# Effects of Scale on Non-Adiabatic Swiss-roll Heat-Recirculating Combustors

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#### Abstracts

Three different scales but geometrically identical 3D numerical models of Swiss-roll combustor were built to study the effects of scale numerically. It is found at low Reynolds number (Re) region, where the heat loss dominates extinction limits due to less thermal throughput, smaller scale combustors unexpectedly showed better performance (in terms of lean extinction limits). This is because, for different scale combustors, internal heat transfer coefficient (for heat exchange)  $U_E$  are inversely proportional to their length scale, while external heat transfer coefficient (for heat loss)  $U_L$  are almost independent to their length scale. Therefore, smaller scale combustors can obtain more heat recirculation due to less heat loss and sustain the reaction in leaner conditions. A dimensionless group  $\alpha = U_L A_L / U_E A_E$  (A<sub>L</sub> and A<sub>E</sub> are heat loss area and heat exchanger area respectively) is defined to show the heat loss effect. If the  $\alpha$  were forced fixed by artificially adjusting external heat transfer coefficient and emissivity based on the length scale, extinction limits for different scale combustors showed almost the same values at low Re region. While at high Re region, the smaller scale combustors showed worse performance. Because when the Re increases (flow velocity increases for the same combustor), the extinction limits are gradually restricted by insufficient resistance time, and also, the reaction time scale is not changed when the size of combustor changes. Therefore, smaller scale combustors have less resident time at the same Re, and more fuel is needed to obtain higher temperature to accelerate the reaction rate. A convergent trend of Damkohler number (Da) at extinction limits for different scale combustors was observed as Re increases. Consequently, Re,  $\alpha$ , and Da are three dimensionless groups that characterize the extinction limits of geometrically identical Swiss-roll combustors.

#### Introduction

Hydrocarbon fuels contain about 50 times more energy per unit mass than state-of-the-art batteries, thus devices converting fuel to electricity at > 2% efficiency represent improvements over batteries for portable electronics [1]. At small scales, however, heat and friction losses become more significant, thus fuel-to-electricity conversion devices based on existing macro-scale systems such as internal combustion engines may be impractical. Consequently, many groups have considered heat-recirculating or "excess enthalpy" combustors for thermal management and thermoelectric, piezoelectric or pyroelectric devices, having no moving parts, for power generation. In heat-recirculating combustors, by transferring thermal energy from combustion products to reactants

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without mass transfer (thus reactant dilution), the total reactant enthalpy (sum of thermal and chemical enthalpy) can be higher than in the incoming cold reactants and therefore can sustain combustion under conditions (lean mixtures, low heating value fuels, large heat losses, high flow rates) that would extinguish without recirculation. One popular type of excess-enthalpy combustor is the double-spiral counter-current "Swiss-roll" heat-recirculating combustor [2, 3] which provides large ratios of (internal) heat exchange area to (external) heat loss area and thus broad extinction limits.

Experiments performed with properly instrumented macroscale devices are useful for predicting the performance of their microscale counterparts via correlations of dimensionless groups. However, due to the difficulty of precisely manufacturing different scale combustors, numerical models are useful tool to obtain quantitative results. In this work, three different-scale but geometrically identical 3D numerical models of Swiss-roll combustor were built to study the effects of different scales.

### Numerical Model

Three different scales (full: 5 cm tall, 3.5 mm channel width; half: 2.5 cm tall, 1.75 mm channel width; double: 10 cm tall, 7 mm channel width) but geometrically identical 3D numerical models of 3.5 turn Swiss-roll combustors were constructed using FLUENT (figure 1). The computational domain includes the gaseous reactants and products, solid combustor walls. A coupled convective (H = 10 W/m<sup>2</sup>-K) and radiative ( $\varepsilon = 0.8$  and 1 for wall and insulation separately) boundary condition was used to describe the external heat loss. Reynolds Stress Model (RSM) was applied to capture the turbulence effect on heat transfer. A 1-step finite rate chemistry with activation energy 40 kcale/ mole and pre-exponential term 9x10<sup>9</sup> in m-sec-kmole units calibrated by experiment at one point (Re ~ 1000) was used to describe the chemical reaction. Details of the numerical model were described in [4].



Figure 1. 3 different-scale but geometrically identical 3D numerical model of Swiss-roll combustors (Half, Full, and Double).

# **Analytical Analysis**

Though the numerical models can obtain quantitative results, to understand physical insight, analytical analysis is necessary. However, the coupling of heat exchange and chemical reaction in the complex shape of Swiss-roll combustor makes it difficult to comprehensively describe their characteristics analytically. Here we choose a simplest heat-recirculating combustor, count-current heat-recirculating combustor (figure 2), and assume the reaction has a constant threshold reaction temperature ( $T_3$ ).





Figure 2. Schematic diagram of counter-current heat recirculating combustor.

From the energy balance of Reactants, Products, and Reaction, we can obtain 3 equations:

$$\begin{split} & \text{mC}_{p}(T_{2} - T_{1}) = Q_{t} - Q_{L1} \\ & \text{mC}_{p}(T_{4} - T_{3}) = -Q_{t} - Q_{L2} \\ & \text{mC}_{p}(T_{3} - T_{2}) = Q_{R} = \frac{f \times Q}{C_{p}} \end{split}$$

where, f is the fuel mass fraction, Q is the heating value, and  $C_p$  is the heat capacity. But there are 6 unknowns (T<sub>2</sub>, T<sub>3</sub>, T<sub>4</sub>, Q<sub>t</sub>, Q<sub>L1</sub>, Q<sub>L2</sub>) in these 3 equations. To solve the equations, simple estimations of Q<sub>t</sub>, Q<sub>L1</sub>, and Q<sub>L2</sub> were made as follow:

$$Q_{t} = UA\left(\frac{T_{4} + T_{3}}{2} - \frac{T_{2} + T_{1}}{2}\right)$$
$$Q_{L1} = U_{L}A_{L}\left(\frac{T_{2} + T_{1}}{2} - T_{1}\right)$$
$$Q_{L2} = U_{L}A_{L}\left(\frac{T_{4} + T_{3}}{2} - T_{1}\right)$$

Therefore, the 3 equations now only have 3 unknowns, T<sub>2</sub>, T<sub>3</sub>, T<sub>4</sub>, which then can be solved. If we set:

$$\frac{U_{L}A_{L}}{UA} = \alpha$$

and define a dimensionless excess enthalpy E = temperature rise from recirculation / temperature rise from reaction, which represents the performance of heat exchange, we can obtain equation:

$$E = \frac{T_2 - T_1}{T_3 - T_2} = \frac{4NTU}{4 + \alpha NTU(4 + NTU(2 + \alpha))}$$
(1)

where

$$NTU = \frac{UA}{\dot{m}C_{p}}$$

is called Number of Transfer Units. For Swiss-roll combustor, since it is combustion system, extinction limits are used to indicate the performance instead of E. Also, since in geometrically identical cases, NTU is proportional to  $Re^{-1}$  (defined here based on inlet flow velocity, viscosity at room temperature, and channel width) at laminar flow region and roughly proportional to  $Re^{0}$  at turbulent flow region, the same Re means almost the same NTU. Therefore, Re is used in this study because of its straightforwardness in terms of flow rate as well as thermal power output.

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# **Results and Discussion**

Figure 3 shows extinction limits for different scale combustors. At lower Re, smaller scale combustor showed better performance (lower extinction limits). Since at the same Re,  $U_E \sim k / w$  (k: thermal conductivity, w: channel width) is inversely proportional to length scale but  $U_L$  is independent to length scale (nature convection  $H_L$  and radiation  $\varepsilon$ ), different scale combustors showed different  $\alpha$  at the same Re. Smaller scale combustors showed less  $\alpha$  and therefore better performance at lower Re, where heat loss dominates the extinction limit behavior. If the  $\alpha$  were forced to be the same value for different scale combustors by adjusting external heat transfer coefficient and emissivity based on the length scale (table 1) the extinction limits are almost the same at low Re region (figure 4). Since all the combustors are geometrically identical, the same Re represents the same NTU. And from the equation (1), for the same NTU and the same  $\alpha$ , E will be the same. Therefore, figure 4 also implies the extinction limits at low Re region are mainly controlled by the performance of the heat exchanger.



Figure 3. Extinction limits vs. Re for different scale combustors.

	Half	Modified Full	Modified Double
$H_L(W/m^2-K)$	10	5	2.5
ε (wall)	0.8	0.4	0.2
ε (insulation)	1	0.5	0.25

Table 1. Modified  $U_L$  (includes  $H_L$  and  $\varepsilon$ ) based on the length scale of different scale combustors.





Figure 4. Extinction limits vs. Re for different scale combustors with modified a.

The effect heat loss coefficient  $\alpha$  can be seen in the figure 5, which shows E vs. NTU for different  $\alpha$  from equation (1). If there is no heat loss, which  $\alpha = 0$ , E is linear with NTU. However, when  $\alpha$  is large, more NTU may not always benefit E.



Figure 5. Dimensionless excess enthalpy vs. NTU for different a.

At higher Re (higher flow rate) larger scale combustors showed lower extinction limits. This is because at this region, the reaction is restricted by insufficient resistance time instead of the amount of heat-recirculation. Reaction time scale is not changed with different scale combustors, and therefore smaller scale combustors have less resident time and more chemical enthalpy is needed to accelerate the reaction rate. Figure 6 shows the Da vs. Re for different scale combustors. Da here is defined by Resistance time scale ( $t_r$ ) / Chemical time scale ( $t_c$ ), where  $t_r$  = overall channel length / input velocity and  $t_c$  = input fuel concentration / maximum reaction rate obtained from FLUENT. Though these definitions may not be appropriate to reflect the true time scale (especially for the using overall channel length in  $t_r$ ), the trend can still be observed. With Re increases, the extinction limits diverge for different scale combustors but their Da converge, which demonstrates the reaction rate plays an more important rule at high Re region.



Figure 6. Da at limits vs. Re for different scale combustors (with the modified  $\alpha$ ).

# Conclusion

The extinction limits of Swiss-roll combustor are determined by both the properties of spiral heat exchanger and combustion chamber. Current study showed Re (NTU),  $\alpha$ , and Da are three dimensionless groups that characterize lean extinction limits of geometrical identical combustors. However, further shrink or expand the combustor may involved more factors that can affect extinction limits, such as wall conduction, wall radical termination, buoyancy effect, flame curved, and etc., and more dimensionless groups are required to characterize the system.

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