterized by a global activation energy E and a pre-exponential factor \mathcal{B} that contains the (constant) oxidizer's concentration.

We introduce dimensionless variables using the state of the fresh mixture (identified by the subscript u) as reference: ρ_u for density, T_u for temperature, p_u for the pressure, and Y_u for the (fuel) mass fraction. If the speed S_L and thermal thickness δ_T of a planar adiabatic flame are used as reference, and the residence time δ_T/S_L is used as a unit of time, the governing equations (in dimensionless form) become:

$$\rho_t + (\rho \, u)_x + (\rho \, v)_y = 0, \tag{1}$$

$$\rho(u_t + uu_x + vu_y) = -p_x + \Pr\left[u_{yy} + \frac{4}{3}u_{xx} + \frac{1}{3}v_{xy}\right]$$
(2)

$$\rho(v_t + uv_x + vv_y) = -p_y + \Pr\left[\frac{4}{3}v_{yy} + \frac{1}{3}u_{xy} + v_{xx}\right]$$
(3)

$$\rho(\theta_t + u\theta_x + v\theta_x) - (\theta_{xx} + \theta_{yy}) = \omega \tag{4}$$

$$\rho(Y_t + uY_x + vY_y) - Le^{-1}(Y_{xx} + Y_{yy}) = -\omega$$
(5)

$$o = 1/(1 + \gamma\theta) \tag{6}$$

where x, y are respectively the longitudinal and transverse coordinates (see Fig. 1), and u, v the corresponding velocity components; subscripts denote partial differentiation. In these equations p is the pressure deviation from the ambient (atmospheric) pressure p_u which, in view of the low Mach number approximation adopted here, is constant to leading order and therefore does not appear explicitly in the equation of state (6). The variable θ is the deviation of the temperature from its ambient value normalized by $T_a - T_u$, where $T_a = T_u + QY_u/c_p$ is the adiabatic flame temperature with Q is the total heat release and c_p the specific heat (at constant pressure) of the mixture. The thermal thickness of the flame is given by $\delta_T = \mathcal{D}_T/S_L$, where \mathcal{D}_T is the thermal diffusivity of the mixture. The parameters appearing in these equations:- the heat release parameter $\gamma = (T_a - T_u)/T_u$, the Prandtl number $\Pr = \nu/\mathcal{D}_T$ representing the ratio of the viscous to thermal diffusivities of the mixture (with ν the kinematic viscosity), and the Lewis number $Le = \mathcal{D}_T/\mathcal{D}_F$ representing the ratio of the fuel \mathcal{D}_F , are all assumed constant.

The reaction rate ω takes the form

$$\omega(\theta, Y) = \frac{\beta^2}{2s_L^2 Le} \left(\frac{1+\gamma}{1+\gamma\theta}\right)^2 Y \exp\left\{\frac{\beta(\theta-1)}{(1+\gamma\theta)/(\gamma+1)}\right\}.$$
(7)

where $\beta = E(T_a - T_u)/\mathcal{R}T_a^2$ is the Zel'dovich number and $\rho_b = \rho_u T_u/T_a$ is the density of the burned gas. Since for any finite β the laminar flame speed needs to be computed numerically, we have introduced, for convenience, the corresponding asymptotic expression

$$(S_L)_{\rm asp} = \sqrt{2Le\mathcal{B}\rho_u \mathcal{D}_T/\beta^2} (\rho_b/\rho_u) e^{-E/2\mathcal{R}T_a}$$

valid for $\beta \gg 1$ and the adjustment factor $s_L = S_L/(S_L)_{asp}$. For a finite value of the Zel'dovich number β and given γ and Le, the factor s_L is determined numerically as the eigenvalue of an appropriate boundary-value problem [1]. The values of s_L for $\beta = 10$ and Le = 1 are 1.0652, 1.0588 and 1.0548 for $\gamma = 3$, 4 and 5, respectively.

The system of equations (1)-(6) are considered in the domain $0 < x < \ell$ and 0 < y < a, where $\ell = L/\delta_T$ and $a = h/\delta_T$ are the channel length and width measured in units of the flame thickness δ_T . At the channel walls we assume no-slip and adiabatic conditions, so that

$$u = v = \partial \theta / \partial y = \partial Y / \partial y = 0$$
, at $y = 0, a$ for $0 < x < \ell$. (8)

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It is also assumed, for simplicity, that the state of the mixture at both ends is uniform and the flow remains parallel to the walls; i.e.,

$$\partial \theta / \partial x = \partial Y / \partial x = v = \partial u / \partial x = 0, \quad \text{at } x = 0, \ell \quad \text{for } 0 < y < a.$$
 (9)

The latter are invariably the appropriate conditions for instants when the flame is sufficiently far from the ends of the channel. The modifications required when the flame is within a close distance $\sim O(a)$ from either end would, for sufficiently long channels, have a small and negligible effect on the entire propagation time. Since following ignition the channel remains open, the pressure at both ends is constant and equal to the ambient pressure, so that

$$p = 0$$
 at $x = 0, \ell$, for $0 < y < a$. (10)

This is consistent with conditions (9) that imply that the pressure variations across the channel at both ends are negligible.

2 Numerical procedure

It is advantageous for numerical calculations to eliminate the pressure from the momentum equations by introducing the vorticity, $\zeta = v_x - u_y$, which satisfies

$$\rho\zeta_t + \rho u\zeta_x + \rho v\zeta_y = Pr(\zeta_{xx} + \zeta_{yy}) + J \tag{11}$$

with J (the vorticity production) given by

$$J = (\rho_y u_t - \rho_x v_t) + \left[(\rho u)_y u_x - (\rho u)_x v_x \right] + \left[(\rho v)_y u_y - (\rho v)_x v_y \right],$$

and express the velocity field as the sum of irrotational and solenoidal components

$$u = \psi_y + \phi_x, \qquad v = -\psi_x + \phi_y \tag{12}$$

by virtue of Helmholtz decomposition theorem. The divergence-free part has been expressed in terms of a stream-like function ψ and the irrotational part in terms of a potential ϕ . The combined mass and energy equations (1) and (4) yield an alternative form for the continuity equation, or an equation for the potential ϕ , in the form

$$\phi_{xx} + \phi_{yy} = \gamma \left[\theta_{xx} + \theta_{yy} + \omega \right]. \tag{13}$$

The stream-like function is readily seen to satisfy

$$\psi_{xx} + \psi_{yy} = -\zeta. \tag{14}$$

The problem then reduces to solving equations (11), (13)-(14) and (4)-(5) for ζ , θ , Y, ϕ and ψ , with ρ given (6). The velocity components are obtained a-posteriori from (12), and the pressure from the momentum equations (2)-(3).

The equations were solved numerically in the domain $0 < x < \ell$ and 0 < y < a/2, by imposing symmetry boundary condition at the centerline, namely

$$\psi = 0, \quad \partial \phi / \partial y = 0, \quad \zeta = 0 \qquad \text{at } y = a/2.$$
 (15)

The remaining boundary conditions, deduced from (8)-(10), are

$$\psi = 0, \quad \partial \phi / \partial y = 0, \quad \partial \psi / \partial y = -\partial \phi / \partial x \quad \text{at} \quad y = 0$$
 (16)

$$\partial \psi / \partial x = \partial^2 \phi / \partial x^2 = 0$$
 at $x = 0$. (17)

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We note parenthetically that evaluating the x-momentum equation (2) along the wall y = 0 implies that

$$p_x = \Pr\left(-\zeta_y + \frac{4}{3}v_{xy}\right) \tag{18}$$

which, when integrated along the channel and using conditions (9)-(10) yields

$$\int_{0}^{\ell} \partial \zeta / \partial y \big|_{y=0} dx = 0.$$
⁽¹⁹⁾

This relation represents the constraint on the velocity field imposed by holding the pressure constant at both ends of the channel. This condition serves resolving the indeterminacy in the determination of the potential ϕ which, by definition, is up to a linear additive of the form $\phi + Ax$. The constraint (19) determines the constant A uniquely.

The initial conditions we have adopted are in the form of a hot spot placed near the left end of the channel, x = 0, in a fluid at rest and in a uniform state (uniform distribution for the mass fraction and density). The position of the flame front $x_f(t)$ was defined as the location where the reaction rate ω reaches its maximum value along the mid-plane; the corresponding propagation speed \dot{x}_f was subsequently determined as its time derivative.

3 Results

Figure 2 displays the main result of this study showing the propagation speed as a function of time for different channel lengths and widths. The abscissa $\hat{t} = t/\ell$ represents the (dimensional) time in units of L/S_L , namely in units of the travel time of a planar adiabatic flame throughout the same channel. The same representation was used in our previous study [1], which was limited to narrow channels ($a \ll 1$).



Figure 2: The flame propagation speed as a function of time for various channel lengths ℓ and widths a.

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The results show that the channel width has a significant effect on flame dynamic. For a channel of width a = 5, the calculated propagation speed is shown for the two values $\ell = 100$ (solid line) and $\ell = 150$ (points identified by circles). The dashed curve, plotted for $\ell = 100$, is based on the analytical expression

$$\dot{x}_f = \left(1 + \frac{\gamma}{\ell}\right) e^{\gamma t/\ell} \tag{20}$$

obtained in [1] under the assumption that $a \ll 1$. Hence, for width comparable to the laminar flame thickness, typically $5\delta_T - 6\delta_T$, the propagation speed increases monotonically and for sufficiently long channels the dependence of \dot{x}_f on the scaled time \hat{t} tends to a universal curve that is well approximated by equ. (20). Note that for a = 5 the flame acceleration is only slightly larger than the value predicted by the asymptotic expression (20). Figure 3 shows the pressure distribution along the channel walls at various times, for a = 5, $\ell = 100$, calculated a-posteriori from (18) after having determined the induced velocity field. The result is indeed consistent with our earlier predictions [1] obtained for $a \ll 1$.

The propagation speed changes drastically in wider channels. The graphs in fig. 2 show the propagation speed calculated for a = 10 and various values of ℓ . During the early stages the behavior remains nearly universal with the dependence of the propagation speed \dot{x}_f on \hat{t} still approximated by the narrow-channel result (20); the small variations due to the ignition event seen for small t disappear very soon thereafter. Beyond a certain time, however, the flame undergoes a sudden rapid acceleration with the propagation speed increasing ten-folds before reaching the end of the channel. The sudden change in propagation speed occurs earlier in the longer channels. It is associated with a change from a nearly-planar to a highly curved flame as seen in Fig. 5. Shown in figures 4 and 5 are flame shape and location at various instants during the propagation in a channel (i) of width a = 5 and length $\ell = 100$, and (ii) of width a = 10 and length $\ell = 200$. The flame in these figures is identified by the spread of constant level curves of the reaction rate ω . We observe that in narrow channels the flame remains nearly planar



Figure 3: Representative pressure distribution along the channel wall at different times; calculated for a channel of width a = 5 and length $\ell = 100$.

during the entire propagation, while in wider channels it changes dramatically to a highly curved flame convex towards the unburned gas. The sudden increase in propagation speed is clearly associated with the increase in flame surface area.



Figure 4: Instantaneous images of the flame, characterized by reaction-rate contours, during propagation in a channel of width a = 5 and length $\ell = 100$.



Figure 5: Instantaneous images of the flame, characterized by reaction-rate contours, during propagation in a channel of width a = 10 and length $\ell = 200$.

4 Conclusions

The dynamics of premixed flames propagating in long two-dimensional adiabatic channels open at both ends and exposed to atmospheric pressure have been studied numerically by fully accounting for thermal expansion resulting from the heat released during combustion. The gas expansion drives a continuous flow of hot burned gas towards the ignition end of the channel that sets as a result of viscous drag at the rigid walls a pressure gradient that enhances the flow of burned gas and simultaneously drives the fresh mixture ahead of the flame towards the other end of the channel, which remains at atmospheric pressure. As a result, the flame accelerates when traveling down the channel. In narrow channels, on the order of the flame thickness, the acceleration is found to be nearly constant. When the channel height is an order of magnitude larger than the flame thickness, the initial propagation speed increases linearly following the behavior observed in narrow channels, but changes rapidly as the flame becomes highly curved, to an exponential-like behavior associated with the Increased flame surface area. The

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dramatic increase in propagation speed results in channels of any length provided the far end of the channel remains open and the ambient held at a constant pressure. Indeed, heat losses towards the cold ambient, not considered here, will modify the flame behavior and propagation speed at distances of the order of the flame thickness of the end of the tube. Effects not considered here include the occurrence of non axisymmetric modes of propagation and the possible development of instabilities, such as Rayleigh-Taylor, associated with the acceleration of the flame surface into a denser fluid.

References

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