Numerical modeling of continuous detonation in a combustor with a plane diffuser with a supersonic flow velocity

Sergey A. Zhdan, Aleksey I. Rybnikov,

Lavrentyev Institute of Hydrodynamics SB RAS,
630090, Novosibirsk, Russia

1 Introduction

Activities aimed at developing a scientific basis for detonation engines where the fuel is continuously burned in a traveling detonation wave [1] (the scheme proposed by Academician Voitsekhovskii [2]) have been performed since the 1960s. The issue of a possibility of extension of the principle of continuous spin detonation (CSD) to a scramjet with a supersonic flow velocity was considered in [3,4], where the numerical simulations of combustion of a hydrogen-oxygen mixture provided a new solution with CSD in an annular cylindrical combustor up to the free-stream Mach number $M_0 = 3$. The goals of the present work are to generalize the problem formulated in [3,4] to the case of shock-wave compression of the incoming supersonic flow in the diffuser and to study numerically the influence of the degree of flow deceleration in the oblique shock wave on the domain of existence and properties of continuous detonation.

2 Problem formulation

A supersonic flow (flow Mach number $M_0 > 1$, pressure p_0, temperature T_0, and ratio of specific heats γ_0) passing through a supersonic diffuser (entrance cross-sectional area S_0 and exit cross-sectional area S_1) and partly decelerated under shock-wave compression to a flow with parameters p_1, T_1, and $M_1 \geq 1$ enters an annular cylindrical channel of length L_1 and width δ and then an annular combustor whose entrance has a connector of length L_2 with linear expansion of the annular channel from δ to Δ (Fig. 1). The combustor diameter is d_c, its length is L_c, the annular channel width is Δ ($\Delta > \delta$), and the free cross-sectional area is S_2. At a distance L_3 from the combustor entrance, the cylindrical channel linearly expands to the free cross-sectional area at the exit S_{ex}. Assuming that the distance between the tube walls is small as compared with its radius ($\delta_0 << d_c$; $\delta < \Delta << d_c$)and the flow parameters in the radial direction remain practically constant, the three-dimensional problem can be simplified [3] to two two-dimensional subproblems with a matching boundary Γ_1 ($x = x_1 = -L_1$ is the entrance to the annular cylindrical part of the diffuser). This simplification reduces to calculations in two rectangular domains with a matching boundary Γ_1: I) two-dimensional gas flow in the domain near the plane diffuser $\Omega_0 = (-L_0 - L_1 < x < -L_1, R(x) < z < \delta_0)$, II) two-dimensional flow in the rectangular domain $\Omega_1 = (-L_1 < x < L_c, 0 < y < l)$ with periodic boundary conditions along the coordinate direction y. The two subproblems were matched at the boundary Γ_1 by averaging gas-dynamic parameters on the left and right of this boundary by the method proposed by Sedov [5] and solving the problem of decaying of an arbitrary discontinuity for the averaged parameters of the problem.

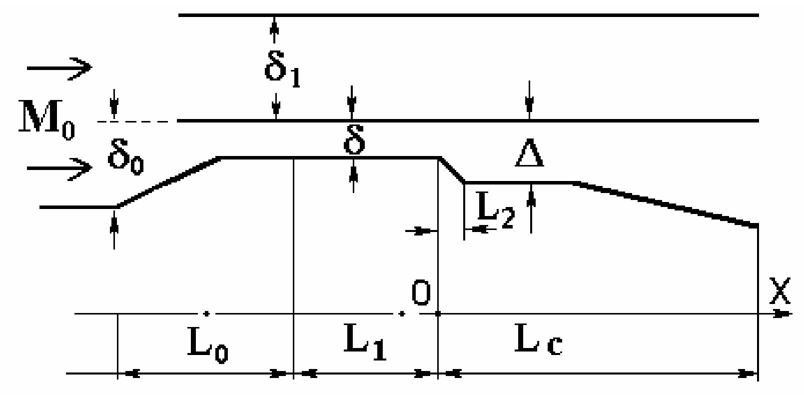

Fig. 1. Sketch of the diffuser and annular combustor.

Here (x, z) are the spatial variables of the orthogonal coordinate system in subproblem I, (x, y) are the spatial variables of the orthogonal coordinate system in subproblem II, and R(x) is the shape of the inner surface of the supersonic diffuser, which is counted at the entrance from zero along the z axis, i.e., $R(-L_0 - L_1) = 0$. The diffuser of length L_0 with the entrance cross-sectional area S_0 (vertical size δ_0) and the output cross-sectional area S_1 (vertical size δ) is defined in the form of a plane wedge with a constant angle of inclination α to the abscissa axis *x*. Then the equation of the lower boundary R(x) of the plane diffuser domain Ω_0 can be presented as

$$R(x) = \begin{cases} (x + L_0 + L_1)\mathrm{tg}\alpha, -L_0 - L_1 < x < -L_0 - L_1 + (\delta_0 - \delta)/\mathrm{tg}\alpha \\ \delta_0 - \delta, x \geq -L_0 - L_1 + (\delta_0 - \delta)/\mathrm{tg}\alpha \end{cases}. \quad (1)$$

Subproblem I in the domain Ω_0 at specified parameters of the supersonic incoming flow M_0, p_0, T_0, and γ_0 at the diffuser entrance, the boundary $\Gamma_0 = \{x = (-L_0 - L_1, 0 < z < \delta_0\}$ and no-slip conditions on the solid walls allows one to obtain the gas-dynamic parameters in the diffuser, including the parameters at the diffuser exit, the boundary $\Gamma_1 = \{x = x_1 = -L_1, \delta_0 - \delta < z < \delta_0\}$. Then, applying Sedov's averaging procedure [5], we find the mean parameters $<\rho(x_1)>$, $<u(x_1)>$, and $<p(x_1)>$ of the one-dimensional flow. In solving the problem of decaying of an arbitrary discontinuity, these parameters allow one to determine the left boundary conditions in the domain Ω_1 for solving the problem of CSD.

Subproblem II, which describes the flow of a reacting hydrogen-oxygen mixture in a combustor with variable heat release in the reaction zone (domain Ω_1) and with the free cross-sectional area of the combustor channel S(x) of the form

$$S(x) = \begin{cases} \delta \cdot l, -L_1 < x < 0 \\ [\delta + (\Delta - \delta)\cdot x / L_2]\cdot l, 0 < x < L_2 \\ \Delta \cdot l, L_2 < x < L_3 \\ [\Delta + (\Delta_{ex} - \Delta)\cdot (x - L_3)/(L - L_3)]\cdot l, L_3 < x < L \end{cases}, \quad (2)$$

was solved in a similar manner [3,4].

The solution of the above-posed problem of CSD in a supersonic incoming flow depends on the input parameters of the supersonic flow (Mach number M_0, pressure p_0, and temperature T_0), the ratio of the cross-sectional areas at the entrance and exit of the supersonic diffuser S_0/S_1, the ratio of the cross-sectional areas of the annular channels of the diffuser and combustor $S_1/S_2 = \delta/\Delta$, the ratio of the cross-sectional areas at the combustor exit and entrance $S_{ex}/S_2 = \Delta_{ex}/\Delta$, and five scale parameters: length of the cylindrical part of the diffuser L_1, length of the initial part of combustor expansion L_2, length of the constant-section part of the combustor L_3, total length of the combustor L_c, and period l. The problem was solved numerically by the Godunov-Kolgan method [6,7] with the number of cells equal to 80,000 in subproblem I and 160,000 in subproblem II.

3 Results of computations

The numerical study was performed for a stoichiometric hydrogen-oxygen gas mixture ($2H_2 + O_2$) at the following values of the constants: T_0 = 300 K, $p_0 = 1.013 \cdot 10^5$ Pa, μ_0 = 12 kg/kmole, $\rho_0 = p_0 \cdot \mu_0/(RT_0)$, and $\gamma_0 = 1.4$. The calculations of subproblem I were performed with the following values of the geometric parameters:

$$S_0/S_1 = \delta_0/\delta = 2,\ L_0 = 2.8\ \text{см},\ \text{tg}\alpha = 0.2 \quad (3)$$

and M_0 = 2.5 – 4. Figure 2 shows a two-dimensional schlieren picture of the pressure distribution in the plane diffuser at M_0 = 2.5 after stabilization of the flow parameters at the diffuser exit. A classical pattern of shock wave interaction with the diffuser walls is seen [8]. An oblique shock wave (white line) at a distance of 0.5 cm from the diffuser entrance is reflected from the upper wall. The supersonic flow passing through a series of shock wave is compressed and partly decelerated to $M_1 \approx 1.554$.

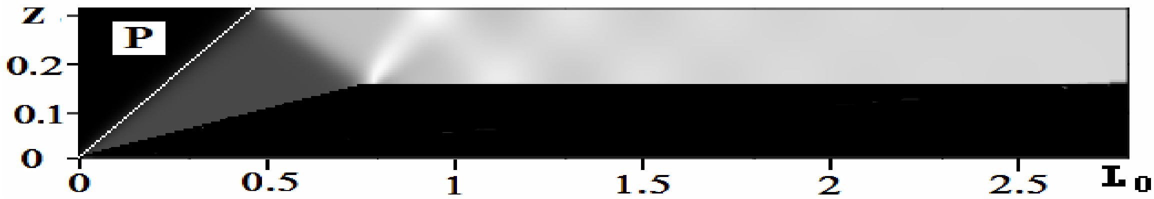

Fig.2. Pressure distribution in the plane diffuser (M_0 = 2.5).

The gas-dynamic flow parameters were averaged in accordance with Sedov's procedure [5] on the boundary Γ_1 and then were transferred to subproblem II, which was calculated at the following geometric parameters of the combustor:

$$L_1 = 2\ \text{см},\ L_2 = 0.2\ \text{см},\ L_3 = L_c = 2\ \text{см},\ \delta/\Delta = 0.5,\ l = 2.5\ \text{см}. \quad (4)$$

At M_0 = 2.5, the initial specific flow rate of the mixture through the combustor is $g_0 = (\delta/\Delta)(S_0/S_1)\rho_0 c_0 M_0$ = 657.25 kg/(s·m^2). It is known [3] that initiation of continuous detonation in an annular combustor leads to generation of a transverse detonation wave (TDW), which rotates in the calculation variant considered here with a velocity $\langle D \rangle = 2.61 \pm 0.02$ km/s. Because of the increase in the total pressure in the combustor, an opposing shock wave propagates toward the supersonic flow. This shock wave converts the supersonic to the subsonic flow and reduces the flow rate of the mixture through the combustor by 20%.

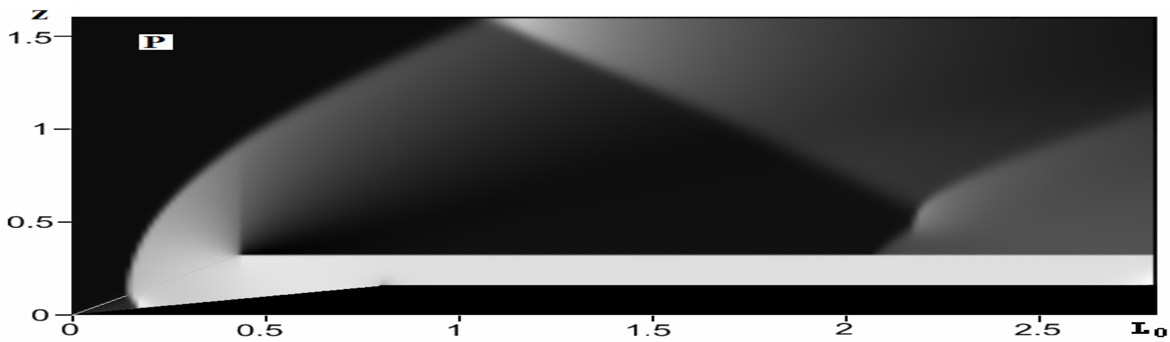

Fig. 3. Pressure distribution at the exit of a two-contour combustor (M_0 = 2.5, δ_1/δ = 8).

Propagation of the opposing shock wave into the diffuser. The opposing shock wave at $t \approx 0.093$ ms reached the left boundary of the domain Ω_1 ($x = -L_1$) and entered the domain Ω_0. To prevent shock wave arrival on the boundary Γ_0, we considered a two-contour combustor in our further calculations. The initial diffuser (domain Ω_0) was supplemented with the second contour (computational domain $\Omega_3 = (-L_0 - L_1 < x < -L_1, \delta_0 < z < \delta_0 + \delta_1)$ (see Fig. 1) separated at $x > x_{SW}$ along the line $z = \delta_0$ from the first contour of the diffuser by a solid wall. Here x_{SW} is the coordinate of the front of the oblique shock wave reflected from the upper wall of the diffuser. The calculations for the two-contour combustor ($\delta_1/\delta = 8$) showed that gas exhaustion into the second contour, attenuation of the opposing shock wave, and changes of gas-dynamic parameters of the flow incoming into the chamber occurred behind the front of the opposing shock wave at $x < x_{SW}$. CSD in the combustor adapted to the new input conditions, and the shock wave structure in the diffuser reached a steady stationary regime at $t \geq 1.8$ ms (see Fig. 3). Reflection of the oblique shock wave from the upper wall ($z = \delta_0 + \delta_1$) is observed in the second contour. Thus, the so-called stationary regime with a "detached" shock wave was formed at $t \geq 1.8$ ms at the diffuser entrance if CSD was initiated in a two-contour combustor at $M_0 = 2.5$ [9].

TDW structure. Let us consider the structure of a steady gas-dynamic flow in the case of TDW propagation in a flow-type combustor ($M_0 = 2.5$). Figure 4 shows the two-dimensional structure of the flow at $t = 1.8$ ms for $l = 2.5$ cm and $L/l = 0.8$. The upper part of the figure ($x < 0$) refers to the flow in the diffuser, and the lower part of the figure ($x > 0$) shows the flow in the combustor.

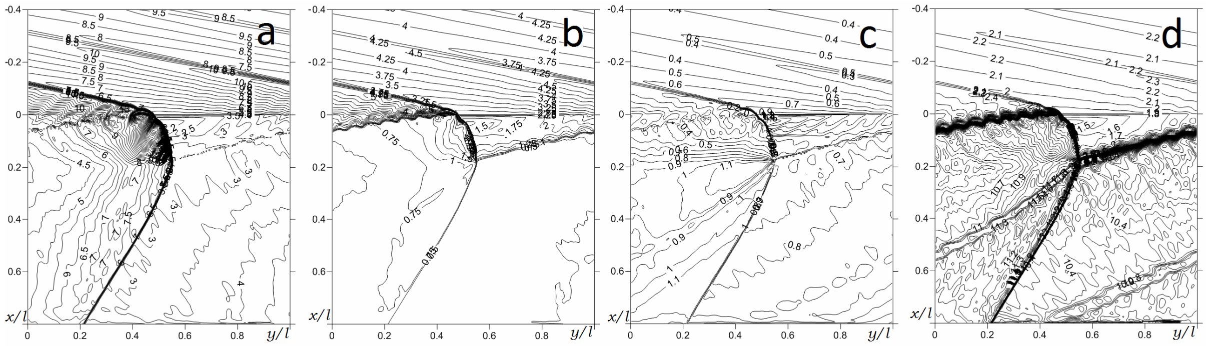

Fig. 4. Calculated two-dimensional structure of CSD in a flow-type combustor: a) pressure contours (p/p_0); b) density contours (ρ/ρ_0); c) Mach number contours ($M_x = u/c$); d) temperature contours (T/T_0). ($M_0 = 2.5$, $l = 2.5$ cm; $S_0/S_1 = 2$; $\delta/\Delta = 0.5$)

The TDW moves from left to right with a velocity $D = 2.58$ km/s over a triangular low-temperature region containing a $2H_2$-O_2 mixture entering from the diffuser. An oblique shock wave moving over a comparatively cold gas ($T \approx 600$ - 660 K) in the diffuser passes from the upper part of the TDW, and an oblique shock wave (tail) moving over hot ($T \approx 3000$-3200 K) CSD products in the combustor passes from the lower part of the TDW. The height of the layer of the combustible mixture ahead of the TDW front at the above-given values of the parameters is $h = 0.425$ cm. Behind the wave, the detonation products gradually expand; if the pressure of the detonation products is lower than the pressure in the diffuser, they are displaced downward by new portions of the gases. Conditions of propagation of a new TDW in the next period are created. The pressure contours (Fig. 4a) and density contours (Fig. 4b) reveal a rapid decrease in pressure and density behind the TDW front. The calculations show that the oblique shock wave adjacent to the TDW moves upstream toward the diffuser and rapidly decays. At $x = 0$, the degree of pressure nonuniformity is $(P_{max} - P_{min})/\langle P \rangle = 2.274$; the corresponding value in the diffuser at a distance equal to the TDW size ($x = -h$) is six times smaller: $(P_{max} - P_{min})/\langle P \rangle = 0.364$; at $x = -2h$, the pressure nonuniformity is thirteen times smaller: $(P_{max} - P_{min})/\langle P \rangle = 0.175$. Figure 4c shows the Mach number contours for the projection of the velocity vector onto the x axis ($M_x = u/c$). It is seen that $M_x < 1$ in the shown part of the diffuser ($-0.4 < x/l < 0$) and in the triangular domain ahead of the TDW front, i.e., the projection of the velocity vector onto the x axis in this flow region is smaller than the velocity of sound. With increasing distance from Γ_1 ($x = 0$) along the x axis up to the relative distance $x/l < 0.2$, the flow is also subsonic. At $x/l > 0.2$, a downstream expanding supersonic zone begins to form behind the tail front; the value of M_x in this zone gradually increases and reaches $M_x = 1.2$. A supersonic flow on the average along the x axis is formed on the lower boundary Γ_2. This means that a transonic transition also occurs in the flow-type combustor with a constant-section channel during TDW propagation [3]. Therefore, sonic perturbations at the combustor exit cannot affect the TDW parameters. The calculated temperature field (Fig. 4d) shows that the gas in the diffuser and ahead of the TDW front in the combustor is comparatively cold ($T \approx 400 - 550$ K), and the maximum temperatures (above 3500 K) are observed behind the TDW front and behind the tail.

Thus, the numerical simulations of CSD in a hydrogen-oxygen mixture show that CSD can propagate in a flow-type annular combustor with a two-contour entrance and parameters (3,4) in a supersonic incoming flow with $M_0 = 2.5$; CSD propagates with a velocity <D> = 2.58 km/s and generates a specific impulse <J> ≈ 1.04 km/s. By varying the parameter l in the periodic problem at fixed values of parameters (3, 4) and flow Mach number $M_0 = 2.5$, we calculated, as it was done in [3], the “minimum” period $l_{min} \approx 2.05$ cm. Note that the physically meaningful period l of the CSD problem should be in the interval $l_{min} < l < 2 \cdot l_{min}$.

Variation of the incoming flow Mach number M_0. We studied the influence of the incoming flow Mach number M_0 on the parameters and structure of the gas-dynamic flow with CSD and considered the issue of the domain of its existence in terms of the Mach number. At fixed geometric parameters of the problem (3,4) and a two-contour combustor ($\delta_1/\delta = 8$), systematic calculations were performed with variations of the incoming flow Mach number M_0. Table 1 gives the calculated data for the specific flow rate <G>, mean pressure <p> at the combustor entrance, “minimum” period l_{min}, relative TDW size $\eta = h/l$, continuous detonation velocity <D>, and mean specific impulse per unit mass of the mixture <J>. Here

$$<J> = 1/l \int_0^l [p(L_c, y, t) + \rho(L_c, y, t) u^2(L_c, y, t) - p_0] dy / <G_{ex}> - c_0 \cdot M_0.$$

Table 1. CSD parameters for several values of M_0 ($S_0/S_1 = 2$; $tg\alpha = 0.2$).

M_0	<G>/g_0	<p>/p_0	l_{min}, (cm)	$\eta = h/l$	<D>, (km/s)	<J>, (km/s)
2.5	0.5	7.0	2.15	0.17	2.58	1.04
3	0.66	11.1	1.75	0.182	2.58	0.925
3.5	0.8	16.0	1.65	0.198	2.54	0.75
4.0	0.95	21.8	0.8	0.21	2.53	0.56

As the incoming flow Mach number increases, the relative flow rate <G>/g_0 and the pressure in the combustor <p> monotonically increase in the constant-section channel of the combustor (see Table 1), whereas the “minimum” period of the problem l_{min} and the specific impulse <J> monotonically decrease. Note that CSD could not be obtained at $M_0 > 4.0$. In this case, a shock wave propagating upstream toward the diffuser was formed at the initial stage after TDW initiation, but the velocity of this shock wave became smaller than the velocity of the incoming supersonic flow with time. Therefore, the shock wave was first entrained to the combustor entrance, then TDW failure occurred, and finally the hot combustion products were entrained by the supersonic flow from the combustor. Thus, for CSD realization in an annular combustor with partial deceleration of the supersonic flow in oblique shock waves, the constraint from above on the Mach number, which was found previously [3], $M_0 < 3/4 \cdot M_{CJ}$, was confirmed ($M_{CJ} = 5.265$ is the Mach number of the Chapman-Jouguet detonation wave for the $2H_2 + O_2$ mixture). Note that this conclusion contradicts the hypothesis put forward in [10], which assumed the possibility of CSD realization in the range of Mach numbers M_0 up to M_{CJ}.

4 Summary

Thus, by an example of a hydrogen-oxygen mixture, the possibility of realization of continuous detonation in an annular combustor with a supersonic ($1 < M_0 \leq 4$) flow velocity at the entrance of a plane diffuser was numerically demonstrated. The influence of the degree of flow deceleration in the oblique shock wave on the domain of existence and properties of continuous detonation was studied. It is found that CSD formation decreases the flow rate of the mixture through the combustor, and a steady regime with a "detached" shock wave is formed at the entrance of the supersonic diffuser. It was found that the domain of CSD realization in the annular combustor is bounded from above in terms of the Mach number of the incoming supersonic flow, which can be presented as $M_0 < 3/4 \cdot M_{CJ}$.

5 Acknowledgement

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