

# Dynamical Motion of Flame Flickering under Swirling Flow

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## Introduction

Periodic and chaotic motions in flame dynamics that can be observed as a result of flame instabilities are of fundamental importance to present-day combustion science. Regarding flame instability issues such as flame oscillations, buoyancy driven by natural convection under terrestrial gravity causes flame front to regularly oscillate at a low frequency. This periodic oscillation of the flame front is referred to as the *flickering flame*, and many aspects of flame oscillations/buoyancy interaction for the flickering flame have been largely investigated at different gravitational levels [1], [2], pressures [2], and oxygen concentrations of the surrounding gas [3], [4]. However, the dynamical motion of flame flickering has not been fully investigated under a swirling flow that is significant flow configurations for fundamental combustion systems.

A rotating cylinder burner that spins on its central axis is one of the most fundamental burners for investigating the influences of both the buoyancy and the centrifugal force on flame dynamics under laminar flow condition [5], [6]. In our preliminary work that used the rotating cylinder burner, we observed that under a high swirling flow condition, the periodic oscillation of the flame front drastically switches to a nonperiodic oscillation that is thought to be chaotic but not turbulent. Our numerical investigation recently showed that vortex motions generated by centrifugal instability associated with a rotating Taylor-Couette flow may cause irregular fluctuations in flame front dynamics [7]. An investigation of the dynamical motion of the flame instability by a sophisticated nonlinear time series analysis based on chaos theory is crucial for gaining a more comprehensive understanding of the complex nonlinear phenomena in flame dynamics. Nevertheless, this interesting issue has not been extensively explored in combustion science areas.

The purpose of this presentation is to investigate experimentally the dynamical motion of flame flickering under a swirling flow produced by the rotating cylinder burner, using the sophisticated nonlinear time series analysis for time series data of the fluctuations in the flame front.

## Experiments

The rotating cylinder burner used in this work is shown schematically in Figure 1. A fuel flows through a diffuser, fine damping screens, a nozzle, and a burner tube (diameter of burner tube,  $d = 10$  mm). The burner tube is supported vertically by bearings and is rotated by a DC motor through a pulley and belt unit. A 15 mm height honeycomb section with a grid diameter of 1.04 mm is fitted inside the burner tube to create the solid-body rotation of the fuel at the burner tube exit. Methane ( $\text{CH}_4$ ) was used as the fuel in this work. The bulk fuel injection velocity (= volume flow rate / cross-sectional area of the burner tube)  $V$  was varied from 0.10 to 0.25 m/s. The Reynolds number  $Re_j$  (=  $Vd/\nu_f$ , where  $\nu_f$  is the kinematic viscosity coefficient of fuel) was between 58 and 144. The rotational speed of the burner tube  $N$ , was varied from 0 to 1400 rpm ( $23.3 \text{ s}^{-1}$ ) at 200 rpm intervals. The rotational Reynolds number  $Re_r$  (=  $v_\theta d/\nu_a$ , where  $v_\theta$  is the tangential velocity at the surface of the burner tube exit ( $r = 5$  mm), and  $\nu_a$  is the viscosity coefficient of the surrounding air) was between 0 and 549. To characterize the balance between the axial momentum and the swirl momentum of the fuel, the Swirl

number  $S (=2\pi\int_0^R \rho_f v_j v_\theta r^2 dr / 2\pi R \int_0^R \rho_f v_j^2 r dr = \omega R/2V$ , where,  $R$  is the radius of the burner tube,  $v_j$  is the axial velocity of the fuel,  $\rho_f$  is the density of the fuel and  $\omega (= \pi N/30)$  is the angular velocity of the burner tube) was introduced.

To investigate the dynamical motion of the flame front, the flame front location  $r_i$  (mm) in the radial direction at a location (at  $z = 20$  mm) close to the flame base was measured as shown in Fig. 1. The flame front around the flame base was recorded by a high-speed video camera (Photron firstcom1024 PCI) of 1000 frames per second. The special resolution of the images was 25 pixels per millimeter. The deviation from the mean flame front location  $\Delta r_f = r_f - \bar{r}_f$  was measured as a function of time  $t$ . Here,  $\bar{r}_f$  is the time-averaged flame front location. The sampling frequency of the obtained time series was 1 kHz, and the data number  $n$  was 10000.

To investigate the change in flow field in the surrounding air generated by burner rotation, a laser tomographic method was used in this work. The light source was an Ar<sup>+</sup> laser (LEXEL Model 95) with a maximum power of 2 W. An optical unit shaped the laser beam into a sheet having a thickness of approximately 0.5 mm, and the laser sheet then traversed the test section. Mie scattered light emitted from TiO<sub>2</sub> particles that was seeded in surrounding air were visualized. The visualized images were recorded with a high-speed video camera (Photron Fastcam 1024PCI) of 1000 frames per second.

## Nonlinear Time Series Analysis Based Chaos Theory

The central idea behind the mathematics of the nonlinear time series analysis employed in this work is briefly described as follows.

The attractor is constructed from the time series of the deviation from the mean flame front location  $\Delta r_f$ . The time-delayed coordinates for the construction of the attractor are expressed as the following equation.

$$X_i = (\Delta r_f(t_i), \Delta r_f(t_i + \tau_0), \dots, \Delta r_f(t_i + (D-1)\tau_0)) \quad \dots (1)$$

where,  $i = 0, 1, \dots, n$  ( $n$  is the data number of the time series),  $X_i$  are the constructed phase space vectors,  $\Delta r_f(t_i)$  is the deviation from the mean value of the flame front location at time  $t_i$ ,  $D$  is the embedding dimension, that is, the dimension of the constructed phase space and  $\tau_0$  is the time lag. In this work,  $\tau_0$  is set to be the time lag that yields a local minimum of mutual information  $I$ .

A standard and classical method for evaluating the complexities of dynamical motion is the Grassberger-Procaccia (GP) algorithm [8], which enables the estimation of the correlation dimension (a kind of fractal dimension) of the geometrical object formed from the trajectories in phase space constructed. Firstly, we calculate the two-point correlation function that is defined as follows.

$$C^D(\varepsilon) = \frac{1}{n^2} \sum_{\substack{i,j=1 \\ i \neq j}}^n H(\varepsilon - \|X_i - X_j\|) \quad \dots (2)$$

Here,  $\varepsilon$  is the small radius of the  $D$ -dimensional hypersphere in the phase space for counting the neighboring points.  $\|X_i - X_j\|$  represents the Euclidean distance between vectors  $X_i$  and  $X_j$  in the phase space.  $H(x)$  is the Heaviside function ( $H(x) = 1$  if  $x > 0$ ,  $H(x) = 0$ ,  $x < 0$ ).

The value of the Heaviside function is unity if the distance between  $X_i$  and  $X_j$  is within  $\varepsilon$ , and is zero otherwise. The hypersphere of a radius  $\varepsilon$  is centered on one of the  $X_i$  vectors. The hypersphere itself is moved from point to point along the trajectories of the phase space. The correlation integral scales with the hypersphere radius  $\varepsilon$  according to the power law of  $C^D(\varepsilon) \approx \varepsilon^{D_c}$ , where,  $D_c$  is the correlation dimension. The correlation dimension can then be estimated from the slope of the linear part of the  $\log C^D(\varepsilon)$  versus  $\log \varepsilon$  curve according to

$$D_c = \lim_{\varepsilon \rightarrow 0} \frac{\log C^D(\varepsilon)}{\log \varepsilon} \quad \dots (3)$$

## Results and Discussion

Changes in flame motion as functions of the Reynolds number of the fuel jet  $Re_j$  and the rotational Reynolds number of burner tube  $Re_r$  are shown in Fig. 2. The periodic oscillation generated by the buoyancy-driven hydrodynamic shear layer instability remains nearly unchanged with increasing  $Re_r$  up to 156. This indicates that the swirling flow produced by burner rotation is not sufficiently large to alter the buoyancy driven hydrodynamic instability mechanism. In contrast, as  $Re_r$  exceeds 312, an interesting flame with a spirally curved configuration (spiral flame) begins to appear, and the flame motion switches back and forth between the non-axisymmetric flickering flame and the spiral flame. With further increase in  $Re_r$ , the formation of the spiral flame becomes increasingly prominent and the flame front fluctuates irregularly with nonperiodic oscillation that is thought to be chaotic.

To quantify these flame motions in terms of the deterministic chaos theory, the three-dimensional attractor  $(\Delta r_f(t_i), \Delta r_f(t_i + \tau_0), \Delta r_f(t_i + 2\tau_0))$  and the correlation dimension  $D_c$  at  $Re_j = 86$  are shown as functions of  $Re_r$  or Swirl number  $S$  in Fig. 3. Under the swirling condition of  $Re_r \leq 156$  ( $S = 0.7$ ), the shape of the attractor remains in a limit cycle with small width and  $D_c$  is approximately unity, indicating a periodic oscillation. At  $Re_r = 237$  ( $S = 1.1$ ), the trajectory of the attractor seems to shift to a torus, and  $D_c$  approaches approximately 2, indicating quasi-periodic oscillation. When the spiral flame clearly starts to appear at  $Re_r = 312$  ( $S = 1.4$ ), the shape of the attractor becomes more complicated and  $D_c$  reaches a noninteger value, indicating a low-dimensional chaotic oscillation. These results show that with increasing  $Re_r$ , the flame motion transits from a periodic oscillation to a low dimensional chaotic oscillation throughout the quasi-periodic oscillation, and the occurrence of the spiral flame becomes significant to progressively induce a chaotic oscillation of the flame front. These results also show that the analytical method used in this work is valid for quantifying complex flame motions.

The effect of flow field on the onset of the spiral flame leading to a low dimensional chaotic oscillation is discussed on the basis of the following results obtained by flow visualization around the burner tube exit. The temporal evolutions of the visualized particle seeded in the surrounding air around the burner tube are shown in Fig. 4. At  $Re_r = 156$ , the trajectory of the seeded particle does not change entirely with time, and is similar to that obtained for the non-swirling flame case ( $Re_r = 0$ ). In contrast, at  $Re_r = 312$ , an interesting vortical structure appears on the surface of the burner tube. The vortex moves in downstream by the entrainment effect due to upward buoyant flow, and interacts with the flame front at the flame base. With further increase in  $Re_r$ , the motion of the vortex structure temporally and spatially seems chaotic, and the vortex interacts irregularly with both the flame front and the interface. Our numerical simulation [7] recently demonstrates that the vortical structure in the surrounding air is attributed to the centrifugal instability associated with a rotating Taylor-Couette flow. The significance of the Taylor-like vortex motion to the notable change in dynamic behavior will be discussed in detail in this presentation.

## Summary

The dynamical motion of the flickering flame under swirling flow has been experimentally investigated from the viewpoint of nonlinear dynamics based on chaos theory. As the rotational Reynolds number  $Re_r$  exceeds 312 ( $S = 1.4$ ), irregular fluctuations of the flame front begin to be produced with the onset of a spiral flame. The trajectory of the attractor changes from a limit cycle to a complicated structure, and the correlation dimension  $D_c$  becomes a noninteger value. This result demonstrates that the dynamical motion of the flickering flame changes from a periodic oscillation to a chaotic oscillation with increasing rotational Reynolds number. The flow visualization shows that the vortex motions generated by centrifugal instability associated with a rotating Taylor-Couette flow have a significant effect on the induction of the chaotic oscillation of the flame front.

## References

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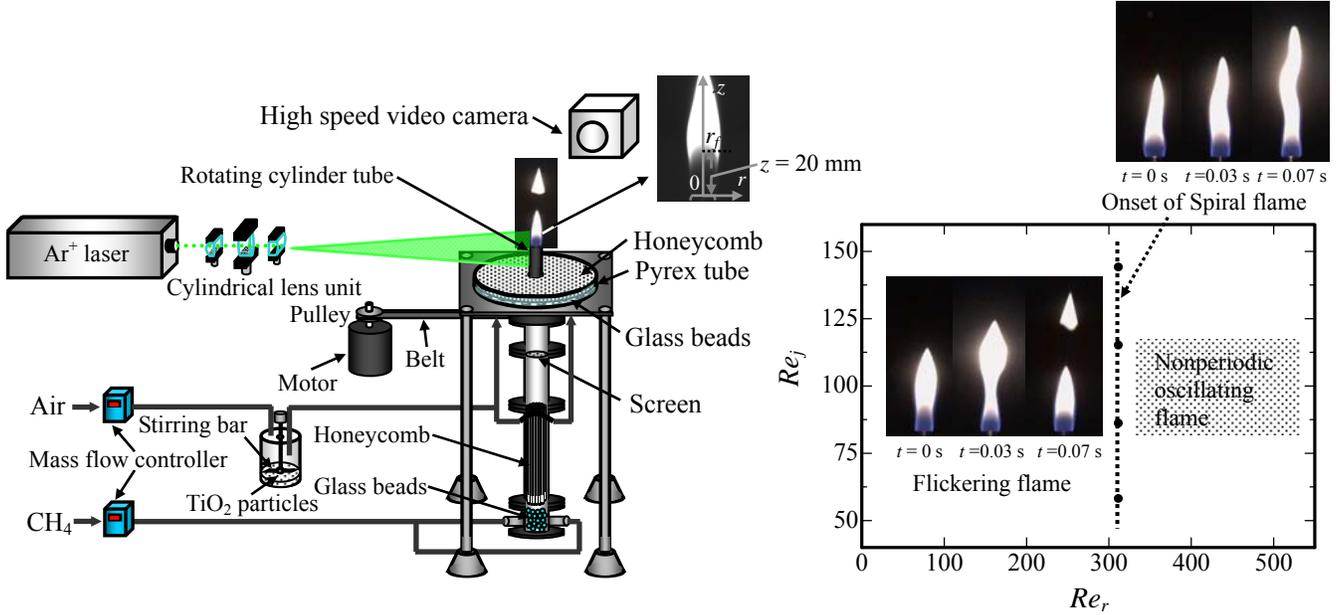


Figure 1: Experimental apparatus

Figure 2: Changes in flame motion as functions of  $Re_r$  and  $Re_j$ .

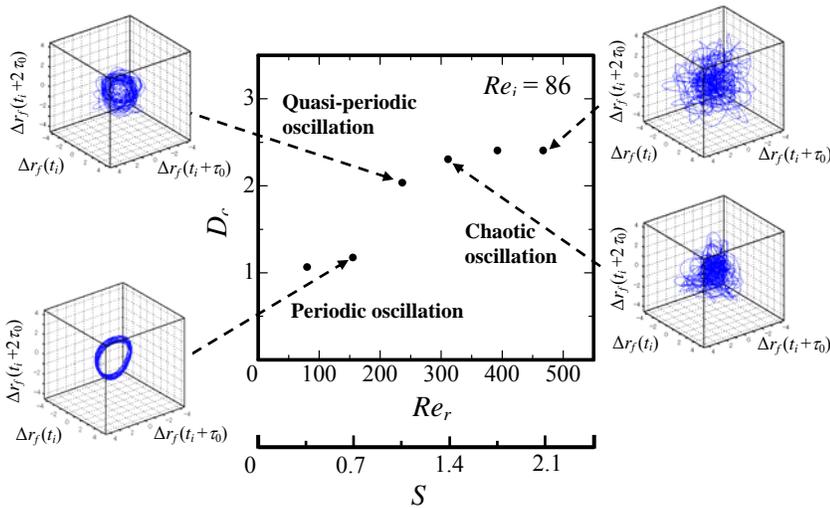


Figure 3:  $D_c$  as a function of  $Re_r$  or  $S$  at  $Re_j = 86$

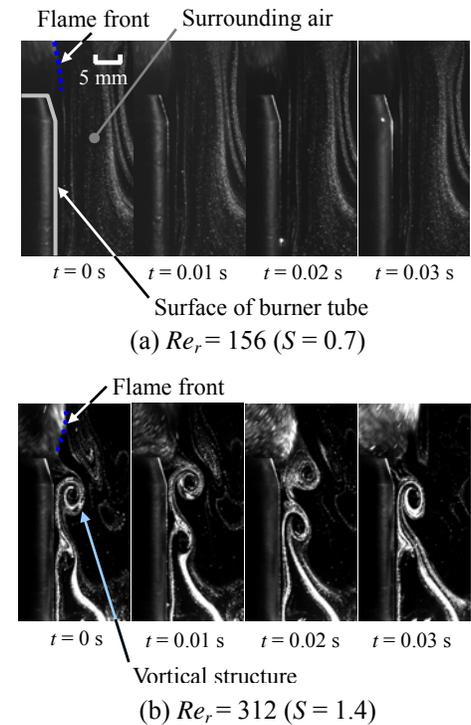


Figure 4: Temporal evolutions of the visualized particle in the surrounding air around the burner tube