Reduced order dynamical modeling of a non-isothermal circulating fluidized bed combustor

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1 Introduction

The fluidized bed combustion (FBC) technology has many advantages over traditional methods of solid fuel combustion, such as higher process efficiency, fuel flexibility and lower pollution. However, there are still many uncertainties in predicting FBC behaviour, which depends on many operational parameters i.e. excess air, solid properties, geometry of the reactor and many others. Prediction of FBC behaviour and parametric studies are conducted by means of mathematical models of different level of complexity [1, 2]. Some sophisticated models – from 1–D plug flow system to 3D models based on Navier Stokes equations [3] – both for bubbling and circulating systems, have been developed. They usually address some specific phenomena and in most cases are limited to steady state operation. Hence there is still need for mathematical modeling and simulation of the dynamic behavior of FB boilers, especially circulating fluidized bed boilers (CFBC) which nowadays come to play a very important role in energy production. A dynamical model could predict the response of the system to sudden changes of process conditions during operation, such as change of the fuel or other operational parameters, or be incorporated in model–predictive control of the unit.

Aim of this work is to report on the development of a 1-D dynamical model of circulating fluidized bed combustors, which integrate simplified hydrodynamics and fuel combustion. Although, in order to emphasize the dynamical aspects, the model details are kept to a minimum, it is well recognized that in many cases the complexity of first principle models of chemically reactive systems require quite significant computation time, due to the high number of ODEs resulting from the discretisation of the original PDE system. Hence, it is desirable to express the original model as a set of ordinary differential equations of the lowest possible order. Spectral methods (e.g. Galerkin) provide an interesting framework in that, by proper choice of the functional basis, one can reduce the number of ODEs necessary to accurately describe the dynamics of the original PDE model. Empirical reduction methods are interesting in that, in principle, they can incorporate information derived both from detailed model simulations and experiments. In this work, in order to build an optimal orthogonal basis, to construct a light yet faithful reduced order model, Proper Orthogonal Decomposition (POD) is used, based on full model simulation data.

2 CFBC model

Starting from the model proposed in [1], i.e. a steady–state model highlighting particle attrition and fragmentation, the system is idealized as a 1-D distributed, unsteady tubular reactor. Quasi-steady approximation is made for interphase momentum exchange. The riser is subdivided vertically into two

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zones i.e. dense bed and dilute region (corresponding to the splashing zone and the freeboard respectively).

Mass balances for the coarse phase, which is depleted both by combustion and attrition, and for the fine phase, enriched by attrition and depleted by combustion, are given respectively, in dimensionless form, by:

$$\frac{\partial \alpha_c}{\partial \tau} + \frac{\partial (v_c \alpha_c)}{\partial \zeta} = -\sigma_a \alpha_c - \sigma_c \alpha_c$$

$$\frac{\partial \alpha_f}{\partial \tau} + \frac{\partial (v_f \alpha_f)}{\partial \zeta} = \sigma_a \alpha_f - \sigma_f \alpha_f$$

The mass balances for gas phase, for O_2 and CO_2, are:

$$\frac{\partial (\varepsilon \alpha_{O_2})}{\partial \tau} + \frac{\partial (v_g \varepsilon \alpha_{O_2})}{\partial \zeta} = -k_2 (\sigma_c \alpha_c + \sigma_f \alpha_f)$$

$$\frac{\partial (\varepsilon \alpha_{CO_2})}{\partial \tau} + \frac{\partial (v_g \varepsilon \alpha_{CO_2})}{\partial \zeta} = k_2 (\sigma_c \alpha_c + \sigma_f \alpha_f)$$

The associated boundary conditions, and the continuity condition for the gaseous species to possibly account for secondary air injection, are given as:

$$v_c(0,\tau)\alpha_c(0,\tau) = k_1 F_c + \eta_c v_c(1,\tau)\alpha_c(1,\tau); \quad v_f(0,\tau)\alpha_f(0,\tau) = \eta_f v_f(1,\tau)\alpha_f(1,\tau)$$

$$\alpha_{O_2}(0,\tau) = 0.21\alpha_{g,prim}; \quad \alpha_{CO_2}(0,\tau) = 0$$

$$\left[\alpha_{O_2}(\zeta,\tau)Q_{tot}(\zeta,\tau)\right]_{\zeta = H_{sec}^+/H_{ex}} = \left[\alpha_{O_2}(\zeta,\tau)Q_{tot}(\zeta,\tau)\right]_{\zeta = H_{sec}^-/H_{ex}} + 0.21 Q_{sec}\alpha_{g,sec}$$

$$\left[\alpha_{CO_2}(\zeta,\tau)Q_{tot}(\zeta,\tau)\right]_{\zeta = H_{sec}^+/H_{ex}} = \left[\alpha_{CO_2}(\zeta,\tau)Q_{tot}(\zeta,\tau)\right]_{\zeta = H_{sec}^-/H_{ex}}$$

Energy balances for solid phases and gas phase are:

$$\frac{\partial (\alpha_c \theta_c)}{\partial \tau} + \frac{\partial (v_c \alpha_c \theta_c)}{\partial \zeta} = \beta \sigma_c \alpha_c - \phi_{c-g} h_c \alpha_c (\theta_c - \theta_g) - \sigma_a \alpha_c \theta_c$$

$$\frac{\partial (\alpha_f \theta_f)}{\partial \tau} + \frac{\partial (v_f \alpha_f \theta_f)}{\partial \zeta} = \beta \sigma_f \alpha_f - \phi_{f-g} h_f \alpha_f (\theta_f - \theta_g) + \sigma_a \alpha_c \theta_c$$

$$\frac{\partial ((1-\varepsilon)\theta_b)}{\partial \tau} + \frac{\partial (v_b (1-\varepsilon)\theta_f)}{\partial \zeta} = -\phi_{b-g} h_b (1-\varepsilon)(\theta_b - \theta_g) - \phi_{b-w} h_w (\theta_b - \theta_w)$$

$$\frac{\partial (\varepsilon \alpha_g c_{pg} \theta_g)}{\partial \tau} + \frac{\partial (v_g \varepsilon \alpha_g c_{pg} \theta_g)}{\partial \zeta} = k_3 \phi_{c-g} h_c \alpha_c (\theta_f - \theta_g) + k_3 \phi_{f-g} h_f \alpha_f (\theta_f - \theta_g) + k_3 \phi_{b-g} h_b (1-\varepsilon)(\theta_b - \theta_g)$$

with boundary and continuity conditions:

$$\theta_c(0,\tau) = \theta_{rec}; \quad \theta_f(0,\tau) = \theta_{rec}; \quad \theta_b(0,\tau) = \theta_{rec}; \quad \theta_g(0,\tau) = \theta_{prim}$$

$$\left[v_c \alpha_c + v_f \alpha_f + v_b (1-\varepsilon)\right]\Big|_{\zeta=0} \theta_{rec} = \left[\eta_c v_c \alpha_c \theta_c + \eta_c v_f \alpha_f \theta_f + v_b (1-\varepsilon)\theta_b\right]\Big|_{\zeta=1}$$

$$\left[\alpha_g Q_{tot} c_{pg}\ \theta_g(\zeta,\tau)\right]_{\zeta = H_{sec}^+/H_{ex}} = \left[\alpha_g Q_{tot} c_{pg}\ \theta_g(\zeta,\tau)\right]_{\zeta = H_{sec}^-/H_{ex}} + \alpha_{g,sec} Q_{sec} c_{pg}\ \theta_{sec}$$

3 Model reduction technique

The objective of the POD approach is to determine an empirical set of orthogonal functions basing on the spatiotemporal simulation data $u_t(x)$ [4]. Sampled data can be represented, in matrix form, as:

$$U = \begin{bmatrix} u_1(x_1) & u_2(x_1) & \cdots & u_M(x_1) \\ u_1(x_2) & u_2(x_2) & \cdots & u_M(x_2) \\ \vdots & \vdots & \ddots & \vdots \\ u_1(x_N) & u_2(x_N) & \cdots & u_M(x_N) \end{bmatrix}$$

where N is the number of position in the spatial domain and M is the number of samples taken in time. The POD basis $\Phi = \{\varphi_1, \varphi_2, \ldots, \varphi_N\}$ is then determined by solving the eigenvalue problem $C\Phi = \lambda\Phi$, where C is the time-averaged autocorrelation matrix. Using the POD modes, the solution can be expressed as:

$$\tilde{u}_t(x) = \sum_{k=1}^{K} a_k(t)\varphi_k(x)$$

where $K<N$ is the number of modes used for truncation and $a_k(t)$ are modal coefficients to be determined by means of Galerkin projection of the system of PDEs onto the POD modes.

4 Results

A finite difference method with staggered grid, employing 501 spatial nodes – resulting in 2000 ODEs to solve – with Adams implicit method for time integration, was used to solve the full model, in order to build the reference solution and collect the snapshots. Typical sets of equally spaced snapshots from transient and steady state collected during simulation of the full order model for solid density are shown in Figure 1.

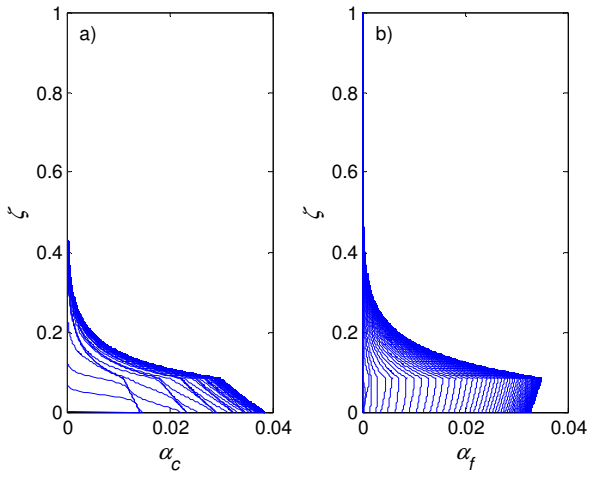

Figure 1. Set of snapshots collected for coarse (a) and fine char (b).

Following the POD procedure, collected data sets are combined into matrices of snapshots for each state variable separately and manipulated to produce four sets of orthogonal basis. The reduced order model was determined by performing Galerkin projection of the governing equations onto the modes. The resulting ODEs were integrated in time using Adams implicit method as for the full model.

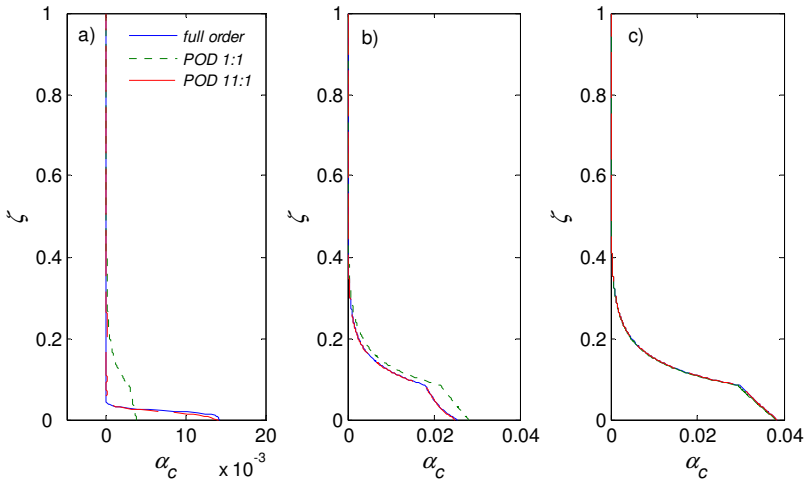

Figure 2. Comparison of vertical concentration profile of coarse in early transient and steady state; τ=40 (a), τ=560 (b) and τ=10000 (c).

Preliminary results referring to isothermal operation are reported here. Axial profiles of coarse char concentration in the early transient and steady state are shown in Figure 2. The cusp in the profiles corresponds to the separation from the dense bed and the freeboard. Two cases are reported: Results from the model obtained by the projection of the PDEs onto one POD mode for each state variable (POD 1:1) and model obtained by the projection of the balances for solids and gas onto 11 and 1 modes (POD 11:1), respectively. POD 1:1 model is unable to capture the solution at the early transient, due to the characteristic shape of the leading POD mode, whereas the higher order model captures quite accurately both transient and steady state behavior.

5 Conclusions

A distributed dynamical model of CFBC has been developed for isothermal operation and a POD/Galerkin method has been illustrated and applied to produce reduced order models. We have shown that, at least in the case of non-oscillatory regime, the system can be successfully projected onto just four POD modes, while retaining all qualitative features of the spatial distribution and, to engineering purposes, even quantitatively at steady–state. To provide accurate results in the transient, 24 modes are found to be sufficient.

The introduction of the energy balance, leading to a non–isothermal system, is expected to exhibit more complex regimes, for which a larger number of modes are expected to be necessary to accurately reproduce the system dynamics.

References

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