Stability of premixed flames in narrow channels

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1 Abstract

This paper studies the stability of premixed flames in a narrow channel with a step-wise, prescribed wall temperature by using the thermo-diffusive approximation. In this work, we focus on the influence of the equivalence ratio ϕ , the mass flow rate m and the fuel Lewis number Le_F on the stability of a premixed flame propagating inside the channel. According to the numerical calculations, the flame presents a range of equivalence ratios ϕ in which the flame becomes unstable, starting an oscillatory movement that leads to a periodic variation of the maximum flame temperature and position of the flame. The region of maximum instability is found to be near $\phi = 1$, with an oscillation amplitude that reduces as the Lewis number increases.

2 Introduction

In the present paper we consider the flow of a mixture of fuel and oxidizer through a narrow channel with a step-wise wall, prescribed temperature. The use of the thermo-diffusive approximation decouples the momentum and continuity equations from those of energy and mass fraction. Even thought the model used in this work reduces the complexity to a minimum, a considerably large number of parameters remain in the problem.

Previously, some authors have study similar problems. Among them, the most relevant are those by Pizza et al. [1], Kurdyumov et al. [2] and Kurdyumov [3]. The article by Pizza et al. studied the stability of a hydrogen flame in a channel with a prescribed step-wise wall by using a detailed chemistry. They identified unstable flame propagation for different values of the channel height and of the Damköhler number. Kurdyumov et al. confirmed the observations of [1] by using a thermo-diffusive model and assuming a very poor mixture. They also compared his numerical results with a one-dimensional stability analysis that provided the map of stability of the problem. The effect of the Lewis number in an adiabatic channel was considered later by the same author in [3]. Unlike previous examples, he carries out a two-dimensional stability analysis by introducing a method that allows the calculation of the first



Figure 1: Sketch of the computational domain with the main boundary conditions at two different times $t_1 = 4.1$ and $t_2 = 5.9$ for a stoichiometric mixture $\phi = 1$, with d = 5, m = 3.5, $\beta = 10$, $\gamma = 0.87$, $\theta_w = 0.6$, s = 4, $Le_O = 1$, and $Le_F = 0.3$. Upper part of the figures includes temperature isocontours for $0.05 \le \theta \le 0.65$ at intervals $\delta\theta = 0.05$ (upper plot) and for $0.1 \le \theta \le 1.2$ at intervals $\delta\theta = 0.1$ (lower plot). Lower part of the figures represents the reaction rate isocontours for $1.2 \le \omega \le 2.8$ at intervals $\delta\omega = 0.4$ (upper plot) and $20 \le \omega \le 80$ at intervals $\delta\omega = 20$ (lower plot). Calculations have been performed.

eigenvalue in a relatively easy fashion.

None of the studies mentioned above have considered the effect that the variation of the equivalence ratio ϕ could have on the stability of the flame. The present study covers that gap by numerically integrating, in the thermo-diffusive approximation, the mass fraction and energy equations in a range that covers from very lean to very rich mixtures.

3 Formulation of the problem

We consider a premixed combustible mixture of fuel and oxidizer flowing in a planar two-dimensional channel of height h, as sketched in Fig. (1). The inlet temperature T_0 at $x' = -\infty$ is fixed, while a velocity profile with mean velocity U_0 is imposed at the inlet section. The wall temperature is supposed to vary in a step-wise fashion, being maintained at the inlet value T_0 if x' < 0 and jumping to a higher value T_w if $x' \ge 0$ (see Fig. 1).

To simplify the problem, we adopt the thermo-diffusive approximation in which density ρ , kinematic viscosity ν , fuel D_F and oxidizer D_O molecular diffusivity, heat capacity c_p , and thermal diffusivity α are defined as constants. The fluid dynamics is not affected by the combustion and the velocity profile at the inlet remains unchanged along the channel.

The chemical reaction is modeled by an irreversible, single-step reaction of the form $\nu_F F + \nu_O O \rightarrow P + Q$, where F, O, P and Q denote the fuel, the oxidizer, the combustion products, and the heat released in the chemical reaction. Hereafter, ν_F and ν_O are the stoichiometric coefficient of the fuel and oxidizer, which have molecular weights W_F and W_O , respectively. The equivalence ratio of the mixture is, therefore, defined as $\varphi = sY_F/Y_O$ where $s = \nu_O W_O/(\nu_F W_F)$ is the mass of oxidizer

needed to burn the unit mass of fuel. To take into account the variation of the stoichiometric ratio, it is useful to write the equations in terms of the excess and deficient species mass fraction, denoted by the subscripts E and D respectively, instead of considering fuel and oxidizer mass fractions as unknowns. The using of this nomenclature provides a system of equations that do not change as we study lean or rich mixtures, introducing an alternative definition for the equivalence ratio $\phi = \nu Y_E/Y_D > 1$, where $\nu = \nu_E W_E/(\nu_D W_D)$. Notice that ϕ is always greater than or equal one and that $\phi = \varphi$, $\nu = s$ and $\phi = \varphi^{-1}$, $\nu = s^{-1}$ in rich and poor mixtures, respectively.

The consumption rate is assumed to follow an Arrhenius law $\Omega = \rho^2 \mathcal{A} Y_F Y_O \exp(-E/R_g T)$. In the previous expression ρ is the density, \mathcal{A} is the pre-exponential factor, Y_F and Y_O are the fuel and oxidizer mass fractions, E is the activation energy, R_g is the universal gas constants, and T is the local temperature of the mixture, respectively. With all these assumptions, the system dynamics is described by three convection-diffusion-reaction equations for the energy and for the fuel and oxidizer mass fractions.

This system of partial differential equations can be non-dimensionalized by scaling the variables with the appropriate reference values. The mass fractions are scaled using their inlet values, while h and h^2/α are chosen as the reference length and time scale, defining x = x'/h, y = y'/h and $t = t'/(h^2/\alpha)$. The non-dimensional temperature is defined as $\theta = (T-T_0)/(T_s-T_0)$, with T_0 representing the inlet temperature and T_s the adiabatic temperature of a stoichiometric planar flame $T_s = T_0 + (Q/c_p)s/(1+s)$.

Introducing the non-dimensional variables defined above, the problem reduced to the integration of

$$\partial_t \theta + m\sqrt{dU}\partial_x \theta = \Delta\theta + d\phi F(\phi)^2 \omega \tag{1}$$

$$\partial_t Y_E + m\sqrt{dU}\partial_x Y_E = (Le_E)^{-1}\Delta Y_E - dF(\phi)\omega$$
⁽²⁾

$$\partial_t Y_D + m\sqrt{dU}\partial_x Y_D = (Le_D)^{-1}\Delta Y_D - d\phi F(\phi)\omega, \qquad (3)$$

where $Le_i = \alpha/D_i$ are the Lewis number of the excess and deficient species $i = E, D, \beta = E(T_s - T_0)/R_gT_s^2$ is the Zel'dovich number, $m = U_0/S_L$ is the mass- flow parameter, $d = h^2/\delta_T^2$ is the Damköler number and S_L and δ_T represent the planar burning velocity and the flame thickness for a stoichiometric mixture $\phi = 1$, respectively. The combustion rate is now written as

$$\omega = \frac{\beta^3}{4Le_E Le_D u_p^2} \exp\left[\frac{\beta(\theta - 1)}{1 + \gamma(\theta - 1)}\right],\tag{4}$$

where $\gamma = (T_s - T_0)/T_s = 0.87$ represents the heat release parameter. In Eq. (1)-(3), $\Delta = \partial_{xx}^2 + \partial_{yy}^2$ is the Laplacian operator, ∂ denotes partial derivative, U = 6y(1 - y) is the Poiseuille velocity profile, and the function $F(\phi) = (1 + s)/(\phi + s)$ depicts the effects of the dilution of the mixture.

Above equations are integrated by imposing boundary conditions upstream $x \to -\infty$ for both temperature and mass fractions

$$\theta = Y_E - 1 = Y_D - 1 = 0, \tag{5}$$

while the flow must reach an equilibrium state far downstream $x \to \infty$

$$\partial_x \theta = \partial_x Y_E = \partial_x Y_D = 0. \tag{6}$$

The physical domain is truncated to a finite computational domain $x \in [x_{min}x_{max}]$, where x_{min} and x_{max} must be carefully chosen in order to assure the fulfillment of boundary conditions. Typically $x_{min} = -3$ and $x_{max} = 10$, but longer values are usually needed when very diffusive fuels and low mass flow rates are considered.

The boundary conditions at the non-porous walls are given by

$$\theta - \theta_w \Theta(x) = \partial_y Y_E = \partial_y Y_D = 0 \text{ at } y = 0, 1$$
(7)

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where $\Theta(x)$ is the Heaviside step function.

The factor $u_p = S_L/U_L$ introduced in Eq. (4) is the ratio of the planar stoichiometric flame speed for finite activation energy and the flame propagation velocity of a stoichiometric mixture in the limit of infinitely high activation energy [4], $U_L^2 = 4\rho\alpha Le_E Le_D A\nu_D W_E s^2 (s+1)^{-1} \beta^{-3} \exp(-E/R_g T_s)$. The factor u_p is obtained numerically by integrating the one-dimensional eigenvalue problem

$$\theta' = \theta'' + \omega \tag{8}$$

$$Y'_{E} = (Le_{E})^{-1}Y''_{E} - \omega$$
(9)

$$Y'_D = (Le_D)^{-1} Y''_D - \omega,$$
(10)

with boundary conditions

$$\begin{aligned} \theta &= Y_E - 1 = Y_D - 1 = 0 \quad \text{at} \quad \xi \to -\infty \\ \theta - 1 &= Y_E = Y_D = 0 \quad \text{at} \quad \xi \to \infty, \end{aligned}$$

where $\xi = x'/(\alpha/S_L)$ is the non-dimensional distance in a reference frame moving with the flame, the prime symbol denotes derivative along ξ , and ω is given by Eq. (4). Notice that with the scales defined above, the factor u_p is only a function of the fuel and oxidizer Lewis numbers. The above onedimensional problem is solved by using a shooting method giving the values of u_p plotted in Fig. (2) for $Le_O = 1$.



Figure 2: Variation of the factor u_p with the fuel Lewis number Le_F

4 Results

4.1 Numerical treatment

The unsteady solution is obtained by integrating Eqs. (1)-(3) using a time-marching procedure, secondorder accurate in both time and space. The solution is started by setting a homogeneous temperature field $\theta = \theta_w$ for x > 0 and $\theta = 0$ for x < 0 and $Y_F = Y_O = 1$ everywhere. Steady solutions are obtained using a Successive Over-Relaxation (SOR) method.

4.2 Effect of the equivalence ratio φ

We have considered in this brief communication the effect of the equivalence ratio φ on the stability of the flame for two different Lewis numbers and fixed mass flow rates m = 3.5 and Damköhler number d = 5. The results of the calculations are represented in both Fig. (1) and Fig. (3). In the latter we have plotted the temporal evolution of the maximum temperature in the channel θ_m for $Le_F = 0.3$ (left plots) and $Le_F = 1$ (right plot) as a function of the equivalence ratio φ . For a highly diffusive fuels $Le_F = 0.3$, the solution presents a Hopf bifurcation at $\varphi \simeq 0.85$ that puts the flame to oscillate, changing its location inside the channel and varying significantly its temperature. Contrary, for $Le_F = 1$ the bifurcation does not emerge at any value of φ and the solutions achieves a steady state after the transient oscillations die out.

In figure (1) we plot temperature and reaction rate isocontours at two different times $t_1 = 4.1$ and $t_2 = 5.9$ for a stoichiometric mixture $\varphi = 1$ with fuel Lewis number $Le_F = 1$. The chosen times, t_1 and t_2 , correspond to the time instants at which the maximum temperature in the channel achieves a minimum and a maximum respectively, as has been indicated with symbols (circles \circ) in the $\varphi = 1$ plot of the left column of figure (3). Remarkably, the maximum reaction rate isosurface presents a discontinuity at $t = t_2$, with an aperture around y = 1/2 where w is ten times smaller that the reaction rate found in lower velocity regions and through where air and fuel can leak to the post-flame neigborhood. This hollow region dissapears as the flame is pushed further downstream by the incoming flow and the flame returns to be a continuous surface (except in a small region close to the channel walls).

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Figure 3: Temporal evolution of the maximum temperature in the channel θ_m for various equivalence ratios ϕ (specified in the figures) and Le = 0.3 (left) and Le = 1.0 (right). The rest of parameters are specified in the caption of figure (1). In the $\varphi = \phi = 1$ figure of the left colum of plots, the symbols (\circ) indicates the times $t_1 = 4.1$ and $t_2 = 5.9$ at which temperature and reaction rates isocontours are represented in figure (1).