Instability of flames in cylindrical tubes

Victor V. Volkov

Odessa National Academy for Food Industry, 65039, str. Kanatnaya 112, Odessa, Ukraine

1 Introduction

Instability of planar flame spreading in an inviscid, incompressible medium was proved by Landau [1]. Viscosity is the main factor stabilizing the process of normal burning [2,3]. The account of viscosity requires simultaneous consideration of the finite thickness of the flame zone. Changing of the flame thickness plays an essential role in stabilizing laminar flame propagation in viscous gas mixtures as well [2,3]. Compressibility of medium is also a stabilizing factor [4], although much less considerable than viscosity. But all those conclusions concern only combustions in the open space. The main aim of this investigation is to research stability of flames propagating in cylindrical tubes.

2 Model

The following mathematical model of combustion is considered. The inviscid compressible medium (gas) moves at a stationary subsonic velocity along a *z*-axis. Plane z = 0 corresponds to a flame front, where chemical transformations occur. Zone "1" (z < 0) is occupied by the combustible gas mixture while zone "2" (z > 0) is occupied by the products of combustion. Physical parameters of the combustible mixture and the products of combustion are related to each other by the conservation laws of mass, momentum, and energy. The flow field is governed by a set of gasdynamic equations:

$$\left| \begin{array}{l} \frac{d\rho}{dt} + \rho \, div\vec{u} = 0 \\ \rho \, \frac{d\vec{u}}{dt} + \overline{grad} \, p = 0 \\ \frac{\partial}{\partial t} \left(pe + \frac{\rho u^2}{2} \right) + div \left[\vec{u} \left(\rho e + p + \frac{\rho u^2}{2} \right) \right] = 0 \end{array} \right| \tag{1}$$

where \vec{u} is the velocity vector, $u^2 = \vec{u}^2$, ρ is the density, p is the pressure, $e = e(p, \rho)$ is the specific internal energy. For the thermally perfect gas:

$$e = \frac{1}{\gamma - 1} \frac{p}{\rho} + \text{const}$$

where γ is the ratio of thermal specific heats. Both the combustible gas and the products of combustion flow in a round cylindrical tube of radius r_0 , therefore Eqs. (1) should be formulated in a cylindrical frame of reference.

Let us investigate the stability of the basic solution of Eqs. (1) in relation to small perturbations u'_{jr} , $u'_{j\varphi}$, u'_{jz} , ρ'_j , p'_j corresponding to radial, tangential and axial components of a velocity vector, pressure and density, in the combustible gas (j = 1) and the products of combustion (j = 2). As a result the following linearized equations can be obtained from Eqs.(1):

$$\begin{cases} \frac{\partial \rho'_{j}}{\partial t} + \frac{\partial}{\partial z} \left(\rho_{j} u'_{jz} + u_{j} \rho'_{j} \right) + \frac{\rho_{j}}{r} \left[\frac{\partial \left(r u'_{jr} \right)}{\partial r} + \frac{\partial u'_{j\varphi}}{\partial \varphi} \right] = 0 \\ \frac{\partial u'_{jr}}{\partial t} + u_{j} \frac{\partial u'_{jr}}{\partial z} + \frac{1}{\rho_{j}} \frac{\partial p'_{j}}{\partial r} = 0 \\ \frac{\partial u'_{j\varphi}}{\partial t} + u_{j} \frac{\partial u'_{j\varphi}}{\partial z} + \frac{1}{\rho_{j}r} \frac{\partial p_{j}}{\partial \varphi} = 0 \\ \frac{\partial p'_{j}}{\partial t} + u_{j} \frac{\partial p'_{j}}{\partial z} - \frac{\gamma_{j}p_{j}}{\rho_{j}} \left(\frac{\partial \rho'_{j}}{\partial t} + u_{j} \frac{\partial \rho'_{j}}{\partial z} \right) = 0 \end{cases}$$

$$(2)$$

where

$$\gamma_{j} = 1 + \left[\left(\frac{\partial e_{j}}{\partial p_{j}} \right)_{\rho_{j}} \rho_{j} \right]^{-1}$$
(3)

In the particular case of thermally perfect gas, γ_j is the ratio of specific heats. The solutions of Eqs.(2) should obey to the condition:

$$u'_{jr}\Big|_{r=r_0} = 0, (4)$$

and the condition of regularity at $r \rightarrow 0$. In view of it, the solution is taken in the form

$$\frac{u'_{jz}}{u_{j}} = y_{1j}(z)F(r, \varphi, t);$$

$$\frac{u'_{jr}}{u_{j}} = y_{2j}(z)r_{0}\xi_{nk}^{-1}\frac{d}{dr}\ln J_{n}(\xi_{nk}rr_{0}^{-1}) F(r, \varphi, t);$$

$$\frac{u'_{j\varphi}}{u_{j}} = y_{2j}(z)ir_{0}r^{-1}\xi_{nk}^{-1}nF(r, \varphi, t);$$

$$\frac{p'_{j}}{p_{j}} = y_{3j}(z)F(r, \varphi, t);$$

$$\frac{\rho'_{j}}{\rho_{j}} = y_{4j}(z)F(r, \varphi, t);$$
(5)

where

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$$F(r, \varphi, t) = \exp\left(\omega r_0^{-1} u_j t + in\varphi\right) J_n\left(\xi_{nk} r r_0^{-1}\right), \tag{6}$$

and ω is a complex value (dimensionless eigenvalue), *n* is the azimuthal wave number (n = 0, 1, 2, ...), $J_n(\xi)$ is the cylindrical functions of the 1-st kind of order *n*, ξ_{nk} is the *k* th root of the equation $dJ_n(\xi)/d\xi = 0$, y_{jk} (k = 1, ..., 4) — dimensionless function of *z*. Functions y_{jk} (k = 1, ..., 4) are determined to be the solutions of system of the linear homogeneous differential equations with constant coefficients and contain uncertain constants A_{jl} (l = 1, 2, 3, 4):

$$y_{1j}(z) = -\sum_{l=1}^{2} \frac{k_{jl}}{\frac{\omega}{\delta_{j}} + k_{jl}} A_{jl} e^{k_{jl}r_{0}^{-1}z} + A_{j3} e^{-\frac{\omega}{\delta_{j}}r_{0}^{-1}z};$$

$$y_{2j}(z) = -\left[i\sum_{l=1}^{2} \frac{e^{k_{jl}r_{0}^{-1}z}}{\frac{\omega}{\delta_{j}} + k_{jl}} A_{jl} + \frac{i\omega}{\delta_{j}} A_{j3} e^{-\frac{\omega}{\delta_{j}}r_{0}^{-1}z}\right];$$

$$y_{3j}(z) = \sum_{l=1}^{2} A_{jl} e^{k_{jl}r_{0}^{-1}z};$$

$$y_{4j}(z) = \frac{M_j^2}{\delta_j^2} \sum_{l=1}^2 A_{jl} e^{k_{jl}r_0^{-l_z}} + A_{j4} e^{-\frac{\omega}{\delta_j}r_0^{-l_z}};$$

where

$$\delta_j = \frac{\rho_1}{\rho_j} = \frac{u_j}{u_1}; \tag{7}$$

$$k_{jl} = \frac{1}{1 - M_j^2} \left[\frac{\omega}{\delta_j} M_j^2 - (-1)^l \sqrt{1 - M_j^2} + \left(\frac{\omega}{\delta_j} M_j \right)^2 \right], l = 1, 2;$$
(8)

The equations of the perturbed flame front is set as $z = \varepsilon(r, \phi, t)$, where $\varepsilon_j = A_{0j} r_0 F(r, \phi, t)$. In the linear approximation, the laws of conservation of mass, momentum and energy at the flame front (z = 0) are given by

$$u_{2}\rho_{2}' + \rho_{2}u_{2z}' - u_{1}\rho_{1}' - \rho_{1}u_{1z}' = (\rho_{2} - \rho_{1})\frac{\partial\varepsilon}{\partial t},$$

$$u_{2r}' - u_{1r}' = (u_{1} - u_{2})\frac{\partial\varepsilon}{\partial r},$$

$$u_{2\varphi}' - u_{2\varphi}' = \frac{1}{r}(u_{1} - u_{2})\frac{\partial\varepsilon}{\partial \varphi},$$

$$p_{2}' + u_{2}^{2}\rho_{2}' + 2\rho_{2}u_{2}u_{2z}' = p_{1}' + u_{1}^{2}\rho_{1}' + 2\rho_{1}u_{1}u_{1z}',$$

$$[(\gamma_{2} - 1)\rho_{2} - (\gamma_{1} + 1)\rho_{1}]p_{2}' + [(\gamma_{2} - 1)p_{2} + (\gamma_{1} + 1)p_{1}]\rho_{2}' =$$

$$= [(\gamma_{1} - 1)\rho_{1} - (\gamma_{2} + 1)\rho_{2}]p_{1}' + [(\gamma_{1} - 1)p_{1} + (\gamma_{2} + 1)p_{2}]\rho_{1}'$$
(9)

The additional boundary condition at the flame front is the condition of Landau [1]

$$u_{1z}' - \frac{\partial \varepsilon}{\partial t} = 0 \tag{10}$$

3 Results

Three different cases are considered:

1) the case of the infinite tube;

2) the case of the cylindrical tube (combustor) with the closed end $z = -L_1$ and the open end $z = L_2$; 3) the case of the tube with both ($z = -L_1$, $z = L_2$) closed ends.

The first case demands limited perturbations when $z \to \infty 0$ and $z \to -\infty$. As a result, one arrives at the equation for definition of eigenvalues

$$F(\omega) = 0, \qquad (11)$$

where function $F(\omega)$ represents function with unstable roots (with a positive real part). The eigenvalue ω is, in general, the complex function of parameter ξ_{nk} . When $M_j^2=0$ (incompressibility) $F(\omega)$ is the polynomial with the unstable root of Landau [1]. The stabilizing effect of compressibility is also proved: this conclusion corresponds well with [4].

In the second case there are two boundary conditions

$$u_{1z}\Big|_{z=-L_1} = 0, (12)$$

and

$$p_{2}'|_{z=L_{2}} = 0 \tag{13}$$

Function $F(\omega)$ in Eq.(11) is quasi-polynomial without the main part. Those polynomials are always unstable.

In the third case, the boundary condition (13) changes to

$$u_{2z}\Big|_{z=L_2} = 0 \tag{14}$$

Function $F(\omega)$ in Eq.(11) is also quasi-polynomial, but it contains the main part. Those polynomials are not always unstable, that proves some degree of stabilization in this case.

The spectrum of values ξ_{nk} satisfying boundary conditions is discrete. And so the eigenvalue function $\omega(\xi_{nk})$ has also a set of discrete values.

References

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