One-dimension Gas Flow through the Porous Media with Large Pressure Gradient

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1 Introduction

Study of gases flow into the porous media has a high priority for many of human activity areas. For example it is needed for rationalization of gas and coal mining, in the geophysical studies, etc. Methods of inhomogeneous media theory are applied for the mathematical studies of such problems [1-2].

This paper is dedicated of study of gas outflow processes from an underground reservoir due to the appearance of large pressure difference. The gas pressure in the reservoir may increase in a moment as a result of different nature, technical, chemical processes. Then a model of process we can define as the model of gas explosion into the underground reservoir.

The two types of process are considered: isothermal and nonisothermal. The problem is solved numerically for each type. Solutions are compared for the different input data. The attention pays on the conditions when the shock wave arises into the porous medium. The rate of stationary state in relation to initial conditions is defined. Changes of the mass gas in the reservoir versus time are examined.

2 Mathematical model

Let's imagine that in underground reservoir filled by gas the pressure much increases immediately. As consequence the gas begins move upward passing through the porous medium in the open air. We write down the mathematical model of this process in terms of inhomogeneous media theory. Let $H$ is the height of the porous medium; $H_0$ is the height of the reservoir. Suppose the porous medium is stationary and homogeneous; and the ideal gas law holds. Then we can get following system for one-dimension process:
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\[
\begin{align*}
\frac{\partial \rho}{\partial t} + \frac{\partial \rho \mathbf{u}}{\partial x} &= 0, \\
\frac{\partial \rho \mathbf{u}}{\partial t} + \frac{\partial \rho \mathbf{u} \cdot \mathbf{u}}{\partial x} &= -\frac{\partial p}{\partial x} - a \frac{\mu}{k} \mathbf{u} - g \rho, \\
(1-a) \rho \mathbf{C}_g \frac{\partial T}{\partial t} &= -\alpha (T - T_s) + (1-a) \lambda \frac{\partial^2 T}{\partial x^2}, \\
a \rho \mathbf{C}_p \left( \frac{\partial T_s}{\partial t} + \mathbf{u} \cdot \frac{\partial T_s}{\partial x} \right) &= \alpha (T - T_s) + a \left( \frac{\partial p}{\partial t} + \mathbf{u} \cdot \frac{\partial p}{\partial x} \right) + a^2 \frac{\mu}{k} \mathbf{u}^2, \\
p &= R \rho T_s, 
\end{align*}
\]

where \( a \) is the porosity, \( C_g \) is the specific heat of solid phase, \( C_p \) is the specific heat at constant pressure, \( g \) is the gravity acceleration, \( k \) is the permeability coefficient of solid phase, \( p \) is the gas pressure, \( R \) is the universal gas constant, \( T \) is the solid phase temperature, \( T_s \) is the gas temperature, \( \mathbf{u} \) is the gas velocity, \( \alpha \) is the constant determining the interphase heat transfer intensity, \( \rho \) is the gas density, \( \rho_s \) is the solid phase density, \( \lambda \) is the thermal conductivity, \( \mu \) is the dynamic viscosity of the gas.

Let denote \( u = a \mathbf{u} \) as filtration velocity and \( q = \rho \mathbf{u} \) as flow rate. The pressure and temperature of the gas are given values at initial moment. The mass gas in the reservoir is changed in a manner:

\[
\frac{dM}{dt} = -q, \text{ where } M = H_0, \rho(t,0).
\]
The gas filtration process is adiabatic:

\[
\left( \frac{p}{p(0,0)} \right)^{\gamma-1} = \frac{T_s}{T_{so}(0,0)}.
\]

At the outlet boundary the pressure is known because the gas flows into the open space. The heat-exchange conditions at the inlet and outlet are also known. Note that the flow rate is unknown for the both boundaries and it have to be found from the solution of problem. So we have the following boundary conditions:

\[
\begin{align*}
p(t, 0) &= p_o(t), \quad T_s(t, 0) = T_{so}(t), \quad \frac{\partial T(t, 0)}{\partial t} = \frac{a_s}{\lambda} (T(t, 0) - T_{so}), \\
p(t, H) &= p_H, \quad \frac{\partial T(t, H)}{\partial t} = \frac{a_s}{\lambda} (T(t, H) - T(t, H)),
\end{align*}
\]

where \( a_s \) is the heat-exchange coefficient.

Initial conditions are found from the conditions of absence of any movement in the porous medium.

3 Numerical experiments

The solution of the problem makes higher demands to a numerical method because it is needed to calculate the fluxes in wide velocity range. The pressure field structure can be a shock wave as well as it can have a quite smooth profile. It depends of the medium properties. Large pressure gradient brings the difficulty in the solution as well. Due to these aspects the use of the most standard schemes leads to unwanted oscillations appearing in the numerical solution. To calculate given problem the Zalesak numerical method was applied. Zalesak method is the upgrade of flux-corrected transport method (FCT) but contrary to FCT it is applied to multidimensional problems easier [3, 4]. The stability of the method was ensured by the Courant stability criterion.

The main parameters that we took for the numerical experience are:

\[
a = 0.3, \quad H = 30 \text{ m}, \quad H_0 = 3 \text{ m}, \quad k = 10^{-8} \text{ m}^2, \quad P_o = 20 \cdot 10^4 \text{ Pa}, \quad T_{g0} = 1500 \text{ K}.
\]
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Computational grid spacing is $\Delta t = 1/18000$ Figures 1 and 2 illustrate trends of the pressure and filtration velocity in a different time moments.

![Figure 1](image1.png)  
**Figure 1.** The gas pressure.

![Figure 2](image2.png)  
**Figure 2.** The filtration velocity.

![Figure 3](image3.png)  
**Figure 3.** The gas density.

![Figure 4](image4.png)  
**Figure 4.** The gas temperature.

The taken coefficient of permeability is quite high for the real media. They usually have less coefficient value. Despite of this the shock wave appearing at the moment of explosion degenerates fast into the ordinary compression wave. Moreover the shock wave will not reach up the outlet boundary even if we take more permeable porous medium. But it is not the case for the isothermal problem, when after explosion the shock wave goes into open space almost with the same intensity. For example, on the figures 5 – 6 we can see the solution of isothermal problem with very high permeable coefficient $k = 10^5 \text{ m}^2$. The medium can be so permeable in consequence of cracking after explosion.

![Figure 5 - 6](image5-6.png)  
**Figure 5 - 6.** The pressure and filtration velocity for isothermal problem with high permeability coefficient.
We calculated how fast the main mass gas goes away the reservoir if the medium has different permeability coefficient. Figures 7 – 8 illustrate a relationship between the time when the 90% of mass gas goes out the reservoir and the permeability coefficient.

If the medium has a high permeability the time of gas outlet for both cases approximately the same. But for the less permeable media the time of gas outlet for isothermal case in a dozen times more because of larger gas density in the reservoir at initial time. Now therefore the temperature changing is important for the media with the medium and high permeability.

The gas flow character depends of the thickness of porous layer as well. By the example of isothermal problem it can be shown that two regimes of gas outflow exist. First regime is when newly formed shock wave comes up to the outlet boundary, second is when it is dissipated earlier. On the figure 9 the nondimensional value $H^2/k$ dependence on the time process is illustrated. The thickness of the porous layer $H$ as a parameter there.

On the left-hand side from the dashed line we have stable shock wave, on the right-hand side we have influence of pressure diffusion [5]. It may be concluded that even for small height porous layer the permeability coefficient $k = 10^{-8}\text{m}^2$ is enough to dissipate the shock wave.

4 Conclusions
The one-dimensional gas outflow problem has been researched. It was supposed that the gas flow was invoked by explosion into the underground reservoir. The problem was solved numerically. It was shown how the permeability of the porous layer influences on the solution. It was demonstrated the
effect of the stationary temperature. The temperature increase has the more diffusion effect and delay the time of the gas outflow from the reservoir considerably. It was found that in the isothermal approximation into the well-permeable media the shock waves typical of the classical gas dynamics are formed. The time when the main mass gas escapes from the reservoir was calculated. It has been found that two regimes of the gas flow exist.

References