Gasdynamics in Turbulent Premixed Combustion: Conditionally Averaged Unclosed Equations and Analytical Formulation of the Problem

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1 Introduction

Instantaneous turbulent combustion in the turbulent premixed flame takes place in thin and strongly wrinkled "flamelet sheet", which often is treated as a wrinkled laminar flame, the BML combustion model [1]. In used below mathematical model the flame speed $S_L = const$ and flame width is zero: a limiting case when the molecular coefficient and chemical time tends to zero, but their ratio tends to a constant. Commonly used theoretical basis for modelling of such turbulent premixed combustion is the Favre averaged unclosed combustion and hydrodynamics equations (Eqs. (1) and (2) below), where modelling of the scalar flux $\overline{\rho\vec{u}''c''}$, stress tensor $\overline{\rho\vec{u}''\vec{u}''}$ and chemical source $\overline{\rho W} = \overline{\rho}\widetilde{W}$ is a challenge. The difficulty of modelling of $\overline{\rho\vec{u}''c''}$ and $\overline{\rho\vec{u}''\vec{u}''}$ is caused by gasdynamics. The point is that combustion generates falling pressure across the flame and it yields different pressure-driven acceleration of relatively heavy reactants and light products. The consequence is that the mean scalar flux $\overline{\rho\vec{u}''c''}$ is usually oppositely directed in comparison with the intuitive expectation: not from products to reactants, but in the opposite direction (the counter-gradient transport). This gasdynamic effect also controls strong increasing of the velocity fluctuations in the flame.

Our statement is that the problem of modelling of $\overline{\rho\vec{u}''c''}$ and $\overline{\rho\vec{u}''\vec{u}''}$ can be eliminated by reformulation of the problem, namely by using conditionally averaged continuity and impulse equations instead of the Favre averaged ones. In stated below unclosed equations, which constitute a basis of proposed alternative formulation of the problem) the requiring modelling unknowns are only the conditional averaged turbulent stresses $(\overline{u_i'u_j'})_u, (\overline{u_i'u_j'})_b$ and mean chemical source $\overline{\rho W} = \overline{\rho}\widetilde{W}$. We present also unclosed equations for these unknowns

2 Formulation of the problem and the main equations

The unclosed Favre averaged equations of the turbulent premixed combustion are as follows:

$$\begin{cases} \partial\,\overline{\rho}\widetilde{c}/\partial t + \nabla\cdot\overline{\rho}\widetilde{\vec{u}}\widetilde{c} + \nabla\cdot\overline{\rho\vec{u}''c''} = \overline{\rho}\widetilde{W}\ (a), \quad \overline{\rho} = \rho_u/[1+\widetilde{c}(\rho_u/\rho_b - 1)]\ (b) & (1) \\ \partial\overline{\rho}\widetilde{\vec{u}}/\partial t + \nabla\cdot\overline{\rho}\widetilde{\vec{u}}\widetilde{\vec{u}} + \nabla\cdot\overline{\rho\vec{u}''\vec{u}''} = -\nabla\overline{p}\ (a), \quad \partial\,\overline{\rho}/\partial t + \nabla\cdot\overline{\rho}\widetilde{\vec{u}} = 0\quad (b), & (2) \end{cases}$$

where the ρ_u and ρ_b are the densities of unburned and burned gases, $\tilde{a} = \overline{\rho a}/\bar{\rho}$ and $a = \tilde{a} + a''$. Eqs. (1) and Eqs. (2) describe correspondingly the combustion and attendant turbulence hydrodynamic sub-problems, which are coupled due to common mean density $\bar{\rho}$ and velocity $\tilde{\vec{u}}$ problems. The requiring modelling unknowns can be presented with known from the literature expressions:

$$\overline{\rho\vec{u}''c''} = \bar{\rho}\tilde{c}(1-\tilde{c})(\bar{\vec{u}}_b - \bar{\vec{u}}_u) \ (a), \quad \bar{\rho}\tilde{W} = \rho_u S_L \bar{\Sigma} \ (b), \tag{3}$$

$$\overline{\rho u_i'' u_j''} = \bar{\rho}(1-\tilde{c})(\overline{u_i' u_j'})_u + \bar{\rho}\tilde{c}(\overline{u_i' u_j'})_b + \bar{\rho}\tilde{c}(\bar{u}_{i,b} - \bar{u}_{i,u})(\bar{u}_{j,b} - \bar{u}_{j,u}), \tag{4}$$

where S_L is defined in reference to reactants and $\bar{\Sigma}$ is the mean area of the wrinkled surface per unite volume. As the conditional velocities $\bar{\vec{u}}_u$ and $\bar{\vec{u}}_b$ are controlled by turbulence and gasdynamics it means that in turn these two mechanisms control $\overline{\rho\vec{u}''c''}$ and $\overline{\rho\vec{u}''\vec{u}''}$, and their modelling in terms of the turbulent diffusion and viscosity coefficient is not theoretically justified. Namely balance between these mechanisms determines the direction of scalar flux (gradient, counter-gradient or neutral) as it was shown in the context of an original gasdynamic model developed in [2]. But stated below unclosed equations in terms of $\bar{\vec{u}}_u$ and $\bar{\vec{u}}_b$ makes their modeling with this gasdynamic model as well as using turbulent type closure of the unclosed $\overline{\rho u_i'' c''}-$ and $\overline{\rho u_i'' u_j''}-$ equations [3].

3. Conditionally averaged hydrodynamics equations

Now we derive conditionally averaged continuity and impulse, which in our formulation of the problem replace Favere averaged Eqs. (2a,b). To avoid invoking the tool of the generalized function we directly split up each of these equations into two conditionally averaged ones. Using our "splitting method" we not only obtain two known mathematically equivalent forms of the impulse equations presented in [3] and [4], but also show existence of the third form of them with less numbers of unknows. The following exact expressions will be used

$$\bar{\rho} = (1-\bar{c})\rho_u + \bar{c}\rho_b \ (a), \quad \tilde{u}_i = (1-\tilde{c})\bar{u}_{i,u} + \tilde{c}\bar{u}_{i,b} \ (b), \quad \bar{c} = \rho_u\tilde{c}/[\rho_b + \tilde{c}(\rho_u - \rho_b)] \ (c). \tag{5}$$

The continuity equations. The identity $\bar{\rho} = \bar{\rho}\tilde{c} + \bar{\rho}(1-\tilde{c})$ and Eq. (5b) present Eq. (2b) as

$$\left\{\partial[\bar{\rho}(1-\tilde{c})]/\partial t + \nabla\cdot[\bar{\rho}(1-\tilde{c})\bar{\vec{u}}_u]\right\}_u + \left\{\partial(\bar{\rho}\tilde{c})/\partial t + \nabla\cdot(\bar{\rho}\tilde{c}\bar{\vec{u}}_b)\right\}_b = 0, \tag{6}$$

where expressions in the braces $\{\}_u$ and $\{\}_b$ refer to reactants and products, respectively. We can easy to check using Eq. (5b) that LHS of Eq. (1a) after implementation of Eq. (3a) is identical to the expression in the braces $\{\}_b$. It means that the expressions in the braces are equal to $\{\}_b = \bar{\rho}\tilde{W}$ and, after using Eq. (6), $\{\}_u = -\bar{\rho}\tilde{W}$. Hence conditionally averaged continuity equations are as follows:

$$\partial[\bar{\rho}(1-\tilde{c})]/\partial t + \nabla\cdot[\bar{\rho}(1-\tilde{c})\bar{\vec{u}}_u] = -\bar{\rho}\tilde{W} \ (a), \quad \partial(\bar{\rho}\tilde{c})/\partial t + \nabla\cdot(\bar{\rho}\tilde{c}\bar{\vec{u}}_b) = \bar{\rho}\tilde{W} \ (b). \tag{7}$$

Eliminating $\bar{\rho}$ and $\tilde{c}$ from Eq. (7) by using $\tilde{c} = \overline{\rho c}/\bar{\rho}$ (Favre averaging) and Eq. (5a) we have

$$\partial[\rho_u(1-\bar{c})]/\partial t + \nabla\cdot[\rho_u(1-\bar{c})\bar{\vec{u}}_u] = -\bar{\rho}\tilde{W} \ (a), \quad \partial(\rho_b\bar{c})/\partial t + \nabla\cdot(\rho_b\bar{c}\bar{\vec{u}}_b) = \bar{\rho}\tilde{W} \ (b). \tag{8}$$

Eqs. (8a, b) are identical to Eqs. (7, 8) from [3] and to Eqs. (23, 24) from [4].

The impulse equations. Quite similar we can present LHS of Eq. (2a) (see also Eq. (35) from [5]) as a sum terms referring to the unburned and burned gases:

$$\begin{aligned}&\left\{\partial[\overline{\rho}(1-\tilde{c})\overline{\vec{u}}_u]/\partial t+\nabla\cdot[\overline{\rho}(1-\tilde{c})\overline{\vec{u}}_u\overline{\vec{u}}_u)]+\nabla\cdot[\overline{\rho}(1-\tilde{c})(\overline{\vec{u}'\vec{u}'})_u]\right\}_u\\&+\left\{\partial[\overline{\rho}\tilde{c}\overline{u}_{i,b}]/\partial t+\nabla\cdot[\overline{\rho}\tilde{c}\overline{\vec{u}}_b\overline{\vec{u}}_b)]+\nabla\cdot[\overline{\rho}\tilde{c}(\overline{\vec{u}'\vec{u}'})_b]\right\}_b=-\nabla[(1-\overline{c})\overline{p}_u]-\nabla(\overline{c}\overline{p}_b),\end{aligned} \tag{9}$$

so the conditionally averaged impulse equations, which follow from Eq. (2b), are as follows:

$$\begin{cases}\partial[\overline{\rho}(1-\tilde{c})\overline{\vec{u}}_u]/\partial t+\nabla\cdot[\overline{\rho}(1-\tilde{c})\overline{\vec{u}}_u\overline{\vec{u}}_u)]+\nabla\cdot[\overline{\rho}(1-\tilde{c})(\overline{\vec{u}'\vec{u}'})_u=-\nabla[(1-\overline{c})\overline{p}_u]+\vec{F}_u\ \ (a),\\ \partial[\overline{\rho}\tilde{c}\overline{u}_{i,b}]/\partial t+\nabla\cdot[\overline{\rho}\tilde{c}\overline{\vec{u}}_b\overline{\vec{u}}_b)]+\nabla\cdot[\overline{\rho}\tilde{c}(\overline{\vec{u}'\vec{u}'})_b]=-\nabla(\overline{c}\overline{p}_b)+\vec{F}_b\ \ (b),\end{cases} \tag{10}$$

where $(\overline{\vec{u}'\vec{u}'})_u$ and $(\overline{\vec{u}'\vec{u}'})_b$ are conditional turbulent stresses, $\vec{F}_u$ and $\vec{F}_b$ are equal by value and oppositely directed forces, which cancel out in the Favre averaged equation $(\vec{F}_u+\vec{F}_b=0)$.

There are three mathematically equivalent forms of RHS of Eqs. (10a,b) , which depend on the used form of the impulse conservation law on the flamelet sheet. The point is that at statistical description we have instantaneous integral transformation of reactants into product, Eq. (10a), which, in fact, takes place in three stages, Eq.(11b):

$$\rho_u,\overline{\vec{u}}_u\Rightarrow\rho_b,\overline{\vec{u}}_b\ \ (a),\qquad \rho_u,\overline{\vec{u}}_u\underset{1}{\Rightarrow}\rho_u,\overline{\vec{u}}_{su}\underset{2}{\Rightarrow}\rho_b,\overline{\vec{u}}_{sb}\underset{3}{\Rightarrow}\rho_b,\overline{\vec{u}}_b\ \ (b) \tag{11}$$

where $\overline{\vec{u}}_{su}$ and $\overline{\vec{u}}_{sb}$ are the mean velocities on the surfaces adjacent to the instantaneous flame. The stage 2 in Eq. (11b) is transformation of parameters in the flame surface, while the stages 1 and 2 take place correspondingly in reactants s and products.

1. The first form of the conditionally averaged impulse equations corresponds to the following form of the impulse conservation law on the instantaneous flame surface, Eq. (13) from [3]:

$$\overline{\vec{u}}_{su}\overline{\rho}\tilde{W}-(\overline{p\vec{n}})_{su}\overline{\Sigma}=\overline{\vec{u}}_{sb}\overline{\rho}\tilde{W}-(\overline{p\vec{n}})_{sb}\overline{\Sigma}. \tag{12}$$

Representing RHS of Eq. (2a) as follows (where the expression in the braces is equal zero, Eq. (12)):

$$=-\overline{\nabla p}=-\nabla\overline{p}=-\nabla[(1-\overline{c})\overline{p}_u]-\nabla(\overline{c}\overline{p}_b)+\{[\overline{\vec{u}}_{sb}\overline{\rho}\tilde{W}-(\overline{p\vec{n}})_{sb}\overline{\Sigma}]-[\overline{\vec{u}}_{su}\overline{\rho}\tilde{W}-(\overline{p\vec{n}})_{su}\overline{\Sigma}]\}$$

and splitting give the first form of RHS in Eqs. (10):

$$=-\nabla[(1-\overline{c})\overline{p}_u]-\overline{\vec{u}}_{su}\overline{\rho}\tilde{W}+(\overline{p\vec{n}})_{su}\overline{\Sigma}\ \ (a),\qquad =-\nabla(\overline{c}\overline{p}_b)+\overline{\vec{u}}_{sb}\overline{\rho}\tilde{W}-(\overline{p\vec{n}})_{sb}\overline{\Sigma}\ \ (b). \tag{13}$$

2. The second form of the conditionally averaged momentum equations corresponds to the following impulse conservation law on the instantaneous flame surface:

$$\overline{\rho}\tilde{W}(\overline{\vec{u}}_{sb}-\overline{\vec{u}}_{su})=-\ \overline{(\nabla \mathrm{p})}_f, \tag{14}$$

where $\overline{(\nabla p)}_f$ is the mean pressure gradient generated by the flame surface. The physical meaning of Eq. (14) is as follows: the rate of change of the mean moment due to transformation of reactants into product with the intensity $\overline{\rho}\widetilde{W}$ in the flame surface (the stage 2 in Eq.(12) is equal to the mean pressure gradient yielding this change. From Esq. (12) and (14) it follows that $\overline{(\nabla \mathrm{p})}_f = [\overline{(p\vec{n})}_{sb} - \overline{(p\vec{n})}_{sb}]\overline{\Sigma}$, i.e. $\overline{(\nabla \mathrm{p})}_f$ expresses in another form an effect difference of the pressure on the adjacent surfaces. It is important that $\overline{(\nabla \mathrm{p})}_f \neq \nabla \overline{\mathrm{p}}_\mathrm{f}$ due to jumping of the pressure $\overline{\mathrm{p}}_\mathrm{f}$.

So we represent RHS of Eq. (2a) as follows (the expressions in the braces are equal to zero, Eq. (14)):

$$
\begin{aligned}
&= -\overline{\nabla p} + \{\overline{(\nabla \mathrm{p})}_f + \overline{\vec{u}}_{sb}\overline{\rho}\widetilde{W} - \overline{\vec{u}}_{su}\overline{\rho}\widetilde{W}\} = -(1-\overline{c})\overline{(\nabla p)}_u - \overline{c}\,\overline{(\nabla p)}_b + \{(1-\overline{c})\overline{(\nabla \mathrm{p})}_{fu} \\
&+ \overline{c}\,\overline{(\nabla \mathrm{p})}_{fb} + \overline{\vec{u}}_{sb}\overline{\rho}\widetilde{W} - \overline{\vec{u}}_{su}\overline{\rho}\widetilde{W}\} = -(1-\overline{c})\overline{(\nabla p)}^*_u - \overline{c}\,\overline{(\nabla p)}^*_b + \overline{\vec{u}}_{sb}\overline{\rho}\widetilde{W} - \overline{\vec{u}}_{su}\overline{\rho}\widetilde{W}
\end{aligned}
\tag{15}
$$

where $\overline{(\nabla p)}^*_u = \overline{(\nabla p)}_u - \overline{(\nabla \mathrm{p})}_{fu}$ and $\overline{(\nabla p)}^*_b = \overline{(\nabla p)}_b - \overline{(\nabla \mathrm{p})}_{fb}$. Splitting of Eq. (15) yields as follows:

$$
= -(1-\overline{c})\overline{(\nabla p)}^*_u - \overline{\vec{u}}_{su}\overline{\rho}\widetilde{W}\ (a), \qquad = -\overline{c}\,\overline{(\nabla p)}^*_b + \overline{\vec{u}}_{sb}\overline{\rho}\widetilde{W}(b). \tag{16}
$$

Eq. (16) results from Eq. (15) as (a) and (b) refer to different gases. We stress that though in Eq. (2a) $\nabla\overline{p} = \overline{(\nabla p)} = (1-\overline{c})\overline{(\nabla p)}_u + \overline{c}\,\overline{(\nabla p)}_b$ in Eq. (14) $(1-\overline{c})\overline{(\nabla p)}^\bullet_u + \overline{c}\,\overline{(\nabla p)}^\bullet_b \neq \overline{(\nabla p)} = \nabla\overline{p}$, Eq. (19e).

3. For deriving the impulse equations in the third form we use the impulse conservation law on the wrinkled dividing surface analysing transformation of reactants into products as one-step process in accordance with Eq. (11a). This equation is as follows:

$$
\overline{\rho}\widetilde{W}(\overline{\vec{u}}_b - \overline{\vec{u}}_u) = -\nabla\cdot\overline{(\rho\vec{u}''\vec{u}'')}_d - \overline{(\nabla \mathrm{p})}_d \quad [= -\nabla\overline{p}_d = -\nabla[(1-\overline{c})\,\overline{p}_{du}] - \nabla(\overline{c}\cdot\overline{p}_{db})], \tag{17}
$$

where $\overline{(\rho\vec{u}''\vec{u}'')}_d$ and $\overline{(\nabla \mathrm{p})}_d$ are the mean stress tensor and pressure gradient generated by the dividing reactants and product surface due to instantaneous transformation of the unburned gas with the parameters $\rho_u, \overline{\vec{u}}_u$ into burned one with the with parameters $\rho_b, \overline{\vec{u}}_b$ with the intensity $\overline{\rho}\widetilde{W}$. The physical reason of appearing of the stresses $\overline{(\rho\vec{u}''\vec{u}'')}_d$ is the velocity fluctuations in the stages 1 and 3, Eq. (11b), which are confined together with the stage 2 in the integral dividing surface.

$$
\begin{bmatrix}
-\nabla\overline{p} = -\nabla\overline{p} + \{\nabla\overline{p}_d + \nabla\cdot\overline{(\rho\vec{u}''\vec{u}'')}_d + \overline{\vec{u}}_b\overline{\rho}\widetilde{W} - \overline{\vec{u}}_u\overline{\rho}\widetilde{W}\} = -\nabla\overline{p}^\circ + \nabla\cdot\overline{(\rho\vec{u}''\vec{u}'')}_d + \overline{\vec{u}}_b\overline{\rho}\widetilde{W} - \overline{\vec{u}}_u\overline{\rho}\widetilde{W} \\
= -\nabla[(1-\overline{c})\,\overline{p}^\circ_u] - \nabla[\overline{c}\cdot\overline{p}^\circ_b) + \nabla\cdot[\rho_u(1-\overline{c})\overline{(\vec{u}'\vec{u}')}_{du}] + \nabla\cdot[\rho_b\overline{c}\,\overline{(\vec{u}'\vec{u}')}_{db}] + \overline{\vec{u}}_b\overline{\rho}\widetilde{W} - \overline{\vec{u}}_u\overline{\rho}\widetilde{W}.
\end{bmatrix}
\tag{18}
$$

Splitting Eq. (18) results RHSs of Eqs. (10a) and (10b) and hence the following equations:

$$
\begin{cases}
\partial[\rho_u(1-\overline{c})\overline{\vec{u}}_u]/\partial t + \nabla\cdot[\rho_u(1-\overline{c})\overline{\vec{u}}_u\overline{\vec{u}}_u)] + \nabla\cdot[\rho_u(1-\overline{c})\overline{(\vec{u}'\vec{u}')}^\circ_u] = -\nabla[(1-\overline{c})\,\overline{p}^\circ_u] - \overline{\vec{u}}_u\overline{\rho}\widetilde{W}\ (a), \\
\partial[\rho_b\overline{c}\overline{\vec{u}}_b]/\partial t + \nabla\cdot[\rho_b\overline{c}\overline{\vec{u}}_b\overline{\vec{u}}_b)] + \nabla\cdot[\rho_b\overline{c}\overline{(\vec{u}'\vec{u}')}^\circ_b] = -\nabla(\overline{c}\cdot\overline{p}^\circ_b) + \overline{\vec{u}}_b\overline{\rho}\widetilde{W}\ \ (b),
\end{cases}
\tag{19}
$$

where $\overline{p}^\circ = \overline{p} - \overline{p}_d$, $\overline{p}^\circ_u = \overline{p}_u - \overline{p}_{du}$, $\overline{p}^\circ_b = \overline{p}_b - \overline{p}_{bd}$. Below we omit the index "$^\circ$" and it is in fact is a physical assumption, permit to use in all equations the same $\overline{(\vec{u}'\vec{u}')}_u, \overline{(\vec{u}'\vec{u}')}_b$ and $\overline{p}_u, \overline{p}_b$.

In our derivation we proceed from the inequality $\overline{(\nabla p_f)} \neq \nabla\overline{p}_f$ in Eq. (14). The reason is that in the context of the mathematical model where the boundary is the flame surface, which divides reactants with $\overline{p}_{su},\overline{\vec{u}}_{su}$ and $\overline{p}_{sb},\overline{\vec{u}}_{sb}$ with $\overline{p}_{su} \neq \overline{p}_{u},\overline{\vec{u}}_{su} \neq,\overline{\vec{u}}_{u}$ and $\overline{p}_{sb} \neq \overline{p}_{b},\overline{\vec{u}}_{sb} \neq,\overline{\vec{u}}_{b}$, it takes place jump of mean parameters (the singularity) on the adjacent surfaces. In Eq. (17) we have $\overline{(\nabla \mathrm{p})}_{d} = \nabla\overline{p}_{d}$ as in the context of the mathematical model where the boundary divides reactants with $\overline{p}_{u},\overline{\vec{u}}_{u}$ and $\overline{p}_{b},\overline{\vec{u}}_{b}$ there is no this singularity; or, more precisely, this singularity is hidden inside the boundary.

Eqs. (10) (after elimination of $\overline{\rho},\tilde{c}$ similar to Eqs. (7, 8)) with the RHS Eqs. (13) and (16) are identical correspondingly Eqs. (25, 26) in [4] and (10, 11) in [3]. They are correct but not fit for our purpose as contains superfluous requiring modelling unknowns connected with parameters on the adjacent surfaces ($(\overline{\vec{u}}_{su},\overline{\vec{u}}_{sb},\overline{(p\vec{n})}_{su},\overline{(p\vec{n})}_{sb}$ in Eqs. (12) and $\overline{\vec{u}}_{su},\overline{\vec{u}}_{sb},\overline{(\nabla p)}^{*}_{u},\overline{(\nabla p)}^{*}_{b}$ in Eqs. (16).

4. Alternative set of equations and analytical formulation of the problem

A system of the unclosed equations of the turbulent premixed combustion, which consists of the combustion $\mathrm{A}-$subsystem and the hydrodynamic $\mathrm{B}-$subsystem, is as follows:

$$\mathrm{A}:\begin{cases}\partial\,\overline{\rho}\tilde{c}/\partial t+\nabla\cdot\overline{\rho}\tilde{\vec{u}}\tilde{c}=-\nabla\cdot\overline{\rho\vec{u}''c''}+\overline{\rho}\tilde{W}\ \ (a),\\ \overline{\rho}=(1-\overline{c})\rho_{u}+\overline{c}\rho_{b}=\rho_{u}/[1+\tilde{c}(\rho_{u}/\rho_{b}-1)]\ \ (b),\\ \overline{c}=\rho_{u}\tilde{c}/[\rho_{b}+\tilde{c}[\rho_{b}+\tilde{c}(\rho_{u}-\rho_{b})]\ \ (c),\ \ \overline{\rho\vec{u}''c''}=\overline{\rho}\tilde{c}(1-\tilde{c})(\overline{\vec{u}}_{b}-\overline{\vec{u}}_{u})\ \ (d).\end{cases}\tag{20}$$

$$\mathrm{B}:\begin{cases}\partial\,\overline{\rho}/\partial t+\nabla\cdot\overline{\rho}\tilde{c}\tilde{\vec{u}}=0\ \ (a),\quad \tilde{\vec{u}}=\overline{\vec{u}}_{u}(1-\tilde{c})+\overline{\vec{u}}_{b}\tilde{c}\ \ (b),\\ \partial(\overline{\rho}\tilde{c}\overline{\vec{u}}_{b})/\partial t+\nabla\cdot(\overline{\rho}\tilde{c}\overline{\vec{u}}_{b}\overline{\vec{u}}_{b})=-\nabla\cdot(\overline{\rho}\tilde{c}(\overline{\vec{u}'\vec{u}'})_{b}-\nabla(\overline{c}\cdot\overline{p}_{b})+\overline{\vec{u}}_{b}(\overline{\rho}\tilde{W})\ \ (c),\\ \partial[\overline{\rho}(1-\tilde{c})\overline{\vec{u}}_{u}]/\partial t+\nabla\cdot[\overline{\rho}(1-\tilde{c})\overline{\vec{u}}_{u}\overline{\vec{u}}_{u}=-\nabla\cdot[\overline{\rho}(1-\tilde{c})(\overline{\vec{u}'\vec{u}'})_{u}]\\ -\nabla[(1-\overline{c})\overline{p}_{u}]-\overline{\vec{u}}_{u}(\overline{\rho}\tilde{W})\ \ (d),\qquad \overline{p}_{u}-\overline{p}_{b}=\dot{m}\,|\,\overline{\vec{u}}_{u}-\overline{\vec{u}}_{b}\,|\ \ (\dot{m}=\rho_{u}S_{L})\ \ (e),\\ \partial\overline{\rho}\tilde{\vec{u}}/\partial t+\nabla\cdot(\overline{\rho}\tilde{\vec{u}}\tilde{\vec{u}})+\nabla\cdot\overline{\rho\vec{u}''\vec{u}''}=-\nabla\overline{p}\ \ (f),\\ \overline{\rho u''_{i}u''_{j}}=\overline{\rho}(1-\tilde{c})(\overline{u'_{i}u'_{j}})_{u}+\overline{\rho}\tilde{c}(\overline{u'_{i}u'_{j}})_{b}+\overline{\rho}\tilde{c}(\overline{u}_{i,b}-\overline{u}_{i,u})(\overline{u}_{j,b}-\overline{u}_{j,u})\ \ (g).\end{cases}\tag{21}$$

Eq. (21d) is the momentum conservation law on the instantaneous boundary between reactants and products. The global momentum equation, Eq. (21f) and the conditional averaged momentum equations, Eqs. (21c, d), are independent as Eq. (21f) follows from Eqs. (21c, d) and Eq. (18), but the latter does not present in the subsystem B. Requiring modelling unknown are $(\overline{\vec{u}'\vec{u}'})_{u},(\overline{\vec{u}'\vec{u}'})_{b}$ and $\tilde{W}$, The variables $\overline{c},\tilde{c},\overline{\rho},\tilde{\vec{u}},\overline{u}_{u},\overline{u}_{b},\overline{p},\overline{p}_{u},\overline{p}_{b},\overline{\rho\vec{u}''c''},\overline{\rho u''_{i}u''_{j}}$ are described by Eqs. (20) and (21).

The unclosed equations for $\tilde{W}$ and $(\overline{\vec{u}'\vec{u}'})_{u},(\overline{\vec{u}'\vec{u}'})_{b}$ are stated as follows:

$$\begin{bmatrix}\partial[\overline{\rho}\tilde{\alpha}(\overline{u'_{i}u'_{j}})_{\psi}]/\partial t+\partial[\overline{\rho}\tilde{\alpha}\cdot\overline{u}_{k,\psi}(\overline{u'_{i}u'_{j}})_{\psi}]/\partial x_{k}+\partial[\overline{\rho}\tilde{\alpha}(\overline{u'_{i}u'_{j}u'_{k}})_{\psi}]/\partial x_{k}\\ =-\overline{\rho}\tilde{\alpha}[(\overline{u'_{j}u'_{k}})_{\psi}\cdot\partial\overline{u}_{i,\psi}/\partial x_{k}+(\overline{u'_{i}u'_{k}})_{\psi}\cdot\partial\overline{u}_{j,\psi}/\partial x_{k}]+\overline{\alpha}[(\overline{u'_{j}\cdot\partial\tau_{i,k}/\partial x_{k}})_{\psi}\\ +(\overline{u'_{i}\cdot\partial\tau_{j,k}/\partial x_{k}})_{\psi}]-\overline{\alpha}[(\overline{u'_{j}\cdot\partial p/\partial x_{i}})_{\psi}+(\overline{u'_{i}\cdot\partial p/\partial x_{j}})_{\psi}]+\sigma(\overline{u'_{i}u'_{j}})_{\psi}(\overline{\rho}\tilde{W}).\end{bmatrix}\tag{22}$$

$$\begin{bmatrix}\partial(\overline{\rho}\tilde{W})/\partial t+\nabla\cdot(\overline{\rho}\tilde{\vec{u}}\tilde{W})+\nabla\cdot(\overline{\rho\vec{u}''W''})=-\rho_{u}S_{L}\nabla\cdot(\overline{\vec{n}W})\\ +\tilde{W}\overline{\rho(\nabla_{t}\cdot\vec{u})}+\overline{\rho W''(\nabla_{t}\cdot\vec{u})''}+2\rho_{u}S_{L}\overline{KW}.\end{bmatrix}\tag{23}$$

In eq.(22) $\alpha=c,\psi=b,\sigma=+1$ (in products) or $\alpha=1-c,\psi=u,\sigma=-1$ (in reactants). In fact eqs.(20) have been derived (but not till the end) in [5], Eqs.(38) and (39). $\nabla_{t}\cdot\vec{u}=\nabla\cdot\vec{u}-\vec{n}\vec{n}:\nabla\vec{u}$ and

$\vec{n} = -\nabla c(\vec{x},t) / |\nabla c(\vec{x},t)|$ in Eq. (23) are the rate of strain acting in the isosurface tangent plane and unit vector normal to the flame, $K = 0.5\nabla \cdot \vec{n}$ is the flame curvature. Eq. (23) was deduced in [6].
In principle we can deduce unclosed equations in terms of all unknowns in Eqs. (22) and (23) and so on, that yields an infinite system that gives an analytical formulation of the analysed problem. This set is asymptotically closed, while every finite subset is unclosed and needs modelling. From more general viewpoint in the light of the analytical formulation of the problem it is obvious that only stated third form impulse equations, RHS Eqs. (18), is fit for the formulation as the feasibility of deducing unclosed equations in terms of superfluous unknowns $\bar{\vec{u}}_{su}, \bar{\vec{u}}_{sb}, (\overline{p\vec{n}})_{su}, (\overline{p\vec{n}})_{sb}, (\overline{\nabla p})^*_u, (\overline{\nabla p})^*_b$, which appear in Eqs. (13) and (16), is questionable. The third form yields the minimal number of unknowns. Though the analytical formulation is obvious in the turbulence theory [7, p.9], it should be discussed for the analogous situation in the theory of turbulent premixed combustion.

5. Conclusion

We state an alternative formulation of the problem of the flamelet regime of premixed combustion based on deduced unclosed conditional averaged equations of continuity and impulse. They include gasdynamics impact on the scalar flux $\overline{\rho \vec{u}'' c''}$ and stress tensor $\overline{\rho \vec{u}'' \vec{u}''}$, and it permit to avoid their modeling. The requiring modeling unknowns in this formulation are only the conditional mean stresses $(\overline{\vec{u}' \vec{u}'})_u$, $(\overline{\vec{u}' \vec{u}'})_b$ and chemical source $\tilde{W}$. We state also the unclosed equations for them.

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