Two-Phase Spray in a Wake of Shattering Fuel Drop

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1 Introduction

Regime of energy release, which provides property of self-sustaining of detonation wave, in gas-droplets systems depends not only on kinetics of chemical reactions, but in more complicated way on liquid atomization, motion and evaporation of stripped mass, mixing of oxidizer with fuel vapors. Calculations show that atomization of parent drop causes the growth of total surface of liquid phase by 2-3 orders, which together with rapid evaporation of finest stripped daughter droplets leads to increasing of rate of fuel mass transfer to gaseous phase by 5-7 orders. So, formation of combustible mixture in a wake of fuel drop must be analyzed with respect to description of pointed processes. Such an investigation runs across obstacles which are caused by lack of knowledge about stripping kinetics: sizes, quantity and moments of tearing-off of stripped droplets. This did't allow to elaborate a mathematical model which would be able to predict in details kinetics of stripping and evolution of stripped mass – its motion and evaporation with due regard to dependencies from values of droplets radii r and regime of streamlining.

Model of shattering which is based on mechanism of dispersing due to action of so-called "gradient-instability" in conjugated boundary layers on drop surface provides all the necessary relations of shattering process [1]. The obtained distribution function $f_{\mathrm{n}}(r,t)$ and history of parent drop mass $m_{\mathrm{p}}(t)$ make it possible to describe quantitatively further processes of rapid acceleration and evaporation of spray of daughter droplets in a wake of shattering parent drop and may serve as a ground for model of formation and combustion of homogeneous inflammable mixture in a wake, and thereby – of heterogeneous detonation wave. The mathematical model build on this basis and some results of calculations of formation of polydispersed liquid-vapor spray in a wake of shattering drop are presented below.

2 Mathematical model of dynamics of liquid-vapor spray

The parent drop is considered as located in spray origin $x = 0$ source of daughter droplets, which are moving in a direction of spray axis OX with velocity $w_{\mathrm{d}}(r,\, x,\, t)$. In turn, each daughter droplet is a moving point source of vapor of capacity $\dot{F}_{\mathrm{v}}(r)$, and their totality forms the distribution of vapor mass $m_{\mathrm{v}}(x,\, t)$. Further evolution of $f_{\mathrm{n}}(r,x,t)$ can be described by equation of dispersed fuel, which was obtained in [2] on a base of analogy with motion of continuum fluid. It describes changing of density of distribution of daughter droplets quantity $f_{\rm n}(r,\,x,\,t)$ in axis OX direction, which proceeds with velocity $dx_{\rm d}/dt = w_{\rm d}(r,\,x,\,t)$ due to droplets acceleration by gas flow, and in direction of r -axis – due to evaporation, which proceeds with velocity dr/dt, determining by evaporation law. Equation of evolution must be supplemented with equation of motion of daughter droplets and with equation of vapors influx, evaporation law accounting for intensification due to streamlining of daughter droplets.

Thus, two-phase flow in spray of shattering drop is described by three dimensionless functions $f_{\rm n}(\tilde r, x', \tau)$, $W_{\rm d}(\tilde r, x', \tau)$, $M_{\rm v}(\tilde r, x', \tau)$, which are solutions of the system of differential equations of spray dynamics

$$\frac{\partial f_{\rm n}}{\partial \tau} + \frac{\partial}{\partial \tilde r}\left(-\frac{\tilde E\,{\rm Nu_d}}{16\tilde r} f_{\rm n}\right) + \frac{\partial}{\partial x'}\left(W_{\rm d} f_{\rm n}\right) = 0\,; \tag{1}$$

$$\frac{\partial W_{\rm d}}{\partial \tau} + W_{\rm d}\frac{\partial W_{\rm d}}{\partial x'} = \frac{3}{2}\frac{\sqrt{\alpha}\,C_{\rm d}}{B_1\,\tilde r}\frac{(1-W_{\rm d})^2}{2}\,; \tag{2}$$

$$\frac{\partial M_{\rm v}}{\partial \tau} + \frac{\partial M_{\rm v}}{\partial x'} = \dot F_{\rm v} f_{\rm n}\,\Delta\tilde r \tag{3}$$

where $\tilde r = r/(R_{\rm p0}B_1),\ x' = x\sqrt{\alpha}/2R_{\rm p0},\ W_{\rm d} = w_{\rm d}/V_\infty,\ \tilde E = 2E/(B_1^2\sqrt{\alpha}V_\infty R_{\rm p0})$, $M_{\rm v} = m_{\rm v}/m_{\rm p0}$, $\tau = t/t_{\rm ch}$, $t_{\rm ch} = 2R_{\rm p0}/\sqrt{\alpha}V_\infty$, $\alpha = \rho_\infty/\rho_l$, $\mu = \mu_\infty/\mu_l$, $B_1 = 0.51\pi\alpha^{1-2\,\xi}{\rm Re}_\infty^{-0.5}$ has a sense of scaling parameter for sizes of stripped droplets, $R_{\rm p}$ – parent drop radius, $\xi = \log_\alpha{(\alpha\,\mu)}^{1/3}$ is parameter of mutual viscous engagement of media in boundary layers [1]. It is assumed in (3), that vapors accelerate instantly to velocity of stream V_∞, and it is neglected by parent drop evaporation. The capacity of spray source was set in parent drop location $x' = 0$ as a given function $\dot F_{\rm s} = \dot f_{\rm n}(\tilde r,\ \tau)$. It is shown in [1], that $f_{\rm n}(\tilde r,\ \tau)$ depends on relation h of rates of two competitive for shattering processes – mass efflux due to dispersion and relaxational reducing of relative velocity of gas stream and parent drop. In the case when they are equal $h = 1$ it has a simple form

$$f_{\rm n}(\tilde r,\tau) = \frac{1-\exp(-A\tau)}{A\,\tilde r^2}\frac{B_2\sin^3\varphi_0(\tilde r)}{(8-2.5\,\tilde r^2\cos\varphi_{\,0}(\tilde r))} \tag{4}$$

where A is initial rate of mass efflux, $B_2 = 0.15{\rm Re}_\infty^{1.5}\alpha^{-3.5(1-2\,\xi)}$ has a sense of scaling parameter for quantity of stripped droplets, $\varphi_0(\tilde r)$ is an inverse to $\tilde r(\varphi)$ function in equation of curve $\tilde r(\varphi, 0) = const$. The dependence of drag coefficient $C_{\rm d}$ from droplets velocities and sizes was taken in the Kurten's form $C_{\rm d} = 24/{\rm Re_d} + 6/{\rm Re_d^{0.5}} + 0.28$, for ${\rm Re_d} < 4000$. To take into account increasing of evaporation rate at streamlining of droplets by speedy flow we used law $dr/dt = -E\,{\rm Nu_d}/16\,r$, ${\rm Nu_d} = 2 + 0.53{\rm Re_d^{0.5}}$ [3] with evaporation constant for hydrocarbon fuels $E = 2.7\cdot 10^{-6}m^2/\sec$. Then $\dot F_{\rm v} = 3/16\tilde r\tilde E\,B_1^3\,{\rm Nu_d}$.

3 Results of calculations

Formulated in this way non-stationary two-dimensional problem was solved numerically with a help of Lax – Vendroff finite-difference scheme. As function (4) is self-similar, diapason of droplets sizes $\tilde r_{\rm min} < \tilde r < \tilde r_{\rm max}$ is invariable and coincides with a base diapason $\tilde r_{l0} \approx \sqrt{3.2} < \tilde r < \tilde r_{\rm r0} = \sqrt{3\pi}$

[1], while magnitude of $f_{\rm n}$ varies in time. This circumstance is necessary condition for liquid-phase jet of spray to be stabilized to certain moment. In general case the base diapason sets up only initial distribution, and then, as shattering proceeds, the bounds of distribution shift decreasing at $h > 1$ and increasing at $h < 1$. The results, presented below, pertain to two variants of values of parameters for kerosene drop in air stream: *variant 1*: $V_\infty = 10^3 m/\sec$, $R_{\rm p0} = 5 \cdot 10^{-4} m$, $\mathrm{Re}_\infty = 9.17 \cdot 10^4$, $B_1 = 5.1 \cdot 10^{-3}$; *variant 2*: $V_\infty = 10^3 m/\sec$, $R_{\rm p0} = 5 \cdot 10^{-5} m$, $\mathrm{Re}_\infty = 9.17 \cdot 10^3$, $B_1 = 1.61 \cdot 10^{-2}$.

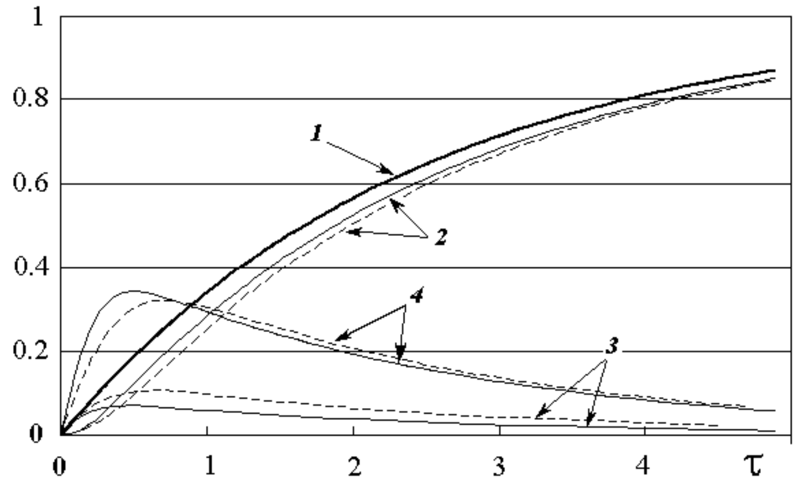
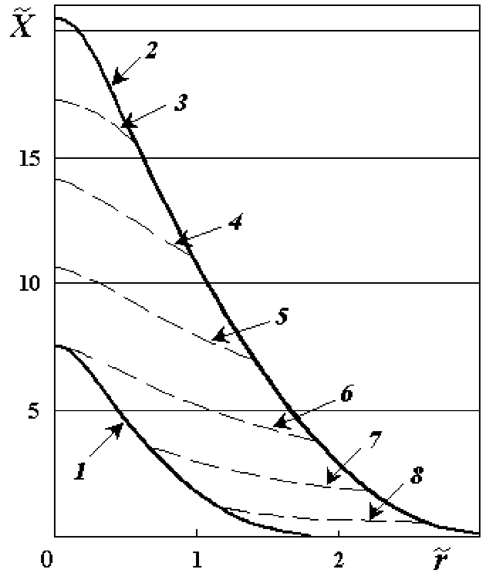


Figure 1: variation of masses in time. Solid lines – $variant\,1$, dashed lines – $variant\,2$.

The general regularities are shown on fig. 1 as dependencies of total stripped mass of liquid $M_{\rm s}(\tau)$ (*curve 1*) and total mass $M_{\rm v}(\tau)$, vaporized by daughter droplets (*curves 2*). They show that for conditions behind shock and detonation waves the intensification of evaporation is substantial, so, the fuel in spray is presented most in vapor phase. Maximum current value of liquid phase mass (*curves 3*) in the first variant $M_{l1} = 0.07$ was reached to $\tau_{l1} \approx 0.45$, while in the second – $M_{l2} = 0.10$ and $\tau_{l2} \approx 0.66$ respectively. At any moment the most part of mass of liquid phase is located in vicinity of parent drop and decreases sharply along spray axis. The mass rate of evaporation $dM_{\rm v}/d\tau$ (*curves 4*) exceeds maximum values to the moments $\tau_{\rm v1} \approx 0.50$, $\tau_{\rm v2} \approx 0.72$, increasing before due to increasing of mass stripped, and decreasing after due to beginning of vanishing (entire evaporation) of daughter droplets, that occur at $\tau_{\min 1} = 0.46$ and $\tau_{\min 2} \approx 0.65$ respectively.

4 Formation of liquid-phase jet

Two-phase spray consist of liquid polydispersed jet and vapor cloud. The moments $\tau_{\min}$ and $\tau_{\max}$ of entire evaporation of droplets of minimum $\tilde{r}_{\min}$ and maximum $\tilde{r}_{\max}$ in distribution radii, which were stripped at $\tau = 0$, are the characteristics of the process of formation of jet of stable length $l_{\rm j}$. The trajectories of these droplets, which were calculated with accordance to fields of $W_{\rm d}(\tilde{r}, x', \tau)$, $\mathrm{Re}_{\rm d}(\tilde{r}, x', \tau)$, $\mathrm{Nu}_{\rm d}(\tilde{r}, x', \tau)$ of system (1)–(3), are shown on fig. 2 (*curves 1, 2*) on ($\tilde{r} - \tilde{X}$) plane, $\tilde{X} = x/R_{\rm p0}$. At any moment they make a lateral bounds of liquid jet, and their tips correspond to entire evaporation of this droplets. The set of droplets from diapason $r_{\min} < r < r_{\max}$, which were stripped at $\tau = 0$, makes a top bound, which marked by dashed lines at various moments. At $\tau_{\min} < \tau < \tau_{\max}$ part of top bound turns into the line of droplets vanishing $\tilde{r} = 0$, and for $\tau > \tau_{\max}$ top bound



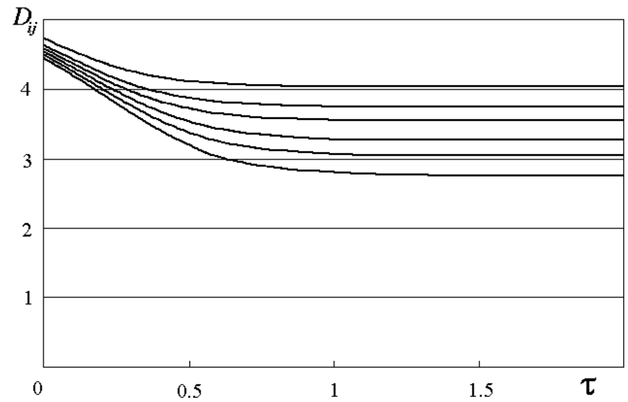


Figure 2: Left– formation of liquid jet; ***1, 2***– trajectories of droplets $\tilde{r}_{\rm min}$, $\tilde{r}_{\rm max}$; top bound (dashed) is given at $\tau_8 = 0.22$, $\tau_7 = 0.44$, $\tau_6 = 0.65$, $\tau_5 = 0.89$, $\tau_4 = 1.13$, $\tau_3 = 1.37$

Figure 3: Right– dependencies $D_{ij}(\tau)$ for *variant 1*

doesn't exist. The tip of jet is sharpen because it consists of smallest droplets, while astern and middle parts are widened, which testify to most polydispersity of jet at these cross sections. After $\tau_{\rm min}$ the vanishing of droplets begins, and middle part becomes the most part of jet, this means that polydispersity of jet increases in time. After $\tau_{\rm max}$ the vanishing of droplets of new radii doesn't occur, and this is the necessary condition for stabilization of length of jet, while quantity of droplets $f_{\rm n}$ decreases in accordance with source law (4). The stabilization proceeds successively along jet from astern part to the tip, which confirms by analysis of dispersive parameters given below. The length of path of droplet of maximum radius is the length of stable liquid jet: $l_{\max 1} \approx 20.5 R_{\rm p0}$ and $l_{\max 2} \approx 24.3 R_{\rm p0}$.

Time $\tau_{\rm max}$ of jet stabilization is much less then breakup time of parent drop: at $\tau_{\rm max}$ the drop loses only $M_{l1} = 0.38$ and $M_{l2} = 0.49$ of mass. This means that life time of stable liquid jet is substantial in comparison with period of its stabilization.

5 Dispersive parameters of spray

The proposed model allows to calculate the dispersive parameters of liquid jet – mean diameters $d_{ij}(x,t)$, – and their evolution in space and time. In considering non-stationary process these values vary in time, as well as along axis of spray. Consider therefore parameters of two kinds: $\Omega_{ij}(x) = (2R_{\rm p0})^{-1} d_{ij}(x, t_{\rm c})$ are defined at each cross section x and characterize spatial structure of jet at fixed moment $t_{\rm c}$, while $D_{ij}(t) = (2R_{\rm p0})^{-1} \int_0^{L_{\rm j}} d_{ij}(x,t)\, dx$ are calculated for totality of droplets at fixed t and describe temporal changing of dispersity of entire spray. The dependencies $D_{ij}(\tau)$ are given on fig. 3 for *variant 1* and show the stabilization of dispersive properties of spray; in each bunch here and on fig. 4 they are located in order (from top to bottom): $D_{43}, D_{32}, D_{31}, D_{30}, D_{20}, D_{10}$. Their initial values are: $D_{43} = 4.74$, $D_{32} = 4.64$, $D_{31} = 4.59$, $D_{30} = 4.54$, $D_{20} = 4.50$, $D_{10} = 4.45$. Sharp initial decreas-

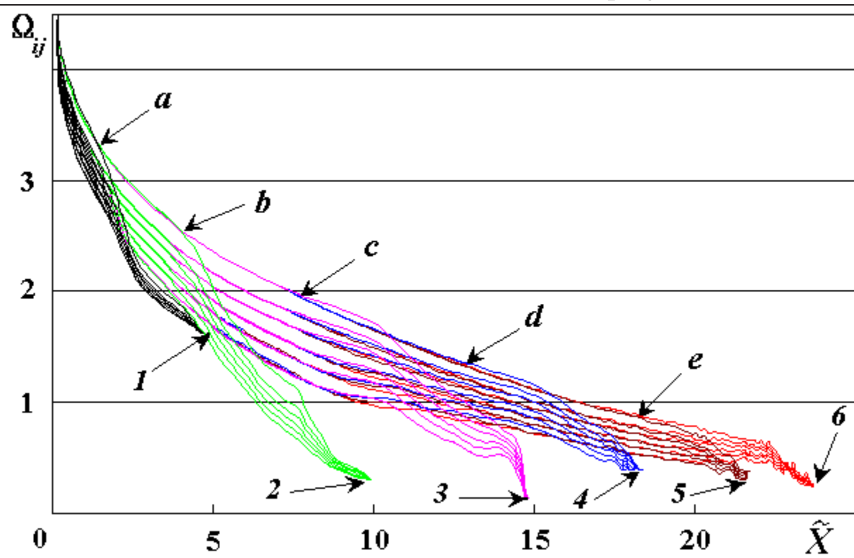


Figure 4: ***1–6*** – locations of tip of jet; $\Omega_{ij}(\tilde{X})$ are given at $\tau_1 = 0.3$, $\tau_2 = 0.5$, $\tau_3 = 0.8$, $\tau_4 = 1.0$, $\tau_5 = 1.2$, $\tau_6 = 1.6$; ***a–e*** – right-point bounds of stabilized part of jet; *variant 1*

ing of D_{ij} is caused by rapid evaporation and acceleration of droplets. Growth of divergence of curves means growth of polydispersity of jet due to non-homogeneity of fields of evaporation and acceleration rates. In time interval $\tau_{\min} < \tau < \tau_{\max}$ the rate of growth decreases, and soon after $\tau_{\max 1} = 1.17$ values D_{ij} remain still. The polydispersity of jet in stable state is much greater then in initial, produced by source.

The dependencies $\Omega_{ij}(\tilde{X})$ are shown on fig. 4 for different moments and they illustrate the evolution of liquid jet. At every moment the set of $\Omega_{ij}(\tilde{X})$ composes the bunch of curves. At the beginning the bunch is narrow, that testifies to low polydispersity because it contains smallest droplets of spray, which rapidly accelerate and evaporate. To the moment $\tau_{\min}$ the tip of jet descends down to $O\tilde{X}$ axis, after that length of jet increases until $\tau_{\max}$, as entire evaporation of all other droplets from diapason $\tilde{r}_{\min} < \tilde{r} < \tilde{r}_{\max}$ requires more time. At the same time the part of jet, where stabilization has finished, increases too (points *a*–*e*). Each curve and bunch as a whole tend to their limit positions as $\tau \to \tau_{\max}$, which approximately coincide with a bunch shown at $\tau = 1.6$. Soon after $\tau_{\max}$ all the parts of $\Omega_{ij}(\tilde{X})$ are stabilized and remain still. It ought to be noted, that near and after $\tau_{\max}$ the part of bunch that adjacent to tip bears corrugations, which can be seen at $\tau = 1.6$.

6 Formation of vapor cloud

The model allow to calculate the values of parameters, which are integral in each cross section of spray, but they may serve to obtain the mean values in each cross section. Let's assume that vapors occupy stream tube of cross area $S_{\text{s.t.}} = \pi\,(2.5R_{\text{p0}})^2$. The vapor cloud formation is illustrated on fig. 5 where the profiles of dimensionless mean density of vapors $\tilde{\rho}_{\text{v.m.}} = \rho_{\text{v.m.}}/\rho_l$ are presented for *variant 1*. At the beginning the capacity of source (4) is highest, therefore the intensification of evaporation due to rapid growth of surface of liquid is so large, that vapor wave appears, which has sharp front similar to that of blast wave. After lose contact with liquid jet this wave has convection drift, keeping its form invariable. Gradual weakening of $\dot{F}_\mathrm{s}$ leads to generation of rarefied wave, so, at distances from front to source $x_\mathrm{f} > 100\,R_\mathrm{p0}$ distribution tends to "triangular" form, which is inherent to blast waves.

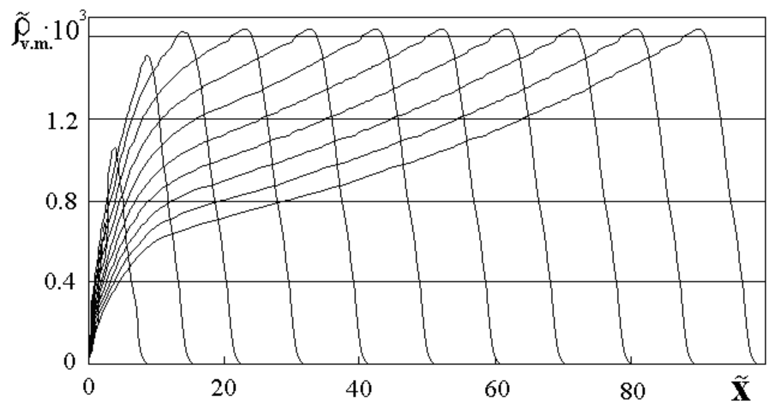


Figure 5: Profiles of $\tilde{\rho}_{\mathrm{v.m.}}(\tilde{X})$ at $\tau = 0.25; 0.46; 0.66; 0.96; 1.26; 1.56; 1.86; 2.16; 2.46; 2.76; 3.06$

These data show that fuel-air mixture in spray of shattering drop behind detonation wave is substantially overriched in average, as vapor density several times exceeds the stoichiometric values, except of regions near borders of spray. Vapor oversaturation leads to supercooling of the combustible mixture. Equation of heat balance for process of fuel-air mixing under considered conditions yields the dropping of temperature in about $280°\mathrm{K}$. With accounting for heat of vaporization such a dropping exceeds $300°\mathrm{K}$, which means that delay of ignition of combustible mixture may jump several orders high.

7 Conclusions

The model of dynamic processes in two-phase polydispersed spray of shattered in gas flow drop is elaborated. It consists of the system of differential equations of motion, evaporation and evolution of distribution function for stripped droplets, which interact with a gas stream. With the use of derived earlier distribution function as source function, calculations permitted to obtain detailed description of spray formation in a wake of shattering drop and to analyze structure of liquid-phase jet and vapor cloud there. The regularities of changing of mean diameters of spray in space and time are revealed. It is found that intensification of liquid mass transfer into vapor phase due to shattering leads to overriching and supercooling in the most part of spray volume. Development of the model will allow to reflect correctly kinetics of further processes of mixing, heating and chemical reactions of burning in a wake of fuel drop.

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