# **Direct Measurement and Relationship between Critical Tube Diameter and Critical Energy for Direct Detonation Initiation**

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## **1** Introduction

Accumulation of dynamic parameter data such as cell size, initiation energy, critical tube diameter and detonability limit for various detonation systems not only is of great fundamental significance to further the understanding of the detonation wave structure and its propagation, but also provides useful information for safety assessment and evaluation of the detonation hazards of explosive mixtures [1, 2]. From the dimensional analysis consideration, it is possible to realize that once a characteristic length scale is determined, the various dynamic parameters can be readily correlated with that particular length scale. Indeed, there are numerous studies attempted to link the detonation cell size  $\lambda$ , as the fundamental chemical length scale, to the critical tube diameter, initiation energy as well as detonation limits [3]. However, a quantitative theory for predicting the cell size is still lacking and experimental determination of a unique characteristic cell size from smoked foils can be difficult and often very subjective due to the inherent instability of the detonation front and cellular irregularity in common explosive mixtures [4].

Beside the cell size  $\lambda$ , critical tube diameter  $d_c$  can be considered as an alternative length scale for various correlations. It can be argued that direct experimental measurement of critical tube diameter are perhaps more trustworthy than the determination of cell size. In fact, many initiation models such as the work done concept and the surface energy model are developed based on the relationship linking the cell size to the critical energy via the critical tube diameter [1, 5, 6]. Therefore, one should look at the direct link between the initiation energy and the critical tube diameter without evaluation of the cell size. The critical tube diameter phenomenon may also provide a better problem from the modeling point of view compared to a quantitative theory of cell size prediction.

In this study, the critical tube diameter, in parallel with the critical energy for direct initiation are measured both experimentally. These two detonation parameters, and the relationship between them, are then investigated for the common hydrocarbon-oxygen-argon mixtures at different initial conditions.

## 2 Experimental Details

The direct measurement of the critical tube diameter and the critical energy has been carried out in a number of mixtures (i.e.  $C_2H_2$ - $O_2$ ,  $C_2H_2$ - $2.5O_2$ ,  $C_2H_2$ - $4O_2$ ,  $C_2H_4$ - $3O_2$ ,  $C_3H_8$ - $5O_2$ ,  $C_2H_2$ - $2.5O_2$ -50%Ar,  $C_2H_2$ - $2.5O_2$ -70%Ar). The schematic of the critical tube diameter experiments is shown Fig. 1. The detonation is first initiated by a high voltage spark ignition source at the top of the vertical circular steel tube and exits into a large spherical bomb chamber at the other end. A photo probe and a shock pin are mounted at the top and bottom of the spherical bomb, which are used to determine the time-of-arrival signal of the wave.



Fig. 1: Schematic of the experimental setup for the critical tube diameter measurement.

Typical traces for a surviving diverging detonation wave, i.e. for  $d > d_c$  and for a detonation failure case in stoichiometric C<sub>2</sub>H<sub>2</sub>-2.5O<sub>2</sub> mixture with the tube diameter of 19.05 mm and an initial pressure of  $p_0 = 12$  kPa and 11 kPa are shown in Fig. 2 and Fig. 3, respectively.



**Fig.2:** Arrival time traces of a planar detonation emerging into the unconfined space: successful initiation of a spherical detonation for  $C_2H_2$ -2.5 $O_2$  mixture at the initial pressure of 12 kPa.

**Fig. 3:** Arrival time traces of a planar detonation emerging into the unconfined space: unsuccessful initiation of a spherical detonation for  $C_2H_2$ -2.5 $O_2$  mixture at the initial pressure of 11 kPa.

It can be seen from Fig. 2 that at an initial pressure of 12 kPa, the arrival time of the expanding wave is 201  $\mu$ s when it reaches the photo probe and 317  $\mu$ s at the shock pin. The velocity of the wave is 2073.4 m/s and 2136.7 m/s in the vertical tube and spherical chamber, which is of 91.1% and 94.4% of CJ detonation velocity, respectively. It shows that at the initial pressure of 12 kPa, the tube diameter is above the critical value, thus the planar detonation can successfully transmit into a spherical detonation.

For an unsuccessful transmission when  $d < d_c$ , Fig. 3 shows that when the initial pressure decreases to 11 kPa, although a detonation wave propagates in the vertical tube at a velocity around

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90% CJ detonation velocity, the detonation fails after exiting into the free space and the velocity of the expanding wave is only 23.6% of CJ velocity value. For each successful and unsuccessful initiation of spherical detonation at least 3 shots are repeated to confirm the critical pressure at which a spherical detonation can form for each tube diameter. In some experiments, the tube inner diameters are also varied via inserting smaller diameter tubes.

Using the same spherical bomb setup, experiments are also carried to determine the energy for direct blast initiation using the same mixtures and initial conditions considered in the critical tube diameter problem. The procedure to distinguish detonation initiation and details to estimate the actual spark discharge energy from the ignition system can be found in authors' previous studies [7, 8].

## **3** Results and Discussion

#### 3.1 Critical tube diameter and cell size

Similar to many previous studies, it is worth correlating the present critical tube diameters with available experimental measured detonation cell size data tabulated in [9]. The curve fit correlations of cell size as a function of initial pressure for the mixtures are illustrated in Table 1. The critical tube diameters for various mixtures as a function of initial pressure obtained by experiment are shown in Fig. 4. The correlations for some mixtures, which are based on the experimental data measured by Matsui and Lee [2], are also included for comparison. It can be seen from Fig. 4 that for the undiluted mixtures, the experimental data from this study is in good agreement with those found by Matsui and Lee [2]. For the stoichiometric acetylene and oxygen diluted with 50% and 70% argon mixtures, there is no previous data to compare. Nevertheless, from the experimental observation, with the increasing amount of argon dilution the initial pressure at which the planar detonation can transmit into a spherical detonation increases consequently using the same tube diameter; in other words, with more argon dilution, the mixture tend to be less sensitive to form a spherical detonation when the planar detonation emerges from the tube into unconfined space.

Mixtures	С	α
C <sub>2</sub> H <sub>2</sub> -O <sub>2</sub>	9.2382	0.9625
$C_2H_2-2.5O_2$	26.262	1.1889
$C_2H_2-4O_2$	54.967	1.1656
$C_2H_4-3O_2$	56.458	0.9736
$C_3H_8-5O_2$	186.55	1.1729
C <sub>2</sub> H <sub>2</sub> -2.5O <sub>2</sub> -50%Ar [14]	61.5	1.12
C <sub>2</sub> H <sub>2</sub> -2.5O <sub>2</sub> -70%Ar [14]	113.8	1.20

**Table 1.** Cell size correlations for mixtures as a function of initial pressure given by  $\lambda[mm] = C(P_0[KPa])^{-\alpha}$ 





**Fig.4** The variation of critical tube diameter as a function of initial pressure.

**Fig.5** The relationship between critical tube diameter and cell size for each mixture.

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From Table 1 and Fig. 4, the relationship between the critical tube and cell size for each mixture can be readily obtained, which is shown in Fig. 5. For the unstable mixtures without Ar dilution the critical tube diameter is found to be around 11 to 14 times of the cell size, which is in agreement with the empirical correlation  $d_c \approx 13\lambda$  [10]. Similar to previous investigation [11-13], the present experiment data also confirms that for mixtures highly diluted with argon, the  $d_c \approx 13\lambda$  correlation breaks down. It is found numerically that the proportional factor equals 21 and 29 for the stoichiometric acetylene-oxygen mixtures with 50% and 70% argon dilution, respectively. Detonations in highly argon diluted mixtures are stable and their propagation mechanism is different from that of cellular detonations in unstable mixtures [14, 15], thus the failure and re-initiation of a diffracting stable detonation emerging from a tube into unconfined space are also different, resulting in the breakdown of the  $d_c \approx 13\lambda$  correlation.

#### 3.2 Critical tube diameter and critical energy

To explore the direct link between the critical tube diameter and the critical energy for direct initiation, we consider some existing initiation models from previous studies. First, with the knowledge of experimentally measured critical tube diameters at different initial pressures, the critical energy can be theoretically estimated by the surface energy model proposed by Lee [5]. The surface energy model is a semi-empirical phenomenological model that relates the point blast initiation mode with the planar wave initiation mode. The link is established based on the minimum surface energy of the critical tube to the surface area of the critical size of the minimum detonation kernel; in other words, the correlation is through equating the surface energy contained in the wave in both cases at criticality. Thus the surface energy contained in the point blast initiated spherical detonation wave at the time when the wave has decayed to the CJ state from its overdriven state is equivalent to the energy in the planar detonation wave in the critical tube diameter situation. This model leads to:

$$E_c = 4\pi\gamma p_o M_{CJ}^2 I \left(\frac{d_C}{4}\right)^3 \tag{1}$$

In this study, a simplified work done model is also investigated [2, 16, 17]. Assuming the energy needed to re-initiate a detonation downstream of the unconfined space in the critical tube diameter problem can be related to the work done delivered by the detonation product in the confined tube (i.e., a fictitious piston) over a period  $t^*$ , the energy can be obtained by:

$$E_{c} = \int_{0}^{t^{*}} p_{CJ} \frac{\pi d_{c}^{2}}{4} u_{CJ} dt$$
 (2)

where  $p_{CJ}$  and  $u_{CJ}$  denote the CJ detonation pressure and particle velocity, respectively.  $t^*$  can be modeled as the time when the rarefaction wave reaches the tube axis, which can be approximately by  $t^* \sim d_c/2a_{CJ}$  with  $a_{CJ}$  being the sound speed of the detonation products. Therefore, a simplified work done model expression can be given as:

$$E_{c} = \frac{p_{CJ} u_{CJ} \pi d_{c}^{3}}{8 a_{CJ}}$$
(3)

The theoretical prediction and experimental measured critical energy as a function of initial pressure for the equimolar  $C_2H_2$ - $O_2$  mixtures is shown in Table 2. From the comparison, it is found that the theoretical critical energies obtained from both models are very close to the experimental measured data. Comparing the theoretical and experimental critical energies of the other mixtures which are shown in Fig. 6 and Fig. 7, similar scenario can be found. Hence, it can be concluded that for either unstable and stable mixtures, both the surface energy model and the simplified work done model provide equally well the direct link between the critical tube problem and direct blast initiation of gaseous detonation with reasonable accuracy (The maximum % deviation from all the experimental results for the surface energy model is 51%, and 92.4% for the work done model).

Zhang, B. et al. Relationship between critical tube diameter and initiation energy Table.2 Theoretical and experimental critical energy for C2H2-O2 at different initial pressure Tube diameter Initial pressure  $E(\mathbf{J})$  $E(\mathbf{J})$  $E(\mathbf{J})$ Mixture (mm) (kPa) (Surface energy model) (Work done model) (Experimental) 19.05 6 0.34 0.57 0.36-0.41 16.1 8 0.28 0.46 0.28-0.32  $C_2H_2-O_2$ 

0.18

0.31

0.17-0.20



**Fig.6** Critical energy as a function of critical tube diameter for a)  $C_2H_2$ - $O_2$ ; b)  $C_2H_2$ - $2.5O_2$ ; c)  $C_2H_2$ - $4O_2$ ; d)  $C_2H_4$ - $3O_2$  and e)  $C_3H_8$ - $5O_2$  mixtures.

12.97

10



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Fig.7 Critical energy as a function of critical tube diameter for a)  $C_2H_2$ -2.5 $O_2$ -50%Ar; b)  $C_2H_2$ -2.5 $O_2$ -70%Ar.

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