

# Short-term prediction of combustion instability in a lean premixed gas-turbine combustor using nonlinear time series analysis

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## 1. Introduction

Lean premixed combustion is highly advantageous in reducing nitrogen oxide (NO<sub>x</sub>) emission from gas-turbine engines without the loss of combustion efficiency by controlling the equivalence ratio to within an appropriate range. This combustion method has attracted considerable attention from developers of gas-turbine combustors. One main drawback of lean premixed combustors, however, is that they are susceptible to flow perturbations. They suffer from combustion instabilities such as thermoacoustic combustion oscillations, lean blowout and flashback. Among these, thermoacoustic combustion instability, caused by the strong coupling between the variations in pressure and heat-release rate, is considered to be a serious problem since it can lead to a reduction in lifespan or even the total destruction of an engine. The physical mechanism underlying the onset of thermoacoustic combustion instability and efficient suppression methods for the combustion instability have been extensively investigated for various types of laboratory-scale gas turbine combustor with swirling flow, which is summarized in detail in a recent review paper edited by Huang and Yang [1].

Regarding the treatment of unstable behaviors in the combustion process induced by the thermoacoustic instability, the power spectral analysis of the pressure and heat-release fluctuations has been performed in most studies [2-5]. This method of linear analysis is capable of detecting the excitation of unstable combustion modes, but may be insufficient for fully understanding the underlying physics of combustion instabilities because they are complex phenomena strongly affected by the inherent nonlinearity associated with chemical reactions, turbulent flow and acoustic perturbations. Therefore, a new approach based on nonlinear dynamics will become significant for the treatment of the combustion instabilities in gas-turbine combustors.

A nonlinear time series approach inspired by chaos theory is becoming an increasingly reliable tool for clarifying the nonlinear properties of complex dynamics, and its importance has been discussed in previous combustion researches [6-9]. In a recent work, we have successfully extracted the deterministic nature in the combustion instability in a lean premixed gas-turbine combustor by using a nonlinear time series analysis [10]. The purpose of this study is to investigate the possibility on whether or not the nonlinear time series analysis is applicable to predict the short-term dynamic behavior of the combustion instability in a lean premixed gas-turbine combustor.

## 2. Experiments

The configuration of the experimental system is shown in Fig. 1. This system is identical to that used in the previous study [11]. The combustion test rig is composed of a blower, an electric heater, a mixing tube, an axial swirler and a combustion chamber. The chamber has a length of 630 mm with a  $100 \times 100$  mm square cross section. The rest of the chamber is composed of a water-cooled stainless-steel duct. Methane gas is used for the main fuel and is injected through multiple orifices at a location 260 mm upstream from the inlet of the combustion chamber. An axial swirler is installed as a flame holder at the inlet of the combustion chamber. The active control by a secondary fuel injection [11] is out of the scope of this work because this work focuses on the investigation of the dynamic behavior of the combustion instability. The inlet air temperature and air mass flow rate are 700 K and 78 g/s. The equivalence ratio  $\phi = 0.45$  is selected in the current experiment because it is under the condition that the intermittent combustion instability occurs (see Fig. 2).

To investigate the dynamic behavior of the combustion instability, the pressure fluctuations are measured by pressure transducers (Kulite Semiconductor Products, Model XTEL-190-15G). A pressure port, PT1, is placed on the wall of the mixing chamber. The other ports, PT2-PT4, are located on the wall of combustion chamber. Signals from the pressure transducers are acquired simultaneously through a multi channel data acquisition system (Ono Sokki, DS-2000). The sampling frequency of the obtained time series is 25.6 kHz. In this work, the nonlinear time series analysis is applied to the time series data of the pressure fluctuations  $p'$  obtained from transducer PT2 because the influence of thermo-acoustic coupling strongly appears in the time series data of the measured location. Note that the nonlinear time series analysis is implemented for the time series data of the pressure fluctuations  $p'$  at the sampling frequency 5.12 kHz.

## 3. Nonlinear time series analysis based on chaos theory

The sensitivity of the time evolution of a system to small changes in initial conditions is a critical characteristic of chaos, which causes the exponential decay of predictability with time. This effect is known as short-term predictability followed by long-term unpredictability. In this work, we use this concept to predict the time variation in the pressure fluctuations.

On the basis of Takens' embedding theorem [6-10], the phase space is constructed from the time series data of the pressure fluctuations  $p'$ . The time-delayed coordinates used for the construction of the phase space are expressed as

$$\mathbf{X}(t_i) = (p'(t), p'(t - \tau), p'(t - 2\tau), \dots, p'(t - (D+1)\tau)) \quad \dots(1)$$

where  $i = 0, 1, \dots, n$  ( $n$  is the data number of the time series),  $\mathbf{X}(t_i)$  is the phase space vectors,  $p'(t)$  is the pressure fluctuations at time  $t_i$ ,  $D$  is the embedding dimension, that is, the dimension of the phase space, and  $\tau$  is a time lag. As used in the previous paper [6-10],  $\tau$  is set to be the time lag that yields a local minimum of mutual information.

We first divide the time series data into first and second parts. The first part is used as a source for generating data library, and the second part is used as reference data for comparison with the predicted time series data. A set of neighbors of the vector  $\mathbf{X}(t_p)$ , which are described by  $\mathbf{X}(t_k)$  ( $k = 1,$

$2, \dots, K$ ), are first searched for from all the points in the phase space, where  $\mathbf{X}(t_p)$  is the final point of trajectory of the phase space constructed from the data library. After  $T$  steps,  $\mathbf{X}(t_k)$  is mapped to the  $T$  step ahead prediction  $\mathbf{X}(t_k + T)$ .  $\mathbf{X}(t_p + T)$  is expressed as follows:

$$\mathbf{X}(t_p + T) = \frac{\sum_{k=1}^K \exp(-d_k) \cdot \mathbf{X}(t_k + T)}{\sum_{k=1}^K \exp(-d_k)} \quad \dots (2)$$

where  $d_k = \|\mathbf{X}(t_p) - \mathbf{X}(t_k)\|$ . The predicted time variation of the pressure fluctuations is obtained inversely from  $\mathbf{X}(t_p + T)$ .

### 3. Results and Discussion

The time variation of the predicted pressure fluctuations, together with the measured pressure fluctuations, is shown in Fig. 3. Note that the pressure fluctuations measured over 21 s are used as the data library to predict the time variation of the pressure fluctuations. The predicted pressure fluctuations follow the measured pressure fluctuations for approximately the first 12 cycles. However, they gradually diverge from the measured pressure fluctuations, showing an inconsistency in the amplitude at  $t \geq 21.06$  s. This result clearly indicates that the nonlinear time series analysis we applied in this work is feasible for predicting the short-term dynamic behavior of the pressure fluctuations. In this work, we predict the time variation of the pressure fluctuations by updating the data library of the phase space before losing the determinism of trajectories in the phase space, which is needed to predict the pressure fluctuations with high accuracy. The schematic of the method of prediction by updating the data library is shown in Fig. 4. As a first step, the prediction of the pressure fluctuations for 10 cycles is implemented. The prediction of the second step is made using the data library, to which the measured pressure fluctuations of the first step have been added. This process is iterated for  $n$  steps, keeping the amount of the data in the data library constant.

The time variation of the predicted pressure fluctuations obtained by updating the data library, together with the measured pressure fluctuations, is shown in Fig. 5. The predicted pressure fluctuations correspond to the measured pressure fluctuations. To quantitatively evaluate the degree of coincidence between the predicted and measured pressure fluctuations, the amplitude ratio  $A_p/A_m$  (where  $A_p$  is the standard deviation of the predicted pressure fluctuations and  $A_m$  is the standard deviation of the measured pressure fluctuations in each 20 cycles starting from  $N = 0$ ) and the phase lag  $\theta$  between the predicted and measured pressure fluctuations in each 20 cycles starting from  $N = 0$  are shown in Fig. 6 as a function of the number of cycles of pressure fluctuations,  $N$ .  $A_p/A_m$  and  $\theta$  is nearly 0.9 and 0, respectively. This result clearly shows that the degree of coincidence between the predicted and measured pressure fluctuations is sufficiently high, and that by updating the data library of the phase space, the nonlinear time series analysis we applied in this work is valid for predicting the intermittent nature of the pressure fluctuations with relatively high accuracy. In addition to the applicability of the nonlinear time series analysis for identifying deterministic chaos [10], the results obtained in this work demonstrate that the nonlinear time series analysis has sufficient potential use for predicting the pressure fluctuations of the combustion instability in a lean premixed gas-turbine combustor, and that it may be worthwhile from a practical viewpoint. We need to examine whether and to what extent the nonlinear time series analysis is feasible for predicting the short-term pressure fluctuations by decreasing the size of the data library. This issue will be presented in this presentation in details.

## Summary

The nonlinear time series analysis based on the concept of orbital instability in phase space has been performed to predict the time variation of the pressure fluctuations for  $\phi = 0.45$ , for which neighboring trajectories in the phase space exhibit a deterministic nature [10]. The time variation of the predicted pressure fluctuations obtained by updating the data library coincides closely with those of the measured pressure fluctuations. This result shows that the nonlinear time series analysis we applied in this work has sufficient potential use for predicting the short-term dynamic behavior of the combustion instability with high accuracy, which has not been previously reported in the fields of combustion science and physics.

## Acknowledgement

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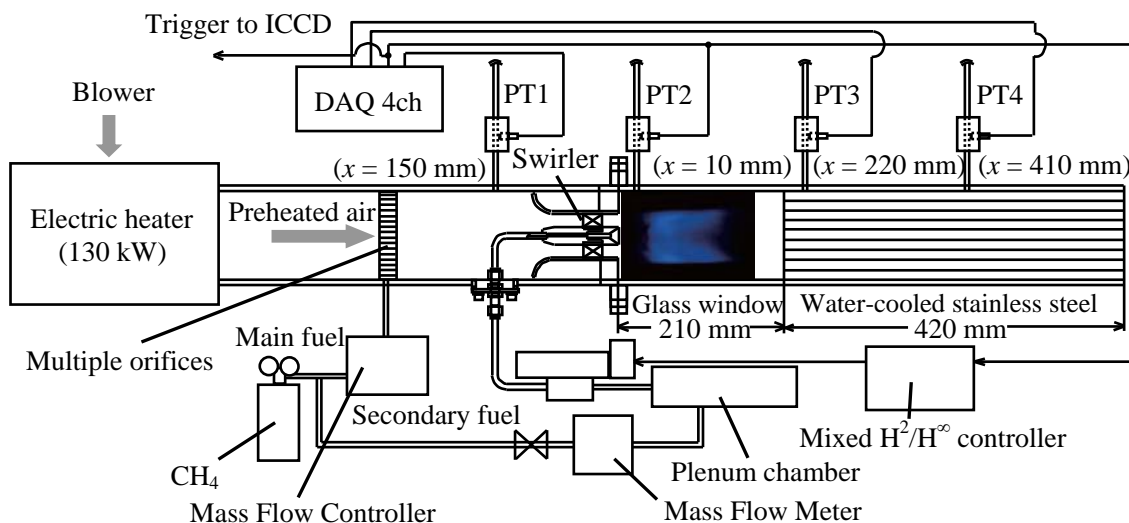


Figure 1. Schematic of experimental system

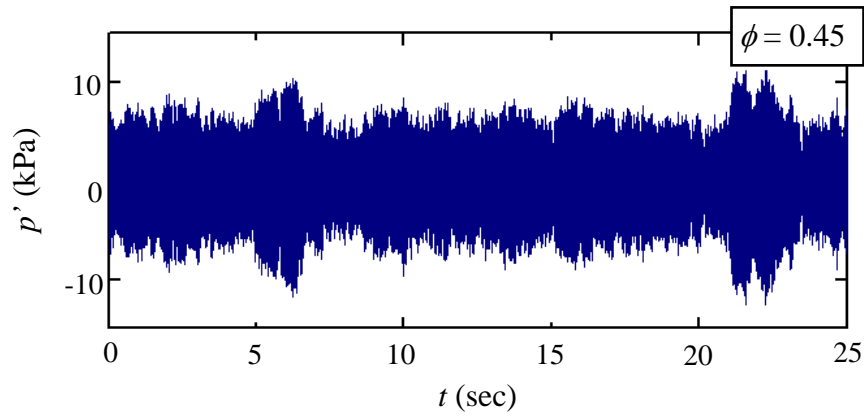


Figure 2. Time variation of pressure fluctuations  $p'$  at equivalence ratios  $\phi = 0.45$ .

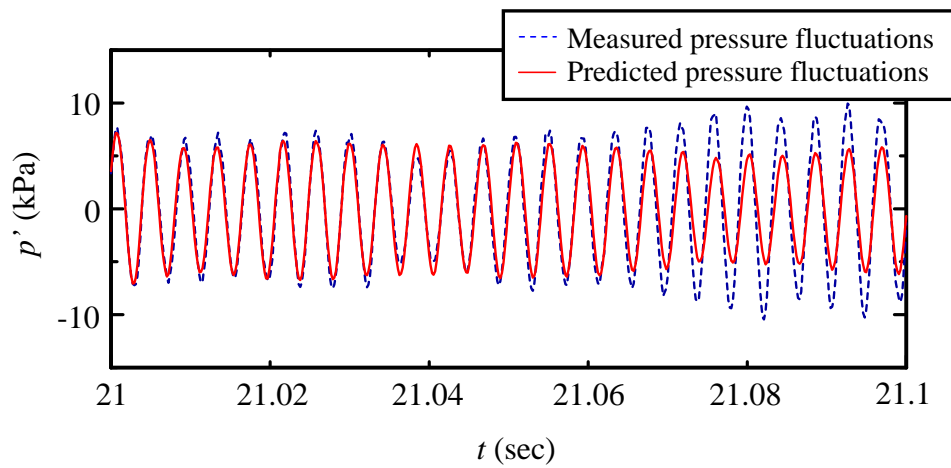


Figure 3. Time variation of pressure fluctuations  $p'$  at equivalence ratios  $\phi = 0.45$ .

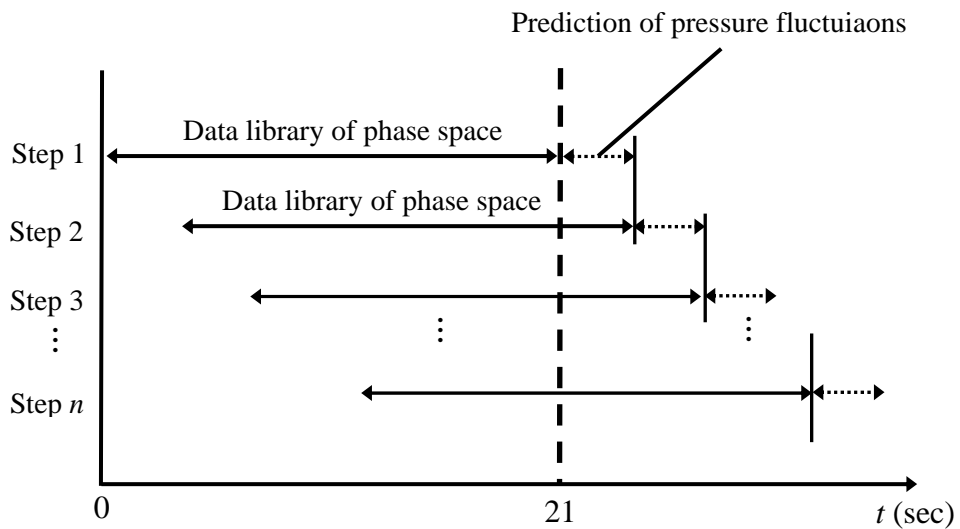


Figure 4. Schematic of the prediction method involving updating the data library of the phase space

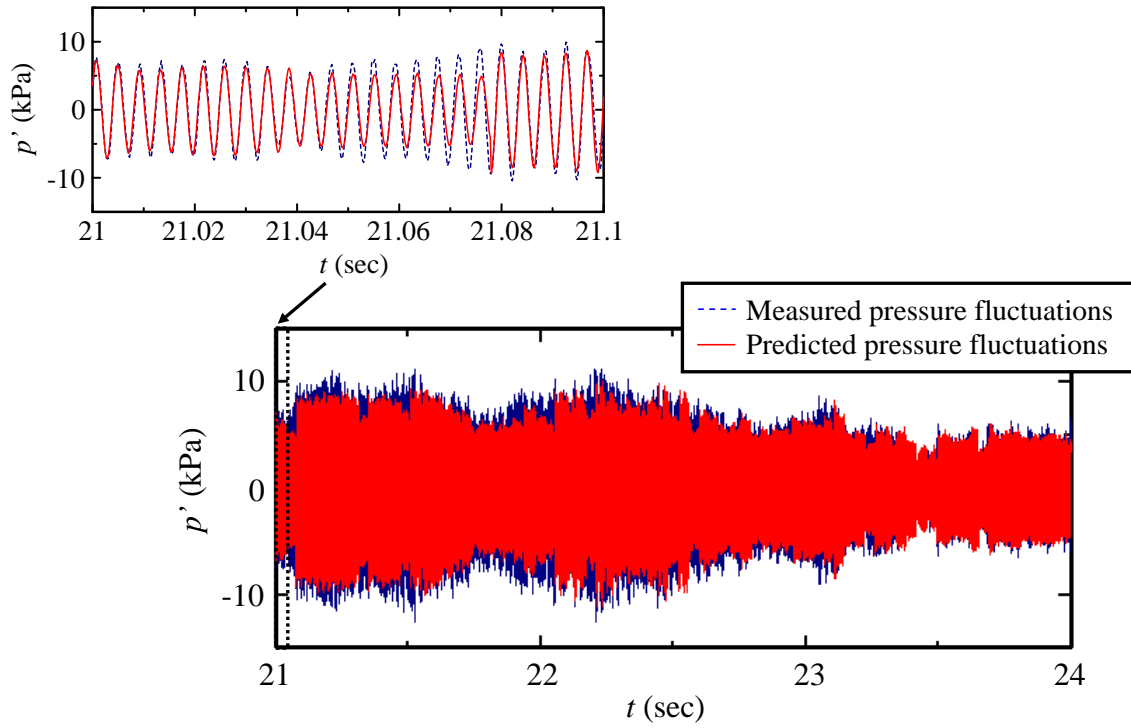


Figure 5. Time variation of pressure fluctuations  $p'$  at equivalence ratios  $\phi = 0.45$ .

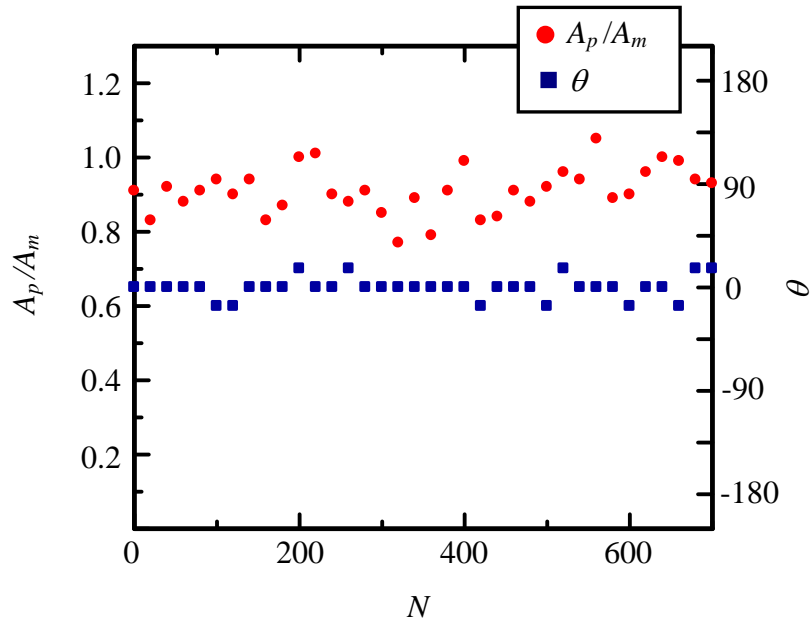


Figure 6. Variations of phase delay  $\theta$  and amplitude ratio  $A_p/A_m$  as a function of the number of cycles of pressure fluctuations,  $N$  at equivalence ratios  $\phi = 0.45$