Deformation of Liquid Layer under the Cool Flame Propagation

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1 Introduction

Experimental study of gravitational flow of thin liquid film with local heating shows the existence of steady-state regimes with two- or three-dimensional flow structure [1-3]. Film deformations are caused by thermocapillary effect. Film thickness increases and liquid flow is slowed down in the region where the temperature distribution at the surface is non-homogeneous. If the power of immovable local heater is lower some critical value, then a steady-state 2-D liquid flow takes place. 2-D flow structure is unstable at high power of heating [4]. The flow becomes 3-D with periodic structure in transverse direction. Such a structure (like "fingers") was observed for the case of combustion wave propagating along immovable thin layer of liquid fuel [5]. The power of heat release in usual combustion wave is high, so 3-D structure only is observed in these experiments. We suppose that 2-D flow structure in thin liquid layer also could exist if power of heat release is low (for example in cool-flame combustion). The paper is devoted to theoretical study of effect of combustion with low heat release on the structure of thin horizontal liquid layer. The liquid is locally heated by a planar cool-flame wave propagating in gas along the layer. The temperature gradient at the free surface is not high. The used model temperature distribution is nonmonotone, describing heating and subsequent cooling down of the liquid. The previous results [6, 7] show that 2-D steady-state flow regime in thin horizontal layer can exist if the heat source is moving and its power doesn't exceed certain critical level. These conditions provide the existence of 2-D steady-state (in the frame moving together with heat source) flow without dry spot formation even in microgravity.

2 Governing equations

Let's consider a thin horizontal layer of incompressible liquid at a flat substrate. We assume that the heat source moves along the layer without dragging of liquid, i.e. momentum transfer caused by viscosity of the gas is neglected. The heat source has finite size in x direction and infinite length in y direction. In other words, we consider the case of planar "thermal wave" propagating along a liquid layer. Characteristic length of the "thermal wave" is much more than thickness of liquid layer far from the heat source: $h_\infty \sim \varepsilon L,\ \varepsilon << 1$. If such a wave propagates in gas, then the temperature distribution at the free surface of the liquid is non-homogeneous. Thermocapillary force induces the flow of the liquid towards the cold liquid and leads to a deformation of the free surface. Let's suppose that the

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characteristic longitudinal scale of film surface deformation equals by the order of magnitude to L and considerably exceeds the amplitude of deformation, which characteristic scale equals by the order of magnitude to h_∞. This condition can be written in the form: $\left|\partial h(t,x)/\partial x\right| \sim \mathrm{O}(\varepsilon)$ (h is film thickness). In this case one can use the boundary layer approximation and Prandtl's equations with gravitational terms [8]. In the accompanying frame of reference moving with the velocity $C = \text{const} > 0$ (C is the velocity of the wave) these equations have the following form for steady-state regime [7]:

$$\frac{\partial p}{\partial x} = \rho\nu\frac{\partial^2 u}{\partial y^2}, \qquad \frac{\partial p}{\partial y} = -\rho\left|\vec{g}\right|, \qquad \frac{\partial u}{\partial x} + \frac{\partial w}{\partial y} = 0, \tag{1}$$

here p is the pressure, u and w are the components of velocity of the liquid, ρ and ν are the density and kinematic viscosity of the liquid, t, x, y are the time and spatial coordinates, $\vec{g}$ is the acceleration due to gravity. The convective terms are neglected here due to low Reynolds number assumption $\left(\mathrm{Re} = Ch_\infty/\nu \sim \mathrm{O}(\varepsilon)\right)$. The dependencies of viscosity and density on the temperature are neglected (ν, $\rho = \text{const}$). Mass transfer is disregarded at the free surface, heat transfer to the substrate is absent.

The boundary conditions for equations (1) include the non-slip condition at the substrate, the kinematic condition at the free surface, and the condition of force balance at the free surface:

$$\begin{gathered} w = 0, \quad u = -C, \quad \text{at } y = 0, \qquad w = u\, dh/dx, \quad \text{at } y = h, \\ \left(p - p^g - \sigma/R\right) n_i = \left(\sigma'_{ik} - \sigma'^{g}_{ik}\right) n_k + \partial\sigma/\partial x_i, \quad \text{at } y = h, \end{gathered} \tag{2}$$

here σ is the surface tension, R is the main radius of the free surface curvature, σ'_{ik} are the components of the viscous stress tensor, n_i are the components of the vector normal to the surface, the superscript g denotes the gas phase. In long-wave approximation $\left(\left|dh/dx\right| << 1\right)$ the system (1), (2) leads to the following dependencies for the film thickness, pressure and velocity components [7]:

$$\frac{h^2}{2h_\infty^2}\frac{h_\infty}{\rho\nu C}\frac{d\sigma}{dx} + \frac{h^3}{3h_\infty^3}\left\{\frac{\sigma}{\rho\nu C}h_\infty^2\frac{d^3h}{dx^3} - \frac{\left|\vec{g}\right|h_\infty^2}{\nu C}\frac{dh}{dx}\right\} = \frac{h}{h_\infty} - 1, \tag{3}$$

$$p = p^g + \rho\left|\vec{g}\right|(h - y) - \sigma\, d^2h/dx^2, \tag{4}$$

$$\frac{u}{C} = -1 + \frac{y}{\rho\nu C}\frac{d\sigma}{dx} + \left(hy - \frac{y^2}{2}\right)\left\{\frac{\sigma}{\rho\nu C}\frac{d^3h}{dx^3} - \frac{\left|\vec{g}\right|}{\nu C}\frac{dh}{dx}\right\}, \tag{5}$$

$$\frac{w}{C} = -\frac{y^2}{2\rho\nu C}\frac{d^2\sigma}{dx^2} - \frac{y^2}{2}\frac{dh}{dx}\left\{\frac{\sigma}{\rho\nu C}\frac{d^3h}{dx^3} - \frac{\left|\vec{g}\right|}{\nu C}\frac{dh}{dx}\right\} - \left(\frac{hy^2}{2} - \frac{y^3}{6}\right)\frac{d}{dx}\left\{\frac{\sigma}{\rho\nu C}\frac{d^3h}{dx^3} - \frac{\left|\vec{g}\right|}{\nu C}\frac{dh}{dx}\right\}. \tag{6}$$

One can note that thermal part of the problem influences on the hydrodynamic part (3)-(6) only through the temperature dependency of the surface tension. If the distribution of the temperature at the free surface is known from the experiment, then it isn't necessary to solve the thermal part of the problem, – the solution is determined by the hydrodynamic part of general problem on the base of the derived equations (3)-(6) and given distribution $\sigma(x)$. Currently we don't have available experimental data on the temperature distribution on the liquid film surface at propagation of a thermal (combustion) wave. So we prescribed the following model distribution of the temperature at $y = h$:

$$T(x,h) - T_\infty = \Delta T \exp\left(-\left(x/L_\pm\right)^2\right),$$

with $L = L_+$ at $x > 0$ and $L = L_-$ at $x < 0$ (see Fig. 1). The branch with characteristic scale L_+ corresponds to heating, and the branch with L_- – to cooling down (slow process due to thermal radiation and heat transfer to outside, where $T = T_\infty$). Taking into account the dependency $\sigma(T)$, one can calculate function $\sigma(x)$ and its derivatives.

3 Simulation

The equation (3) on the film thickness was solved numerically with iteration method. The derivatives were approximated by the finite differences. The implicit finite-difference scheme and three-point double-sweep algorithm were used. As the boundary conditions the constant values for the sought solution were specified far from the heat source $\left(h\left(t, x \to \pm\infty\right) = h_\infty\right)$. The initial approximation was taken as follows: $h\left(t = 0, x\right) = h_\infty$. The iteration process was in progress till squared difference summed over all the calculation grid nodes reached given small quantity (< 10^{-6}).

The following parameters were given at simulations: $h_\infty = 2 \cdot 10^{-5}$ m, $C = 10^{-2}$ m/s, $p^g = 10^5$ Pa, $\left|\vec{g}\right| = 9.8$ m/s^2, $T_\infty = 303$ K, $\Delta T = 6.7$ K, $L_+ = 50\, h_\infty$, $L_- = 250\, h_\infty$. The physical properties of the liquid considered in the simulations corresponded to 25% solution of ethyl alcohol in water (25% C_2H_5OH + + 75% H_2O): $\sigma_\infty = 0.034$ kg/s^2, $\nu = 1.8 \cdot 10^{-6}$ m^2/s, $\rho = 956$ kg/m^3, $d\sigma/dT = -1.1 \cdot 10^{-4}$ kg/(s^{2}K).

In accordance with the calculated solution $h(x)$ the dependencies $p(x, y)$, $u(x, y)$, $w(x, y)$ were determined and stream function was found (see Fig. 1).

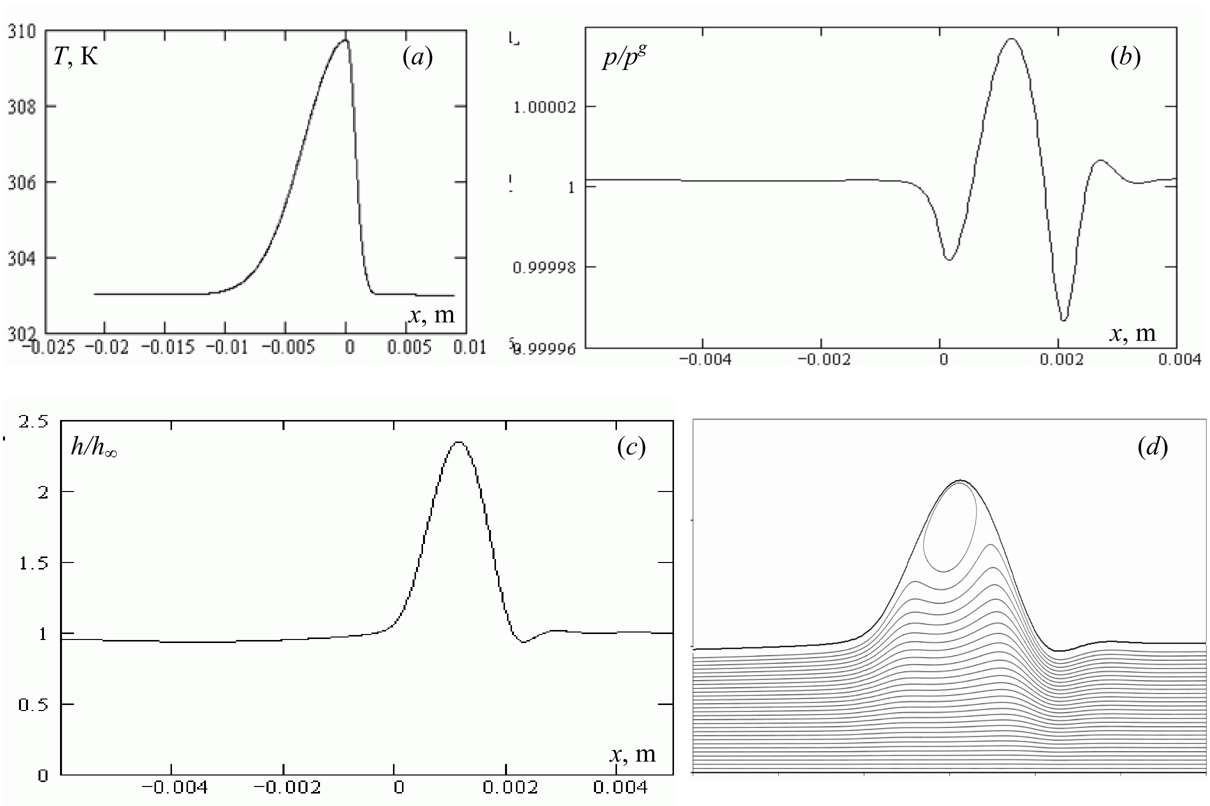

Figure 1. Distributions of temperature at y=h (a), pressure at y=0 (b), thickness of the layer (c) and stream lines (d).

4 Discussion and conclusions

Relative to the previous works (for example, [6, 7]) the presented numerical results additionally take into account the process of liquid cooling. It can be governed by different physical mechanisms. Which of them is dominant – depends on physical conditions. Here we didn't specify the mechanism, describing qualitatively the temperature slow decrease behind the heat source. It is shown that the cooling of liquid leads to nonmonotone variation of the thickness behind the main roll. This effect wasn't revealed earlier. Its nature is evidently explained by thermocapillarity. The amplitude of the thickness decrease depends on the characteristic time of the process. If the cooling is fast then the gradient of surface tension is high and the local layer thickness becomes significantly less than the undisturbed value h_∞.

The obtained theoretical results confirm that steady-state 2-D flow structure can exist in horizontal liquid layer under the propagation of thermal wave with low heat release power (for example – cool-flame combustion wave). It is interesting to try to discover such a regime experimentally.

Acknowledgements: the work is supported by Russian Foundation for Basic Research, project No 07-08-00296, Program of Siberian Branch of Russian Academy of Sciences and Program of Federal Agency of Education.

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