Non-linear theory of conservation laws of solid medium

Victor G. Grudnitsky

Dorodnicyn Computing Centre of RAS, 119333, Moscow, Russia

Non-linear character of the processes in a compressible solid medium is a defining factor inherent to flows with shock waves. The main theoretical apparatus applying for flow considerations is based on the conception of flow characteristics. This approach is used in calculations of flows with shock waves for analyses of difference schemes quality (stability, approximation and so on) and also for schemes construction. The apparatus is quasi linear because it is founded on the quasi linear equations. Here the shocks have a sense of singularities and are not the natural elements of the solution. Algebraic descriptions are widely employed for shocks evolution, the latter being not directly connected with the considered differential equations. Accepted divergent and quasi linear forms for the system of differential equations describing discontinuous solutions are not adapted to shock representation. These equations don't exist on the shocks because in these equations the discontinuous functions increments (and the increments of their fluxes) are related to the increments of the continuous quantities (independent time and space variables).

This work includes the results from [1]-[4]. The purpose of the research is to fill in the methodological gaps outlined above. The characteristic apparatus is derived by means of identity substitution without losing any type of the solutions. It is established that this transformation is based on the exact solution of the Riemann problem. In generalized characteristic form for the equations obtained in this manner the fluxes increments are related to the corresponding functions increments (i.e. the increments of the discontinuous quantities are related to the same ones) and the increments of the continuous quantities (independent time and space variables) are related to the corresponding increments of the continuous functions. Equations obtained allow a limit passage to the infinitely small increments of independent variables everywhere including the shocks, and characteristic velocities (directions) include the velocities of shock waves and contact discontinuities in addition to the velocities of quasi linear characteristics.

With the use of the non-linear approach it succeeded to establish the universal methodology of stability proving for the difference schemes based on the conservation laws in the divergent form. It is pointed out that satisfaction of derived stability constraints guarantees monotone character of the numerical solution in the area close to shock front (at least for the conservation laws in Cartesian coordinates). Sufficient conditions of a difference scheme stability have been obtained for several types of flows such as non-steady one- and two-dimensional flows, flows with viscosity, two-dimensional steady flows, supersonic over one coordinate flows.

Non-linear apparatus of the investigations of flows with shock waves gives a possibility of the correct investigation of a numerical solution quality (especially near shock wave front) and construction of more precise stable and monotone difference schemes.

Correspondence to: penyaz@dnp.itmo.by

References

[1] Grudnitsky VG. (1993). Generalized characteristics for the Euler system of equations. Algorithms for numerical investigation of discontinuous flows. Proc. CC RAS.

[2] Grudnitsky VG. (1998). Sufficient stability condition for explicit construction of shock solutions of the Euler system of equations. Dokl. RAN. 3:362.

[3] Grudnitsky VG. (2005). About the sufficient stability conditions for the S. K. Godunov scheme. Mathematical modeling. 12:17.

[4] Grudnitsky VG. (2008). Sufficient stability conditions in the calculations of steady supersonic flows by marching method and non-steady flows with viscosity . Mathematical modeling. 2: 20.