# Detonation Propagation as a System of Randomized Discrete Energy Sources

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#### 1 Introduction

Detonation waves in gaseous and homogeneous liquid explosives (e.g., nitromethane) are known to be inherently unstable, with the detonation front comprised of an ensemble of interacting wavelets. While the structure of the detonation front may be characterized by a single cell size, the overall dynamics of the front instabilities is largely stochastic, particularly for mixtures with a high effective activation energy. Detonation waves in highly heterogeneous explosives (e.g., blasting slurries with large prills) are similarly dominated by the random nature of the media itself. All of these systems exhibit anomalous behavior when scaling results from axisymmetric (cylindrical) geometry to two-dimensional (planar) geometry. For example, in a study of the critical diameter  $(d_c)$  and slot width  $(t_c)$  required for a gaseous detonation to emerge from a channel and continue to propagate in an unconfined environment by Benedick et al. [1], the scaling from  $d_c$  to  $t_c$  was found to be approximately 4:1, rather than the expected value of 2:1 as predicted by front curvature theory. Recently, a systematic study by Petel et al. [2] of critical diameter to critical thickness of explosive necessary for successful propagation showed a similar departure from the expected scaling: The critical diameter to thickness ratio was found to be on the order of 3:1 to 4:1 for four of the five different explosives studied. Only condensed-phase explosives with very fine-scale heterogeneities exhibited the expected 2:1 scaling of critical diameter to thickness, suggesting that detonation in these compositions is indeed governed by front curvature. A hypothesis to account for this deviation from the expected scaling for unstable and heterogeneous explosives was advanced by Petel et al. [3] as follows: For a composition in which the detonation front is inherently unstable or in a random heterogeneous media, the detonation front is able to exploit local fluctuations in energy release to continue propagation in high aspect ratio (wide) two-dimensional charges. In other words, in a high aspect ratio slab, the detonation is able to find new "avenues" of propagation across the front, while a cylindrical stick is more susceptible to failure.

In order to further investigate this hypothesis, this paper considers a "model problem" of detonation propagation in a system of point-like energy release centers embedded in an inert media. This problem of detonation in a system of discrete energy release is analogous to the problem of deflagration propagation in a system of discrete heat release centers [4], which can be solved analytically by superimposing the solution to the problem of a single-point heat release (Green's function). The detonation problem, in comparison, is complicated by the fact that the spherically expanding shock waves emanating from the point sources (i.e., spherical blast waves) interact in a highly nonlinear fashion and cannot be linearly superimposed. In general, this interaction can only be handled via a computational simulation of the flowfield. For this preliminary 3-D investigation, however, the model will assume linear superposition of blast waves in order to obtain a feel for the dynamics of this system. Although the model is entirely analytic, superimposing the blast waves from thousands of sources and determining the sequence of initiation of the next sources requires implementation on computer, similar to cellular automata.

## 2 Model

The media under consideration is assumed to have a energy release of  $E_o = 2.5 \times 10^6$  J/kg, with an initial pressure and density of 100 kPa and 1 kg/m<sup>3</sup>. This energy release corresponds to a CJ detonation velocity of 2253 m/s. For a one-dimensional system of planar of energy release (where the energy release is collapsed onto planar sheets of energy separated by inert gas), Higgins [5] previously showed that the propagation velocity for a sequence of "sympathetic detonations" in such a system is very close (within 6%) of the CJ detonation speed of the equivalent homogenized system. In the current work, the energy release will be collapsed into point sources, which release their energy instantaneously to generate a spherical blast wave. The blast wave flowfield is modeled using the first-order solution of Sakurai [6]. As the blast wave propagates outward, the blast encounters other sources, and these new sources are triggered if the blast overpressure reaches the initiation criterion. Different initiation scenarios are considered: (i) peak pressure (ii) integrated pressure (impulse) and (iii) critical fluence (defined as  $\int p^2 dt$ ). These initiation criteria are more applicable to condensed-phase media; future efforts can include using an Arrhenius-based kinetics model that would be more relevant to gases.



Figure 1: Pressure profiles for blast waves generated in a gaseous medium ( $\gamma = 1.4$ ) with total energy density  $E_o = 2.5 \times 10^6$  J/kg. Planar source in purple, cylindrical sources in cyan, spherical sources shaded gray. Note shock arrival time, peak pressure, and pressure profile in vicinity of l = 0.1 m (the average source spacing) are approximately equivalent for the three different geometries.

The blast waves that are generated by the triggered sources are linearly superimposed using a simple rule  $\Delta p = \sum_{i=1}^{N_{sources}} \Delta p_i$ . Other methods to superimpose blast pressures are reviewed by Kandula and Freeman [7], and these approaches can easily be implemented as well. Some justification for the use of linear superposition of blast waves is provided in Fig. 1, which shows the planar blast wave from a sheet of energy release obtained by collapsing the energy into sheets with l = 0.1 m spacing. Also shown is the linear superposition of line sources (cylindrical blast waves) with a regular spacing of  $l \times 1 = 0.1 \text{ m} \times 0.1 \text{ m}$ , as well as the linear superposition of points sources (spherical blast waves) arrayed as a regular grid with  $l \times l \times l$  spacing. The superposition of the line and point sources should be expected, in the far field, to match the planar source. The timing of the front arrival is not, in general, well predicted by the linear superposition approach, since the more concentrated sources (spherical) generate the strongest blast waves initially, but then decay quickly. In the far field, the planar blast wave has the lowest rate of decay and therefore leads. Interestingly, however, the shock trajectory and velocity of the blast fronts (planar, cylindrical, spherical) cross over at a location approximately equal to the average source spacing l. In addition, the linear superposition approach was found to do an adequate job of predicting the peak pressure and plateau region behind the blast wave.

### 3 Results

Using the approach outlined above, a simulation was initialized with 981 points of energy release randomly distributed in a cylindrical cloud with diameter d = 0.5 m, with a density to maintain the equivalent homogeneous energy release of  $E_o$ . The initiation criterion of the particular simulations discussed



Figure 2: 3-D simulation of detonation propagation in a system point sources randomly located in a cylindrical cloud. Solid points indicate sources that have been initiated.

here was that a peak pressure of  $p_{init} = 2.5p_o$  was required to initiate a source. The resulting dynamics shows a wave of sympathetic initiation events propagating through the cloud of sources at an average velocity of 1400 m/s. When viewed side-on, the front of initiated sources exhibits a front curvature due to the delayed initiation or quenching of sources on the edge of the charge, reminiscent of the front curvature in actual rate-stick experiments. The front exhibits a roughness that is not accounted for in continuumbased front curvature models. Simulations were performed in which the diameter of the cloud was varied from 0.4 m to 1.0 m. As the radius became small, the blast waves were not of sufficient density to continue initiating new sources, and the wave quenched. As the diameter increased, the wave velocity exhibited less of a velocity deficit, in qualitative agreement with experimental detonation dynamics (Fig. 3).

Simulations were also performed with a rectangular slab of points, as shown in Fig. 4. The slab was maintained at an aspect ratio (width to height) greater than 10, and the width was sufficient to ensure that lateral effects did not affect the core of the slab over the distance of propagation. The dynamics of the front clearly exhibits a "percolating" type behavior that permits the wave to exploit local fluctuations in source density that in turn allows the detonation to propagate around regions of local failure. These results are also plotted in Fig. 3. It can be seen that the expected 2:1 scaling is not obtained, and the ratio of critical diameter to critical slab thickness obtained is 3.5:1. This result bears a remarkable similarity to actual measurements of detonation wave velocity vs. diameter and thickness of a highly heterogeneous emulsion-based explosive performed by Petel et al. [3]



Figure 3: Wave propagation velocity vs. the reciprocal of thickness or diameter. For a detonation governed by front curvature, the two curves should approximately coincide.

## 4 Conclusions

The simple model developed here captures the behavior observed in highly unstable gaseous and liquid explosives and highly heterogeneous explosives. In particular, the stochastic nature of energy release in the detonation front (or in the media itself) permits the wave to propagate in much thinner twodimensional slabs than would be predicted by scaling the cylindrical charge results using front curvature



Figure 4: 3-D simulation of detonation propagation in a system point sources randomly located in a rectangular cloud. Solid points indicate sources that have been initiated. Note appearance of zones of failure ("failure waves").

theory. It could be argued that this model makes a number of highly idealized assumptions, with the use of linear superposition of blast waves being the most egregious. However, the results obtained and particularly the  $d_c: t_c$  scaling have proven highly insensitive to the particular details of the model, such as the initial pressure, source spacing, total energy release, etc. While computational simulations will be necessary to properly model the salient shock dynamics, it is suggested here as a tentative hypothesis that the observed deviations from front curvature theory obtained with unstable detonations or with detonations in media with a wide spectrum of heterogeneities can be predicted by a universality class of behavior for reactive waves in random media. Undeniably, additional experimentation and modeling are necessary to verify or disprove this hypothesis.

### References

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